3.1 Law of Sines

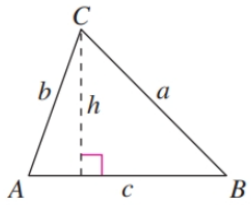
Law of Sines

If ABC is a triangle with sides a, b, and c, then

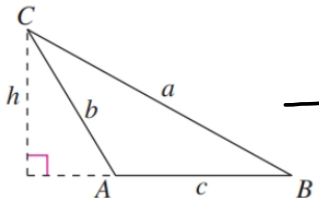
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \text{OR} \quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$

3.1 questions?

Do not use rounded numbers in calculations!

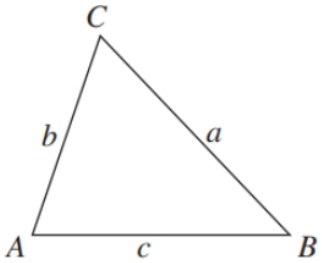


A is acute.



A is obtuse.

Benjamin suggest I share the screen.



A triangle is uniquely determined by any of the following:

Law of Sines
1. Two angles and any side (AAS or ASA)
2. Two sides and an angle opposite one of them (SSA) ASS

Law of Cosines
3. Three sides (SSS)
4. Two sides and their included angle (SAS)

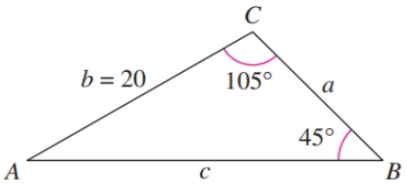
I never remember which goes to which. But I can always tell when I don't have enough for Law of Sines in a problem situation, just by trying it!

Method: ASA, SAS, AAS - Follow your nose.

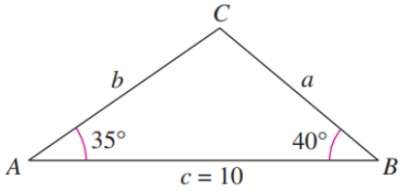
ASS - The ambiguous case. Check *h* and see if there's no solution, one solution, or two solutions.

Using the Law of Sines **In Exercises 5–22, use the Law of Sines to solve the triangle. Round your answers to two decimal places.**

5.



7.



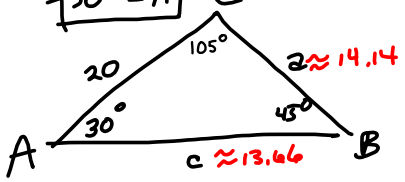
13. $A = 35°, \quad B = 65°, \quad c = 10$

14. $A = 120°, \quad B = 45°, \quad c = 16$

(5) $A = 180° - 105° - 45°$

$= 180° - 150°$

$= 30° = A$



$\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow$

$a = \frac{b \sin A}{\sin B} = \frac{20 \sin(30°)}{\sin(45°)}$

$= \frac{20(\frac{1}{2})}{\frac{1}{\sqrt{2}}} = 10\sqrt{2} = a$

≈ 14.14213562

$\approx 14.14 \approx a$

$\frac{c}{\sin(C)} = \frac{b}{\sin B}$

$c = \frac{b \sin(C)}{\sin B}$

```
10√(2)
      14.14213562
```

$c = \frac{20 \sin(105°)}{\sin(45°)}$

≈ 13.66025404

$\approx 13.66 \approx c$

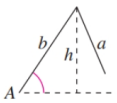
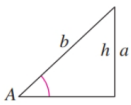
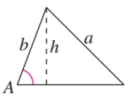
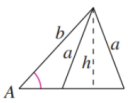
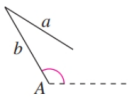
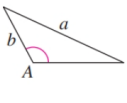
Noooooooooooo!!!!

I'm in radians! This exercise is in degrees!

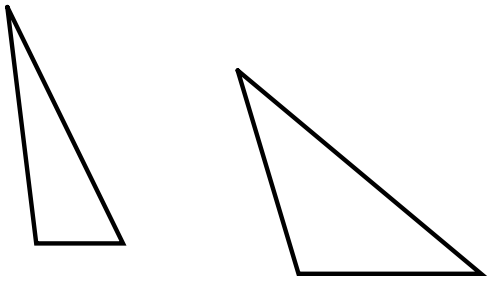
```
10sin(105)/sin(4
5)
     -11.40593799
10sin(105)/sin(4
5)
      13.66025404
```

The Ambiguous Case (SSA)

Consider a triangle in which a, b, and A are given. $(h = b \sin A)$

	A is acute.	A is acute.	A is acute.	A is acute.	A is obtuse.	A is obtuse.
Sketch						
Necessary condition	$a < h$	$a = h$	$a \geq b$	$h < a < b$	$a \leq b$	$a > b$
Triangles possible	None	One	One	Two	None	One

Using the Law of Sines **In Exercises 23–32, use the Law of Sines to solve (if possible) the triangle. If two solutions exist, find both. Round your answers to two decimal places.**

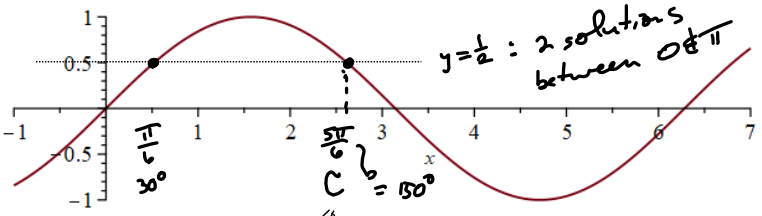


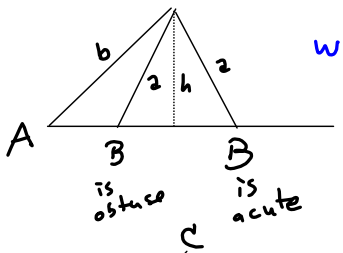
23. $A = 110°, \quad a = 125, \quad b = 100$

24. $A = 110°, \quad a = 125, \quad b = 200$

25. $A = 76°, \quad a = 18, \quad b = 20$

26. $A = 76°, \quad a = 34, \quad b = 21$



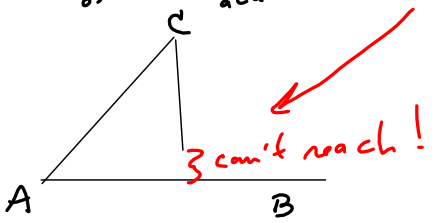


When $a < b$ AND $a > h$ then 2 solns.

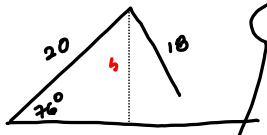
$a = h$ Right triangle!

$a < h$ No Solution!

$a > b \Rightarrow$ 1 solution!



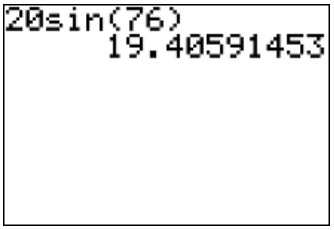
#25



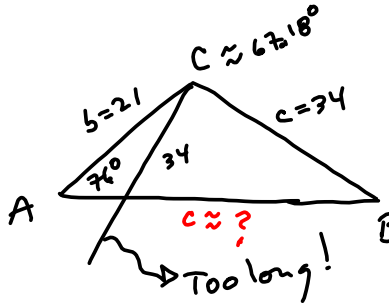
$\frac{h}{20} = \sin(76°) \Rightarrow$

$h = 20 \sin(76°) \approx 19.40591453 > 18 = a$

$\Rightarrow$ No solution!



$h = 20 \sin(76°) = 19.\ldots$



only one triangle possible

$$\frac{\sin B}{b} = \frac{\sin A}{a} \Rightarrow$$

$$\sin B = \frac{b \sin A}{a} = \frac{21 \sin(76^\circ)}{34}$$

```
21sin(76)/34
      .5993003015
sin⁻¹(Ans)
      36.81980185
```

$\approx .5993003015$

$\Rightarrow B \approx \arcsin(\text{prev}) \approx 36.81980185$

$\approx 36.82^\circ \approx B$

$C = 180^\circ - A - B$

$= 180^\circ - 76^\circ - 36\ldots$

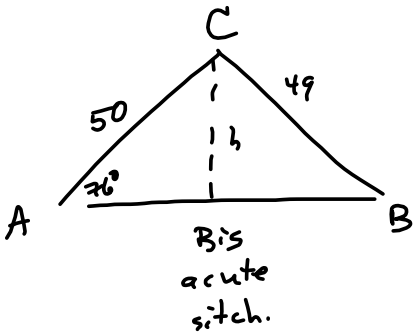
$\approx 67.18019815 \approx 67.18^\circ \approx C$

```
21sin(76)/34
      .5993003015
sin⁻¹(Ans)
      36.81980185
180-76-Ans
      67.18019815
```

$A = 76^\circ$

$a = 49$

$b = 50$



B is acute sitch.

$\frac{h}{50} = \sin 76^\circ$

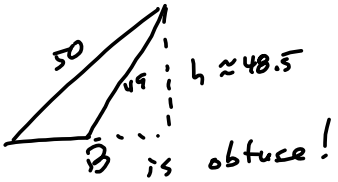
$h = 50 \sin 76^\circ$

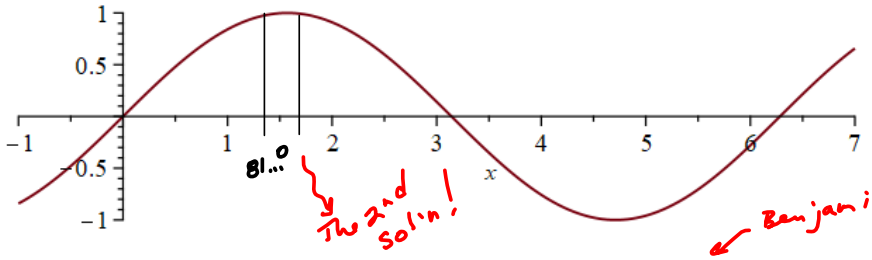
≈ 48.51478631

< 49

& $49 < 50$

2 sol'ns!



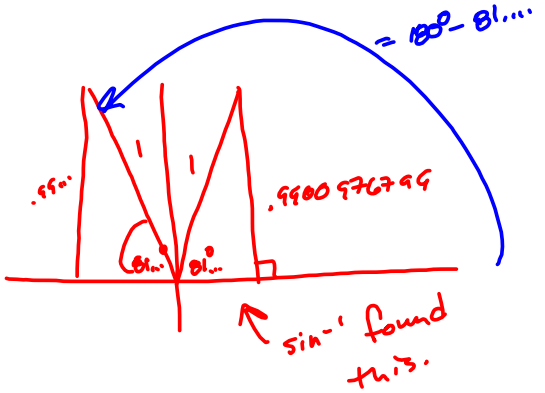


Benjamin

$\frac{\sin B}{b} = \frac{\sin A}{a} \Rightarrow \sin B = \frac{50 \sin A}{a} = \frac{50 \sin(76^\circ)}{49} = .99\ldots$

$\Rightarrow B_1 \approx 81.930\,15598$

$= 180^\circ - 81\ldots^\circ$



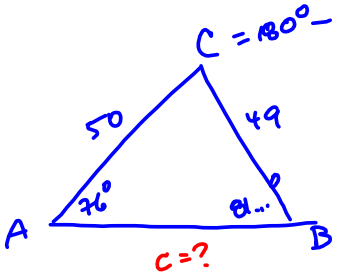

```
      29.10887179
50sin(76)
      48.51478631
50sin(76)/49
       .9900976799
sin⁻¹(Ans
      81.93015598
```

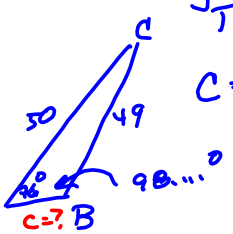
```
      48.51478631
50sin(76)/49
       .9900976799
sin⁻¹(Ans
      81.93015598
180-Ans
      98.06984402
```

The obtuse B version

$C = 180^\circ - 76^\circ - 98\ldots^\circ$

$C = 180^\circ - 76^\circ - 81\ldots^\circ$

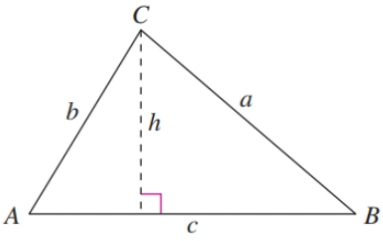




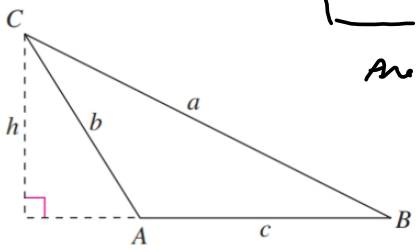
Area of an Oblique Triangle

The area of any triangle is one-half the product of the lengths of two sides times the sine of their included angle. That is,

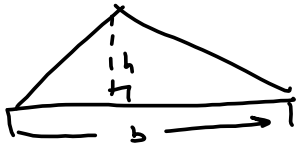
$$\text{Area} = \frac{1}{2}bc \sin A = \frac{1}{2}ab \sin C = \frac{1}{2}ac \sin B.$$



A is acute.



A is obtuse.



Area $= \frac{1}{2}bh$

$\frac{h}{a} = \sin B$

$h = a \sin B$

$\frac{1}{2}ch$

$= \frac{1}{2}c(a \sin B)$

$= \frac{1}{2}ac \sin B$

Area $= \frac{1}{2}ch = \frac{1}{2}c(a \sin B) = \frac{1}{2}ac \sin B = \text{Area}$

~~$\frac{h}{b} = \sin B$~~

~~$h = b \sin B$~~

No $\frac{h}{a} = \sin B$

$h = a \sin B$

Have a great Spring Break!