$$\sqrt{A^2} = |A|$$

$\frac{1}{2}$- angle:

$$\cos(2x) = 2\cos^2(x) - 1$$

$$\Rightarrow \quad 2\cos^2(x) = 1 + \cos(2x)$$

$$\Rightarrow \quad \cos^2(x) = \frac{1+\cos(2x)}{2}$$

$$\sqrt{\cos^2(x)} = \sqrt{\frac{1+\cos(2x)}{2}}$$

$$|\cos(x)| = \sqrt{\frac{1+\cos(2x)}{2}}$$

$$\cos(x) = \pm\sqrt{\frac{1+\cos(2x)}{2}}$$

$$\cos\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1+\cos(u)}{2}}$$

Likewise

$$|\sin(x)| = \sqrt{\frac{1-\cos(2x)}{2}}$$

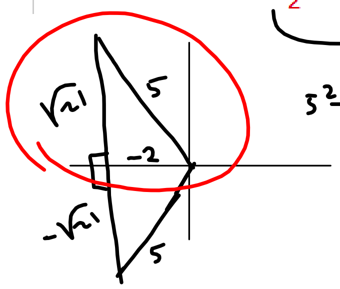
$$\sin\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1-\cos(u)}{2}}$$

7. 0/6 points

Use the given conditions to find the values of all six trigonometric functions. (If an answer is undef

$\sec(x) = -\frac{5}{2}, \quad \tan(x) < 0$

$\cos(x) = -\frac{2}{5}$ QII or QIII



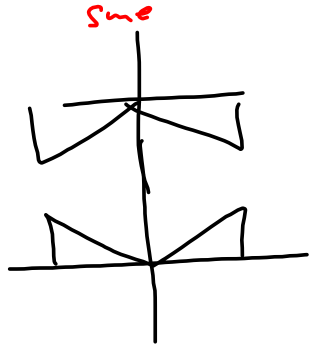
$5^2 - 2^2 = 25 - 4 = 21$

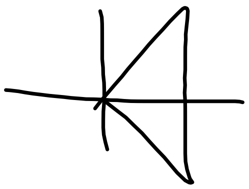
$\sin(x) = \frac{\sqrt{21}}{5}$ $\quad \csc(x) = \frac{5}{\sqrt{21}}$

$\cos(x) = -\frac{2}{5}$ $\quad \sec(x) = -\frac{5}{2}$

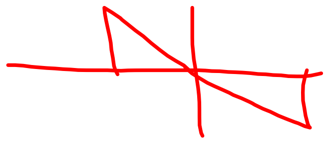
$\tan(x) = \frac{\sqrt{21}}{-2}$ $\quad \cot(x) = -\frac{2}{\sqrt{21}}$

sine





tangent



$$\cot^2(x) - \csc^2(x)$$

$$= \frac{\cos^2(x)}{\sin^2(x)} - \frac{1}{\sin^2(x)} = \frac{\cos^2(x) - 1}{\sin^2(x)} = \frac{-(1-\cos^2(x))}{\sin^2(x)}$$

$$= \frac{-\sin^2(x)}{\sin^2(x)} = -1$$

11. 0/1 points

Perform the addition or subtraction and use the fundamental identities to simplify. There is more answer.

$$\frac{\cos(x)}{1+\sin(x)} - \frac{\cos(x)}{1-\sin(x)}$$

$$\text{LCD} = (1+\sin(x))(1-\sin(x))$$

$$= \left(\frac{\cos(x)}{1+\sin(x)}\right)\left(\frac{1-\sin(x)}{1-\sin(x)}\right) - \left(\frac{\cos(x)}{1-\sin(x)}\right)\left(\frac{1+\cos(x)}{1+\cos(x)}\right)$$

$$= \frac{\cos(x) - \sin(x)\cos(x) - (\cos(x) + \cos^2(x))}{\text{LCD}}$$

$$= \frac{\cancel{\cos(x)} - \sin(x)\cos(x) - \cos(x) - \cos^2(x)}{\text{LCD}}$$

$$= \frac{-\sin(x)\cos(x) - \cos^2(x)}{1-\sin^2(x)} = \frac{-\cos(x)[\sin(x) + \cos(x)]}{\cos^2(x)}$$

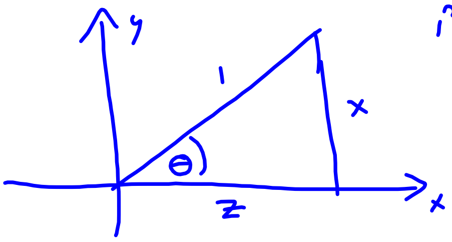
$$= \frac{-(\sin(x) + \cos(x))}{\cos(x)} = \frac{-\sin(x) - \cos(x)}{\cos(x)}$$

$$= -\frac{\sin(x)}{\cos(x)} - 1 = -\tan(x) - 1$$

16. 0/2 points

Verify the identity.

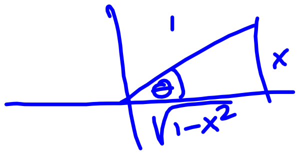
$9\cos\theta = 9\cos(\sin^{-1} x) = \sqrt{81 - 81x^2}$



$1^2 - x^2 = z^2 \rightarrow$

$z = \pm\sqrt{1-x^2}$

keep those in QI



$9\cos\theta = 9\left(\frac{\sqrt{1-x^2}}{1}\right)$

$= 9\sqrt{1-x^2}$

$= \sqrt{9^2}\sqrt{1-x^2}$

$= \sqrt{9^2(1-x^2)}$

$= \sqrt{81 - 81x^2}$

8. 0/3 points

Use the given conditions to find the exact values of sin(2u), cos(2u), and tan(2u) using the

$\sec(u) = -2, \quad \pi/2 < u < \pi$

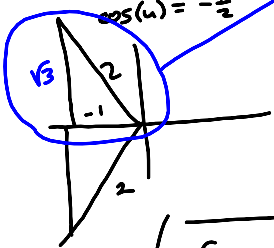
$\sin(2u) =$	$-\frac{\sqrt{3}}{2}$
$\cos(2u) =$	$-\frac{1}{2}$
$\tan(2u) =$	$\sqrt{3}$

$\sec(u) = -2$

$\cos(u) = -\frac{1}{2}$

$\frac{\pi}{2} < u < \pi$

$\pi < 2u < 2\pi$



$\sin(2u) = 2\sin(u)\cos(u) = 2\left(\frac{\sqrt{3}}{2}\right)\left(-\frac{1}{2}\right) = \boxed{-\frac{\sqrt{3}}{2} = \sin(2u)}$

$\cos(2u) = 2\cos^2(u) - 1 = 2\left(-\frac{1}{2}\right)^2 - 1 = 2\left(\frac{1}{4}\right) - 1 = \frac{1}{2} - 1 = \boxed{-\frac{1}{2} = \cos(2u)}$

$\tan(2u) = \dfrac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = \left(-\frac{\sqrt{3}}{2}\right)\left(-\frac{2}{1}\right) = \sqrt{3}$

25. 0/1 points LarTrig10 2.3.071. [388277

Use the Quadratic Formula to find all solutions of the equation in the interval $[0, 2\pi)$. (Enter your answers as a comma-separated list. Round each answer to four decimal places.)

$20 \sin^2(x) - 26 \sin(x) + 8 = 0$

$x =$ ✖ 0.5236, 0.9273, 2.2143, 2.618

```
26²-4*20*8
                36
36
```

$20\sin^2(x) - 26\sin(x) + 8 \Rightarrow$

$20u^2 - 26u + 8 = 0$, where $u = \sin(x)$.

$u = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a} = \dfrac{26 \pm 6}{2(20)} = \dfrac{13 \pm 3}{20}$

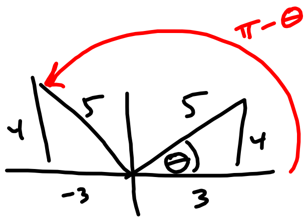
$\dfrac{16}{20} = \dfrac{4}{5} = u = \sin(x)$

$\dfrac{10}{20} = \dfrac{1}{2} = u = \sin(x)$

$b^2 - 4ac = (-26)^2 - 4(20)(8)$

$= 36 \Rightarrow \sqrt{36} = 6$

$\sin(x) = \dfrac{4}{5}$



If you were asked for an EXACT answer, you'd get:

$\arcsin\left(\frac{4}{5}\right),\ \pi - \arcsin\left(\frac{4}{5}\right)$

$\sin^{-1}\left(\frac{4}{5}\right),\ \pi - \sin^{-1}\left(\frac{4}{5}\right)$

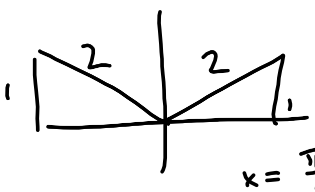
```
sin⁻¹(4/5)
      .927295218
π-Ans
      2.214297436
```

$= \theta \Rightarrow$

$= \pi - \theta$

$\Rightarrow \theta \approx .9273,\ 2.2143,\ .5236,\ 2.6180$

Also, $u = \sin(x) = \dfrac{1}{2}$



$x = \dfrac{\pi}{6},\ \dfrac{5\pi}{6}$

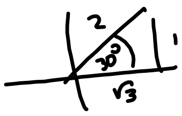
$=$

```
      .927295218
π-Ans
      2.214297436
π/6
      .5235987756
5π/6
      2.617993878
```

$\sin(15^\circ)$, $\cos(15^\circ)$, $\tan(15^\circ)$

$15^\circ = \frac{30^\circ}{2}$ $\quad 15^\circ \in QI \Rightarrow$ sine, cosine, tangent > 0

$$\sin(15^\circ) = \sqrt{\frac{1-\cos(30^\circ)}{2}} = \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2-\sqrt{3}}{4}}$$



$$= \frac{\sqrt{2-\sqrt{3}}}{\sqrt{4}} = \boxed{\frac{\sqrt{2-\sqrt{3}}}{2} = \sin(15^\circ)}$$

$$\cos(15^\circ) = \sqrt{\frac{2+\sqrt{3}}{4}} = \boxed{\frac{\sqrt{2+\sqrt{3}}}{2} = \cos(15^\circ)}$$

$$\tan(15^\circ) = \frac{\sin(15^\circ)}{\cos(15^\circ)} = \frac{\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}} \overset{?}{=} \frac{1-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{2-\sqrt{3}}{2}\cdot\frac{2}{1} = 2-\sqrt{3}$$
?
?

$$\frac{\text{sqrt}(2-\text{sqrt}(3))}{\text{sqrt}(2+\text{sqrt}(3))}$$

$$\frac{\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}}{\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}}$$

evalf(%)

0.2679491928

$2-\text{sqrt}(3)$

$2-\sqrt{3}$

evalf(%)

0.267949192

Same?!
Yes!

$$\frac{\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}} = \sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}} = \sqrt{\left(\frac{2-\sqrt{3}}{2+\sqrt{3}}\right)\left(\frac{2-\sqrt{3}}{2-\sqrt{3}}\right)}$$

$$= \frac{\sqrt{(2-\sqrt{3})^2}}{2^2-\sqrt{3}^2} = \frac{|2-\sqrt{3}|}{4-3} = |2-\sqrt{3}|$$

$$= 2-\sqrt{3} \ !$$