35. 0/1 points LarTrig10 2.3.053. [388287

Use a graphing utility to approximate (to three decimal places) the solutions of the equation in the interval $[0, 2\pi)$. (Enter your answers as a comma-separated list.)

$\cos(x) = x$

$x =$ ✖ 0.739

$Y_1 = \cos(x)$

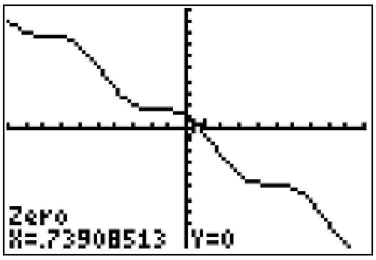
$Y_2 = x$

Find Intersection

$Y_1 = \cos(x) - x$

Find zeros

(x-intercepts)



Using Calculate menu on a TI-84. Select "Zero."

39. 0/1 points LarTrig10 2.3.068. [4242445]

Solve the equation. (Enter your answers as a comma-separated list. Use n as an integer constant. Enter your response in radians.)

$$\sec^2(x) + 5\sec(x) - 14 = 0$$

$x =$ ✖ $2\pi n + \frac{\pi}{3}, 2\pi n + \cos^{-1}\left(-\frac{1}{7}\right), 2\pi n - \cos^{-1}\left(-\frac{1}{7}\right), 2\pi n + \frac{5\pi}{3}$

Let $u = \sec(x)$

$$u^2 + 5u - 14 = 0$$

$$(u+7)(u-2) = 0$$

$u = -7$ or $u = 2$

$\sec(x) = -7$

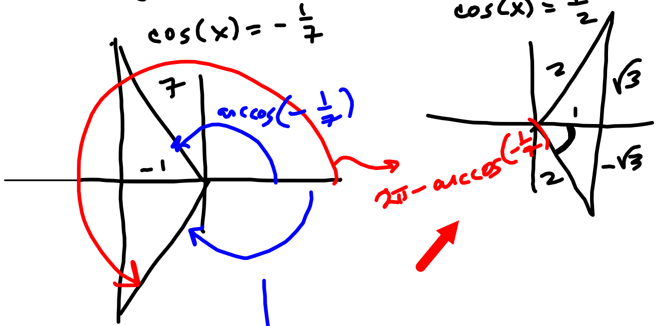
$\cos(x) = -\frac{1}{7}$

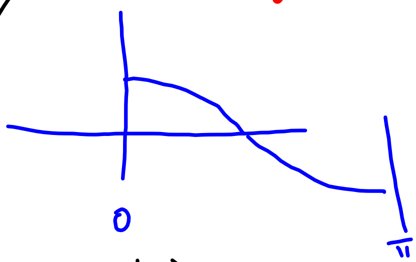
$\sec(x) = 2$

$\cos(x) = \frac{1}{2}$

$2\pi - \frac{\pi}{3} = \frac{6\pi - \pi}{3} = \frac{5\pi}{3}$

$$\boxed{x = \frac{\pi}{3}, \frac{5\pi}{3}}$$





$x = \arccos\left(-\frac{1}{7}\right), 2\pi - \arccos\left(-\frac{1}{7}\right)$ is the only way to express this exactly. All else is an approximation.

$$A = \left\{\frac{\pi}{3}, \frac{5\pi}{3}, \arccos\left(-\frac{1}{7}\right), 2\pi - \arccos\left(-\frac{1}{7}\right)\right\}$$

$$S = \{x + 2n\pi \mid x \in A, n \in \mathbb{Z}\}$$

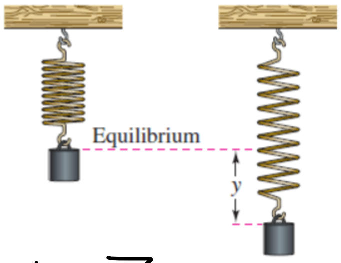
48. 0/1 points LarTrig10 2.3.089. [3882829]

A weight is oscillating on the end of a spring (see figure). The displacement from equilibrium of the weight relative to the point of equilibrium is given by

$$y = \frac{1}{12}(\cos(8t) - 3\sin(8t))$$

where y is the displacement (in meters) and t is the time (in seconds). Find the times when the weight is at the point of equilibrium $(y = 0)$ for $0 \le t \le 1$. (Enter your answers as a comma-separated list. Round your answers to two decimal places.)

$t =$ ✗ 0.04, 0.43, 0.83 s



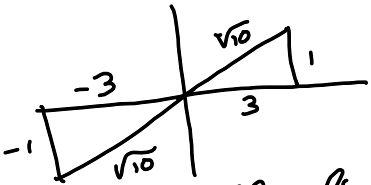
Solve

$y = \frac{1}{12}(\cos(8t) - 3\sin(8t)) \overset{\text{SET}}{=} 0$

$\Rightarrow \cos(8t) = 3\sin(8t)$

$\frac{\cos(8t)}{\sin(8t)} = \cot(8t) = 3$

$\tan(8t) = \frac{1}{3}$



$0 \le t \le 1 \rightarrow$

$0 \le 8t \le 8$

$\arctan\left(\frac{1}{3}\right) \approx .32$

$\pi + \arctan\left(\frac{1}{3}\right) \approx 3.46$

$2\pi + \arctan\left(\frac{1}{3}\right) \approx 6.60$

Find all sol'ns $8t$, such that $0 \le 8t \le 8$

$8t = .32\ldots$ $\quad t = \frac{.32\ldots}{8}$

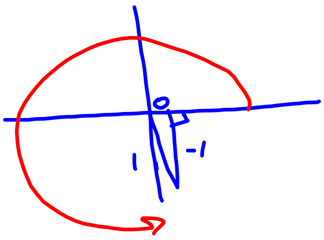
$8t \approx .32 + \pi$ $\quad .32\ldots + \frac{\pi}{8}$

$8t \approx .32 + 2\pi$ $\quad .32\ldots + \frac{\pi}{4}$

```
tan-1(1/3)
        .3217505544
Ans+π
        3.463343208
Ans+π
        6.604935862
```

2.3 #29 $\quad \sin\left(\frac{\pi x}{8}\right)+1 \overset{\text{SET}}{=} 0$

$\Longrightarrow \sin\left(\frac{\pi x}{8}\right) = -1$



on $[0,2\pi]$, we have

$$\frac{\pi x}{8} = \frac{3\pi}{2}$$

$$x = 12$$

$$\frac{\pi x}{8} = \frac{3\pi}{2} + 2n\pi \Longrightarrow$$

$$x = 12 + 2n\pi \cdot \frac{8}{\pi}$$

$$= 12 \pi + 16n$$

$$\frac{\pi x}{8} = \frac{3\pi}{2} \Longrightarrow$$

$$x = \frac{3\pi}{2} \cdot \frac{8}{\pi} = 12$$

Link to Cheat Sheet: 2.5 Stuff

https://harryzaims.com/public_html/122/1420-spring-23/cheat-sheet-test-2.pdf

Double-Angle Formulas

$\sin 2u = 2 \sin u \cos u$

$\tan 2u = \dfrac{2 \tan u}{1 - \tan^2 u}$

$\boxed{\cos 2u = \cos^2 u - \sin^2 u}$

$= 2\cos^2 u - 1$

$= 1 - 2\sin^2 u$

$\sin(a+b) = \sin(a)\cos(b) + \sin(b)\cos(a)$

$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$

$\cos(2u) = \cos(u+u)$

$= \cos(u)\cos(u) - \sin(u)\sin(u)$

$= \cos^2(u) - 1$

Solving a Multiple-Angle Equation

In Exercises 7–14, solve the equation.

7. $\sin 2x - \sin x = 0$
8. $\sin 2x \sin x = \cos x$
9. $\cos 2x - \cos x = 0$
10. $\cos 2x + \sin x = 0$
11. $\sin 4x = -2 \sin 2x$
12. $(\sin 2x + \cos 2x)^2 = 1$
13. $\tan 2x - \cot x = 0$
14. $\tan 2x - 2 \cos x = 0$

$\sin(2x) - \sin(x) = 0$

$2\sin(x)\cos(x) - \sin(x) = 0$

$\sin(x)[2\cos(x) - 1] = 0$

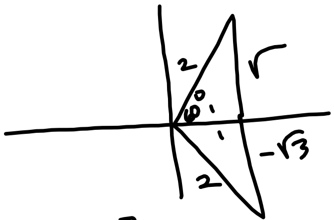
At first, I didn't think they needed to specify where u resided. But it matters. Because the $\sin(u)$, $\cos(u)$, and $\tan(u)$ values depend on what quadrant the u is in.

$\sin(x) = 0$ OR $2\cos(x) - 1 = 0$

$2\cos(x) = 1$

$\cos(x) = \frac{1}{2}$

$x = 0, \pi$



$x = \frac{\pi}{3}, \frac{5\pi}{3}$

$x = 60°, 360° - 60° = 300°$

All sol'ns in $[0, 2\pi]$:

$0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}$

odt: $\boxed{n\pi,\ n \in \mathbb{Z}}$

$\frac{\pi}{3}, \frac{5\pi}{3}$: $\boxed{\frac{\pi}{3} + 2n\pi,\ n \in \mathbb{Z} \quad \frac{5\pi}{3} + 2n\pi,\ n \in \mathbb{Z}}$

$A = \{ n\pi \mid n \in \mathbb{Z} \}$

$B = \{ \frac{\pi}{3} + 2n\pi \mid n \in \mathbb{Z} \}$

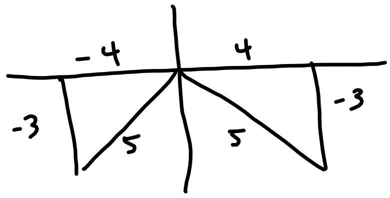
$C = \{ \frac{5\pi}{3} + 2n\pi \mid n \in \mathbb{Z} \}$

Sol'n Set $S = A \cup B \cup C$.

Evaluating Functions Involving Double Angles In Exercises 21–24, use the given conditions to find the exact values of $\sin 2u$, $\cos 2u$, and $\tan 2u$ using the double-angle formulas.

21. $\sin u = -3/5, \quad 3\pi/2 < u < 2\pi$
22. $\cos u = -4/5, \quad \pi/2 < u < \pi$
23. $\tan u = 3/5, \quad 0 < u < \pi/2$
24. $\sec u = -2, \quad \pi < u < 3\pi/2$

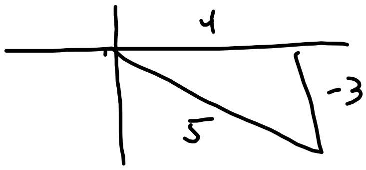
$\sin(u) = -\frac{3}{5}$



$5^2 - 3^2 = 25 - 9$
$= 16 = x^2$
$x = \pm 4$

But they go further & say

$\frac{3\pi}{2} < u < 2\pi$, so:



$\sin(2u) = 2\sin(u)\cos(u)$

$= 2\left(-\frac{3}{5}\right)\left(\frac{4}{5}\right) = \boxed{-\frac{24}{25} = \sin(2u)}$

$\cos(2u) = \cos^2(u) - \sin^2(u)$

$= \left(\frac{4}{5}\right)^2 - \left(-\frac{3}{5}\right)^2$

$= \frac{16-9}{25} = \boxed{\frac{7}{25} = \cos(2u)}$

$\tan(2u) = \frac{\sin(2u)}{\cos(2u)}$

$= -\frac{24}{25} \cdot \frac{25}{7} = \boxed{-\frac{24}{7} = \tan(2u)}$

From Cheat Sheet

$\frac{2\tan u}{1 - \tan^2 u} = \frac{2\left(-\frac{3}{4}\right)}{1 - \left(-\frac{3}{4}\right)^2} = \frac{-\frac{3}{2}}{1 - \frac{9}{16}} = \frac{-\frac{3}{2}}{\frac{16-9}{16}} = \frac{-\frac{3}{2}}{\frac{7}{16}}$

$= -\frac{3}{2} \cdot \frac{16}{7} = \frac{-3(4)}{7} = \boxed{-\frac{24}{7} = \tan(2u)}$

Power-Reducing Formulas

$$\boxed{\begin{aligned}\sin^2 u &= \frac{1-\cos 2u}{2}\\ \cos^2 u &= \frac{1+\cos 2u}{2}\end{aligned}}$$

$$\tan^2 u = \frac{1-\cos 2u}{1+\cos 2u}$$

Memorize

$\cos(2u) = 2\cos^2(u) - 1$

$2\cos^2(u) - 1 = \cos(2u)$

$\Rightarrow 2\cos^2(u) = \cos(2u) + 1 = 1 + \cos(2u)$

$\cos^2(u) = \dfrac{1+\cos(2u)}{2}$

This comes up a lot in CALC II.

$\int \cos^5(x)\,dx$ can be broken into smaller powers

$$\int \cos^3(x)\underbrace{\cos^2(x)}_{\frac{1+\cos(2x)}{2}}dx$$

Half-Angle Formulas

$$\sin\frac{u}{2} = \pm\sqrt{\frac{1-\cos u}{2}} \qquad \cos\frac{u}{2} = \pm\sqrt{\frac{1+\cos u}{2}}$$

$$\tan\frac{u}{2} = \frac{1-\cos u}{\sin u} = \frac{\sin u}{1+\cos u}$$

The signs of $\sin\frac{u}{2}$ and $\cos\frac{u}{2}$ depend on the quadrant in which $\frac{u}{2}$ lies.

$$\sin(u) = \pm\sqrt{\frac{1-\cos(2u)}{2}}$$

RE-LABEL

$$\sin\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1-\cos(u)}{2}}$$

I'll show you how to figure it out, analytically, and how to cheat it with a calculator.

Using Half-Angle Formulas In Exercises 35–40, use the half-angle formulas to determine the exact values of the sine, cosine, and tangent of the angle.

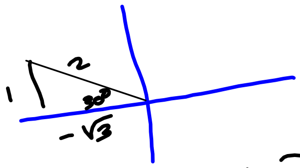
35. $75°$ **36.** $165°$

37. $112° \, 30'$ **38.** $67° \, 30'$

39. $\pi/8$ **40.** $7\pi/12$

$$75° = \frac{150°}{2}$$

$$\sin(75°) = \pm\sqrt{\frac{1-\cos(150°)}{2}} = \pm\sqrt{\frac{1-\left(-\frac{\sqrt{3}}{2}\right)}{2}}$$



$$= \pm\sqrt{\frac{\frac{2+\sqrt{3}}{2}}{2}} = \pm\sqrt{\frac{2+\sqrt{3}}{4}}$$

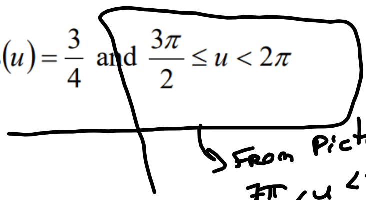
$$= \pm\frac{\sqrt{2+\sqrt{3}}}{\sqrt{4}} = \pm\frac{\sqrt{2+\sqrt{3}}}{2}$$

What about $\pm$?
$75° \in QI$ sine, cosine, & tangent positive.

$$= +\frac{\sqrt{2+\sqrt{3}}}{2}$$

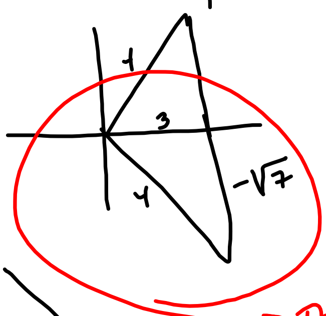
$$\cos(75°) = +\sqrt{\frac{1+\left(-\frac{\sqrt{3}}{2}\right)}{2}} = \dots \frac{\sqrt{2-\sqrt{3}}}{2}$$

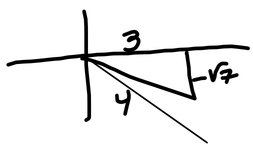
2. (20 pts) Find $\sin\left(\frac{u}{2}\right)$, $\cos\left(\frac{u}{2}\right)$, and $\tan\left(\frac{u}{2}\right)$, given that $\cos(u) = \frac{3}{4}$ and $\frac{3\pi}{2} \le u < 2\pi$



From picture,
$\frac{7\pi}{4} < u < 2\pi$

$\cos(u) = \frac{3}{4}$





This is the one between $\frac{3\pi}{2}$ & 2π

$\frac{6\pi + \pi}{4} = \frac{7\pi}{4}$

Now $\frac{3\pi}{2} \le u \le 2\pi$

$\frac{3\pi}{4} \le \frac{u}{2} \le \pi$

$\cos(\frac{u}{2}) < 0$ $\quad -\sqrt{}$

$\sin(\frac{u}{2}) > 0$ $\quad +\sqrt{}$

$\tan(\frac{u}{2}) < 0$ (Don't even need.)

Important

Product-to-Sum Formulas

$$\sin u \sin v = \frac{1}{2}[\cos(u-v) - \cos(u+v)]$$

$$\cos u \cos v = \frac{1}{2}[\cos(u-v) + \cos(u+v)]$$

$$\sin u \cos v = \frac{1}{2}[\sin(u+v) + \sin(u-v)]$$

$$\cos u \sin v = \frac{1}{2}[\sin(u+v) - \sin(u-v)]$$

Less important?

Sum-to-Product Formulas

$$\sin u + \sin v = 2\sin\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$

$$\sin u - \sin v = 2\cos\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$

$$\cos u + \cos v = 2\cos\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$

$$\cos u - \cos v = -2\sin\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$

This is hard: $\int \sin(5x)\cos(7x)\,dx$

$$= \frac{1}{2}\int [\sin(12x) + \sin(-2x)]\,dx$$

$$= \frac{1}{2}\int (\sin(12x) - \sin(2x))$$ is very easy