Section 4.1 Complex #s

Section 4.2 Theory of Polynomials

Write the polynomial as the product of linear factors:

$f(x) = 3x^3 - x^2 + 54x - 18$

Rational zeros Theorem $\frac{p}{q} = \frac{\text{factor of } 18}{\text{factor}}$ if $f(p/q) = 0$. Turns out $x = \frac{1}{3}$ works

$\pm1, \pm3, \pm2, \pm6, \pm9, \pm18$

$\pm\frac{1}{3}, \pm\frac{2}{3}, \cancel{\pm1}, \cancel{\pm\frac{6}{3}}, \cancel{\pm\frac{9}{3}}, \cancel{\pm\frac{18}{3}}$

GRAPHER FOR the $x = \frac{1}{3}$ guess

Divide by $x - \frac{1}{3}$

$$\begin{array}{c|cccc} \frac{1}{3} & 3 & -1 & 54 & -18 \\ & & 1 & 0 & 18 \\ \hline & 3 & 0 & 54 & 0 \\ & x^2 & x^1 & c & r \end{array}$$

Sweet!

This says $f(x) = (x - 1/3)(3x^2 + 54)$

Depressed Polynomial

(Irreducible) Quadratic Factor.

$3x^2 + 54 \overset{\text{SET}}{=} 0$

$3x^2 = -54$

$x^2 = -\frac{54}{3} = -18 = x^2$

$\Rightarrow \sqrt{x^2} = \sqrt{-18}$

$|x| = i\sqrt{18} = i \cdot 3\sqrt{2}$

$$\begin{array}{r|l} 2 & 18 \\ 3 & 9 \\ & 3 \end{array}$$

$= 3i\sqrt{2}$

$= 3\sqrt{2}\, i$

$\Rightarrow x = \pm 3i\sqrt{2}$

DEFINITION:

The principal square root is the positive square root

$\sqrt{9} = 3$

Principle square root of

$\sqrt{-9} = \sqrt{-1}\sqrt{9} = \sqrt{-1} \cdot 3$

$= i \cdot 3 = 3i$

i = imaginary unit.

$\sqrt{-1} = i$

$\Rightarrow i^2 = -1$

NOT QUITE $f(x) = \underbrace{(x - \tfrac{1}{3})(x - 3i\sqrt{2})(x + 3i\sqrt{2})}$ They're Linear factors!

= factored form!

$= x^3 + \ldots$ when you multiply it out!?

We want $3x^3 + \ldots = f(x)$:

$f(x) = 3(x - \tfrac{1}{3})(x - 3i\sqrt{2})(x + 3i\sqrt{2})$

$= (3x - 1)(x - 3i\sqrt{2})(x + 3i\sqrt{2})$

I think WebAssign will accept this form. If not, do this trick:

16. 0/1 points LarTrig10 4.2.054. [3883162]

Use the given zero to find all the zeros of the function. (Enter your answers as a comma-separated list. Include the given zero in your answer.)

Function	Zero
$g(x) = 5x^3 + 29x^2 + 44x - 10$	$-3 + i$

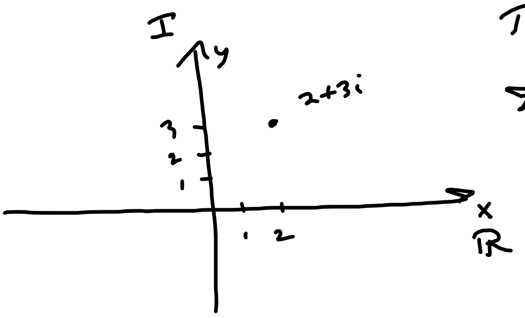
$x =$ ✗ $\frac{1}{5}, -3+i, -3-i$

Conjugate Pairs Theorem

$f(a+bi)=0$

& all of f's coefficients are real, then

$f(a-bi)=0$



$2+3i$ "=" $\langle 2,3\rangle$ "=" $(2,3)$

There's no "multiplication of vectors"

$\star : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$

We do have dot product

$\bullet : \mathbb{R}^2 \times \mathbb{R}^2 \longrightarrow \mathbb{R}$

$\langle 1,2\rangle \bullet \langle 3,4\rangle = 3+8=11$

$f : \mathbb{R} \longrightarrow \mathbb{R}$

$f(x) = x^2$

$x \longmapsto x^2$

Distance:

$d : \mathbb{R}^2 \times \mathbb{R}^2 \longrightarrow \mathbb{R}$

$d\left((x_1,y_1),(x_2,y_2)\right)$

$= \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2} \in \mathbb{R}$

Complex Numbers are

More than "just" vectors.

$\mathbb{C}$ is a number field

$+, -, \times, \div$

Associative, Commutative, Distributive Law,

$(ab)c = a(bc)$, $ab=ba$, $a(b+c) = ab+ac$

$(a+bi)+(c+di) = (a+c)+(b+d)i$

$3+2i + 7-11i = 10-9i$

Real Part Imaginary Part

$3+2i$ is imaginary

$2i$ " pure imaginary

3 is real

Multiplication

$(a+bi)(c+di)$ $\qquad\qquad$ $bidi = bdi^2 = -bd$

$= ac + adi + bic + bidi$

$= ac - bd + (ad+bc)i$

$(3+2i)(7-5i)$

$= 21 - 15i + 14i - 10i^2$

$= 21 + 10 - i = 31 - i$

If $z = a+bi$, then $\bar{z} = a - bi$ is the complex conjugate of z.

Fact: $z\bar{z} = a^2+b^2 = |z|^2$

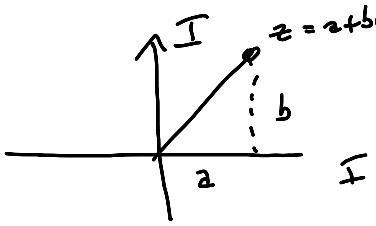
$(a+bi)(a-bi) = a^2 - abi + bia - (bi)(bi)$

$= a^2 \underline{+abi - abi} - b^2i^2$

$= a^2+b^2$

$= |z|^2,$

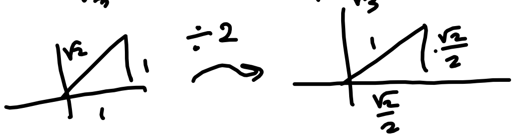
since $|z| = \sqrt{a^2+b^2}$



$z_1 = \frac{\sqrt{3}}{2} + \frac{1}{2} i$

$\theta_1 = 30^\circ$

$z_2 \quad \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i$



$\theta_2 = 45^\circ$

.2588190451
+ .9659258263

$(z_1 z_2) = \left(\frac{\sqrt{3}}{2} + \frac{1}{2} i\right)\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i\right)$

$= \frac{1}{2} \cdot \frac{1}{2} (3 + i)(\sqrt{2} + \sqrt{2} i)$

$= \frac{1}{4} (3\sqrt{2} + 3\sqrt{2} i + \sqrt{2} i + \sqrt{2} i^2)$

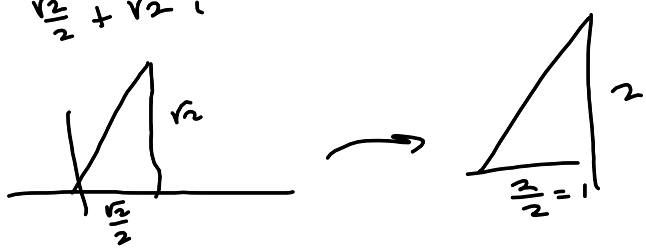
$= \frac{1}{4} (3\sqrt{2} - \sqrt{2} + (3\sqrt{2} + \sqrt{2}) i)$

$= \frac{1}{4} (2\sqrt{2} + 4\sqrt{2} i)$

$= \frac{\sqrt{2}}{2} + \sqrt{2} i$

I wanted to show you that we <u>Add</u> the angles!

75°
$= \frac{\pi}{6} + \frac{\pi}{4} = \frac{(2+3)\pi}{12}$
$= \frac{5\pi}{12}$
= angle = argument of the product



Angles (arguments) add when we multiply

18. 0/1 points LarTrig9 4.2.080. [2456305]

Find a cubic polynomial function *f* with real coefficients that has the given complex zeros and *x*-intercept. (There are many correct answers.)

Complex Zeros	x-Intercept
$x = -4 \pm \sqrt{3}i$	$(-2, 0)$

$f(x) =$ ✗ $x^3 + 10x^2 + 35x + 38$

$(x+2)(x-(-4+\sqrt{3}i))(x-(-4-\sqrt{3}i))$

Full credit for written work

But WebAssign wants it multiplied all the way out.

$(x+4-\sqrt{3}i)(x+4+\sqrt{3}i)$

$= x^2 + \sqrt{3}ix + (4-\sqrt{3}i)x + (4-\sqrt{3}i)(4+\sqrt{3}i)$

$(x+4-\sqrt{3}i)(x+4+\sqrt{3}i)$

$= x^2 + 4x + \sqrt{3}ix + 4x + 16 + 4\sqrt{3}i - \sqrt{3}ix - 4\sqrt{3}i - (\sqrt{3}i)(\sqrt{3}i)$

$= x^2 + 4x + 4x + 16 - 3i^2$

$= x^2 + 8x + 19$

$(x+2)(x^2+8x+19)$

$= x^3 + 8x^2 + 19x$
$\quad\;\; 2x^2 + 16x + 38$

$x^3 + 10x^2 + 35x + 38$

In the case of a repeated root, that root gets counted twice (or however many times it's repeated.

$(x-3)^3$

$x=3$, multiplicity 3

Use the given zero to find all the zeros of the function. (Enter your answers as a comma-separated list. Include the given zero in your answer.)

Function	Zero
$g(x) = 5x^3 + 29x^2 + 44x - 10$	$-3 + i$

$x =$ $\boxed{\frac{1}{5}, -3+i, -3-i}$

Divide by $x-(-3+i)$:

$$
\begin{array}{r|llll}
-3+i & 5 & 29 & 44 & -10 \\
 & & -15+5i & -47-i & +10 \\
\hline
-3-i & 5 & 14+5i & -3-i & 0 \\
 & & -15-5 & +3+i & \\
\hline
 & 5 & -1 & 0 &
\end{array}
$$

$(x-(-3+i))(x-(-3-i))(5x-1)$

Roots:

$-3 \pm i$, $\frac{1}{5}$

$(14+5i)(-3+i)$

$= -42 + 14i - 15i + 5i^2$

$= -47 - i$