9. 0/1 points

Verify the identity. (Simplify at each step.)

$$4\cos^2\beta - 4\sin^2\beta = 4 - 8\sin^2\beta$$

$4\cos^2\beta - 4\sin^2\beta = \left(4 - \boxed{}\right.$ ✖ $\boxed{4\sin^2(\beta)}\left.\right) - 4\sin^2\beta = 4 - 8\sin^2\beta$

$\sin^2\beta + \cos^2\beta = 1$

$4(1-\sin^2\beta) - 4\sin^2\beta = 4 - 4\sin^2\beta - 4\sin^2\beta = 4 - 8\sin^2\beta$

11. 0/2 points

Verify the identity by converting the left side into sines and cosines. (Simplify at each step.)

$$\frac{\cot^2(t)}{4\csc(t)} = \frac{1-\sin^2(t)}{4\sin(t)}$$

$\dfrac{\cot^2(t)}{4\csc(t)} = \dfrac{\cos^2(t)/\left(\boxed{}\right.$ ✖ $\boxed{\sin^2(t)}\left.\right)}{4/\sin(t)}$

$= \dfrac{\cos^2(t)}{\boxed{}}$ ✖ $\boxed{4\sin(t)}$

$= \dfrac{1-\sin^2(t)}{4\sin(t)}$

$$\frac{\frac{\cos^2 t}{\sin^2 t}}{4\left(\frac{1}{\sin(t)}\right)} = \frac{\frac{\cos^2 t}{\sin^2(t)}}{\frac{4}{\sin t}}$$

$$= \frac{\cos^2 t}{\sin^2 t} \cdot \frac{\sin(t)}{4}$$

$$= \frac{\cos^2 t}{4\sin t} = \frac{1}{4} \cdot \frac{1}{\sin t} \cdot (1-\sin^2 t)$$

$$= \frac{1-\sin^2 t}{4\sin t}$$

15. 0/5 points LarTrig10 2.2.024. [3882208]

Verify the identity algebraically. Use the *table* feature of a graphing utility to check your result numerically. (Simplify at each step.)

$$4\cos(x) - \frac{4\cos(x)}{1-\tan(x)} = \frac{4\sin(x)\cos(x)}{\sin(x)-\cos(x)}$$

$$4\cos(x) - \frac{4\cos(x)}{1-\tan(x)} = \frac{(4\cos(x))\left(\boxed{}\ \boxed{1-\tan(x)}\right) - 4\cos(x)}{1-\tan(x)}$$

$$= \frac{(-4\cos(x))\left(\boxed{}\ \boxed{\tan(x)}\right)}{1-\tan(x)}$$

$$= \left(\frac{(-4\cos(x))\left(\frac{\boxed{}\ \boxed{\sin(x)}}{\cos(x)}\right)}{1-\left(\frac{\boxed{}\ \boxed{\sin(x)}}{\cos(x)}\right)}\right)\left(\frac{\cos(x)}{\cos(x)}\right)$$

$$= \frac{(-4\cos(x))\left(\boxed{}\ \boxed{\sin(x)}\right)}{\cos(x)-\sin(x)}$$

$$= \frac{4\sin(x)\cos(x)}{\sin(x)-\cos(x)}$$

$$= \frac{4\cos x}{1}\cdot\frac{1-\tan x}{1-\tan x} - \frac{4\cos x)}{1-\tan x}$$

$$= \frac{4\cos x - 4\cos x\tan x - 4\cos x}{1-\tan x}$$

$$= \frac{-4\cos x\tan x}{1-\tan x}$$

$$= \frac{-4\cos x\left(\frac{\sin x}{\cos x}\right)}{1-\frac{\sin x}{\cos x}}\cdot\frac{\cos x}{\cos x}$$

$$= \frac{-4\cos x\sin x}{\cos x - \sin x}\cdot\frac{-1}{-1}$$

$$= \frac{4\cos x\sin x}{\sin x - \cos x}$$

My Way

$$= \frac{-4\sin x}{\left(\frac{\cos x - \sin x}{\cos x}\right)}$$

$$= -4\sin x\cdot\left(\frac{\cos x}{\cos x - \sin x}\right)$$

$$= \frac{-4\sin x\cos x}{\cos x - \sin x} = \frac{4\sin x\cos x}{\sin x - \cos x}$$

20. 0/2 points

Verify the identity. (Simplify at each step.)

$\tan\theta =$ $\tan\left(\sin^{-1}\frac{x-1}{3}\right) = \frac{x-1}{\sqrt{9-(x-1)^2}}$

Let $\theta = \sin^{-1}\frac{x-1}{3} \Rightarrow \sin\theta = \frac{x-1}{3}$. Thus,

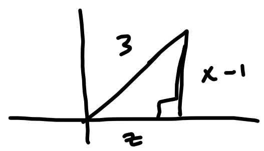
$\tan\left(\sin^{-1}\frac{x-1}{3}\right) = \tan\left(\boxed{} \times \boxed{\theta}\right)$

$= \dfrac{\dfrac{\boxed{} \times \boxed{x-1}}{3}}{\dfrac{\sqrt{9-(x-1)^2}}{3}}$

$= \dfrac{x-1}{\sqrt{9-(x-1)^2}}.$

$\tan\left(\arcsin\left(\frac{x-1}{3}\right)\right) = \frac{x-1}{\sqrt{9-(x-1)^2}}$

$\arcsin\left(\frac{x-1}{3}\right)$



$3^2 - (x-1)^2 = z^2$

$\sqrt{9-(x-1)^2} = \sqrt{z^2} = |z|$

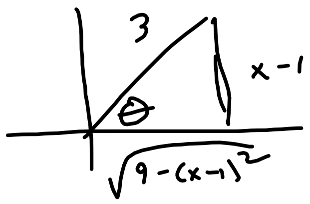
$= z^*$

$z^2 + (x-1)^2 = 3^2$

* because $z>0$ by picture

(assume QI)

This gives



$\theta = \arcsin\left(\frac{x-1}{3}\right)$

$\Rightarrow \tan(\theta) = \frac{x-1}{\sqrt{9-(x-1)^2}}$

18. 0/2 points LarTrig10 2.2.502.XP. [388

Verify the identity. (Simplify at each step.)

$$\sqrt{\frac{9-9\cos\theta}{1+\cos\theta}} = \frac{3(1-\cos\theta)}{|\sin\theta|}$$

"conjugate"

$$\sqrt{\frac{9-9\cos\theta}{1+\cos\theta}} = \sqrt{\left(\frac{9-9\cos\theta}{1+\cos\theta}\right)\left(\frac{\square\ 1-\cos(\theta)}{1-\cos\theta}\right)}$$

$$= \sqrt{\frac{(3)^2\left(\square\ 1-\cos(\theta)\right)^2}{1-\cos^2\theta}}$$

$$= \frac{3(1-\cos\theta)}{|\sin\theta|}$$

$$\sqrt{\frac{9-9\cos\theta}{1+\cos\theta}\cdot\frac{1-\cos\theta}{1-\cos\theta}} = \sqrt{\frac{9(1-\cos\theta)(1-\cos\theta)}{1^2-\cos^2\theta}}$$

$$= \sqrt{\frac{9(1-\cos\theta)^2}{\sin^2\theta}} = \frac{\sqrt{9}\sqrt{(1-\cos\theta)^2}}{\sqrt{\sin^2\theta}} = \frac{3|1-\cos\theta|}{|\sin\theta|}$$

$$= \frac{3(1-\cos\theta)}{|\sin\theta|}$$

b/c

$-1 \le \cos\theta \le 1$

$1 \ge -\cos\theta \ge -1$

$+1:\quad 2 \ge \boxed{1-\cos\theta \ge 0}$

OR

$\cos\theta \le 1$

$-\cos\theta \ge -1$

$1-\cos\theta \ge 0$

$A^2 = B \implies$

$\sqrt{A^2} = \sqrt{B}$

$|A| = \sqrt{B}$

$A = \pm\sqrt{B}$

$\sqrt{A}^2 = A$

$\sqrt{(-3)^2} = \sqrt{3^2} = 3$

$\sqrt{(-3)^2} = |-3|$, $\sqrt{3^2} = 3$

13. 0/1 points LarTrig10 2.3.033. [388

Find all solutions of the equation in the interval $[0, 2\pi)$. (Enter your answers as a comma-separated list. If there is no solution, enter NO SOLUTION.)

$\sin^2(x) = 3\cos^2(x)$

$x =$ ✗ $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

S 2.3 #13

$$\Rightarrow \frac{\sin^2(x)}{\cos^2(x)} = 3$$

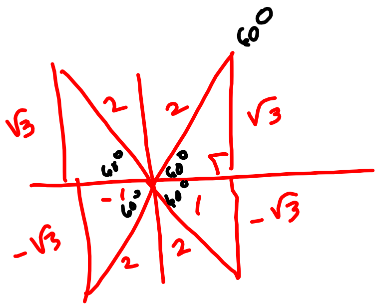
$$\tan^2(x) = 3$$

$$\sqrt{\tan^2(x)} = \sqrt{3}$$

$$|\tan(x)| = \sqrt{3}$$

underlying truths

$$\tan(x) = \pm\sqrt{3}$$



$\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

$\pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$\pi + \frac{\pi}{3} = \frac{4\pi}{3}$

$2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

```
NORMAL  SCI  ENG
FLOAT  0 1 2 3 4 5 6 7 8 9
RADIAN
FUNC  PAR  POL  SEQ
CONNECTED  DOT
SEQUENTIAL  SIMUL
REAL  a+bi  re^θi
FULL  HORIZ  G-T
↓NEXT↓
```

Degrees Mode

```
sin⁻¹(3/(2√(3))
                60
tan⁻¹(√(3))
                60
```

$= \frac{\pi}{3}$ radians

(via $(60^\circ)\left(\frac{\pi}{180^\circ}\right) = \frac{\pi}{3}$

```
NORMAL  SCI  ENG
FLOAT  0 1 2 3 4 5 6 7 8 9
RADIAN  DEGREE
FUNC  PAR  POL  SEQ
CONNECTED  DOT
SEQUENTIAL  SIMUL
REAL  a+bi  re^θi
FULL  HORIZ  G-T
↓NEXT↓
```

```
sin⁻¹(3/(2√(3))
                60
tan⁻¹(√(3))
                60
tan⁻¹(√(3))
       1.047197551
```

Decimal radians suck!

```
                60
tan⁻¹(√(3))
       1.047197551
Ans/π
       .3333333333
Ans▸Frac
               1/3
```

Reverse-engineering decimal radians to exact Pi radians.

Divide out the Pi, and then make a fraction out of the repeating decimal.

Convert .3333333333..... to a fraction:

$x = .333...$

$10x = 3.333...$

$10x - x = 9x = 3$

$\Rightarrow x = \frac{3}{9} = \frac{1}{3}$

13. 0/2 points LarTrig10 2.2.030. [388

Verify the identity algebraically. Use a graphing utility to check your result graphically. (Simplify at each step.)

$$\frac{4\sec(\theta)-4}{1-\cos(\theta)} = 4\sec(\theta)$$

$$\frac{4\sec(\theta)-4}{1-\cos(\theta)} = \left(\frac{4(\sec(\theta)-1)}{1-\frac{1}{\sec(\theta)}}\right)\left(\frac{\boxed{}\ ✗\ \boxed{\sec(\theta)}}{\sec(\theta)}\right)$$

$$= \frac{4\sec(\theta)(\sec(\theta)-1)}{\left(\boxed{}\ ✗\ \boxed{\sec(\theta)}\right)-1}$$

$$= 4\sec(\theta)$$

$$\frac{4\sec\theta-4}{1-\cos\theta}$$

$$= \left(\frac{4\sec\theta-4}{1-\frac{1}{\sec\theta}}\right)\left(\frac{\sec\theta}{\sec\theta}\right)$$

$$= \frac{4\sec^2\theta-4\sec\theta}{\sec\theta-1}$$

$$= \frac{4\sec\theta(\sec\theta-1)}{\sec\theta-1}$$

$$= 4\sec\theta$$

$$\cancel{\frac{\frac{4\sec\theta-4}{\cos\theta}}{\frac{\frac{4}{\cos\theta}-4\cdot\frac{\cos\theta}{\cos\theta}}{\cos\theta}}}$$

$$\cancel{= \frac{\frac{4-4\cos\theta}{\cos\theta}}{\cos\theta}} = \frac{4-4\cos\theta}{\cos^2\theta}$$

Want $4\sec\theta$? /

$$= \frac{4-\frac{4}{\sec\theta}}{\frac{1}{\sec^2\theta}} =$$

Do it in $\sec\theta$ s

$$\cancel{\frac{4\sec\theta-4}{\frac{1}{\sec\theta}} = (4\sec\theta-4)\sec\theta = 4\sec^2\theta-4\sec\theta}$$

$$\frac{4\sec\theta-4}{1-\cos\theta}$$

$$= \frac{4(\sec\theta-1)}{1-\frac{1}{\sec\theta}} = \frac{4(\sec\theta-1)}{\frac{\sec\theta-1}{\sec\theta}} = \frac{4\cancel{(\sec\theta-1)}}{\cancel{\sec\theta-1}}(\sec\theta)$$

$$= 4\sec\theta$$

Link to Cheat Sheet: 2.5 Stuff

https://harryzaims.com/public_html/122/1420-spring-23/cheat-sheet-test-2.pdf

Double-Angle Formulas

$\sin 2u = 2 \sin u \cos u$

$\tan 2u = \dfrac{2 \tan u}{1 - \tan^2 u}$

$\cos 2u = \cos^2 u - \sin^2 u$
$\quad = 2\cos^2 u - 1$
$\quad = 1 - 2\sin^2 u$

Solving a Multiple-Angle Equation
In Exercises 7–14, solve the equation.

7. $\sin 2x - \sin x = 0$
8. $\sin 2x \sin x = \cos x$
9. $\cos 2x - \cos x = 0$
10. $\cos 2x + \sin x = 0$
11. $\sin 4x = -2 \sin 2x$
12. $(\sin 2x + \cos 2x)^2 = 1$
13. $\tan 2x - \cot x = 0$
14. $\tan 2x - 2 \cos x = 0$

$\sin(2x) - \sin(x) = 0$

$2\sin(x)\cos(x) - \sin(x) = 0$

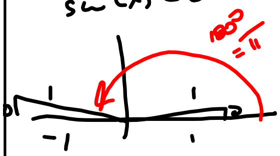
$\sin(x)[2\cos(x) - 1] = 0$

At first, I didn't think they needed to specify where *u* resided. But it matters. Because the sin(*u*), cos(*u*), and tan(*u*) values depend on what quadrant the *u* is in.

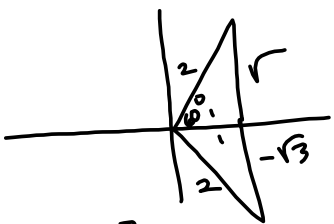
$\sin(x) = 0$ OR $2\cos(x) - 1 = 0$

$2\cos(x) = 1$

$\cos(x) = \frac{1}{2}$



$x = 0, \pi$



$x = \frac{\pi}{3}, \frac{5\pi}{3}$

$x = 60°,\ 360° - 60° = 300°$

All sol'ns in $[0, 2\pi]$:

$0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}$

$0, \pi$: $\boxed{n\pi,\ n \in \mathbb{Z}}$

$\frac{\pi}{3}, \frac{5\pi}{3}$: $\boxed{\begin{array}{l} \frac{\pi}{3} + 2n\pi,\ n \in \mathbb{Z} \\ \frac{5\pi}{3} + 2n\pi,\ n \in \mathbb{Z} \end{array}}$

$A = \{ n\pi \mid n \in \mathbb{Z} \}$

$B = \{ \frac{\pi}{3} + 2n\pi \mid n \in \mathbb{Z} \}$

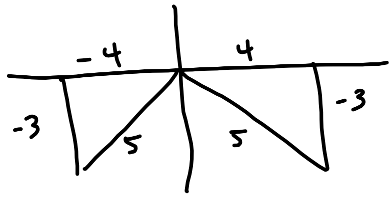
$C = \{ \frac{5\pi}{3} + 2n\pi \mid n \in \mathbb{Z} \}$

Sol'n Set $S = A \cup B \cup C$.

Evaluating Functions Involving Double Angles In Exercises 21–24, use the given conditions to find the exact values of $\sin 2u$, $\cos 2u$, and $\tan 2u$ using the double-angle formulas.

21. $\sin u = -3/5, \quad 3\pi/2 < u < 2\pi$
22. $\cos u = -4/5, \quad \pi/2 < u < \pi$
23. $\tan u = 3/5, \quad 0 < u < \pi/2$
24. $\sec u = -2, \quad \pi < u < 3\pi/2$

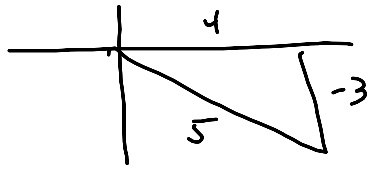
$\sin(u) = -\frac{3}{5}$



$5^2 - 3^2 = 25 - 9$
$= 16 = x^2$
$x = \pm 4$

But they go further & say

$\frac{3\pi}{2} < u < 2\pi$, so:



$\sin(2u) = 2\sin(u)\cos(u)$
$= 2\left(-\frac{3}{5}\right)\left(\frac{4}{5}\right) = \boxed{-\frac{24}{25} = \sin(2u)}$

$\cos(2u) = \cos^2(u) - \sin^2(u)$
$= \left(\frac{4}{5}\right)^2 - \left(-\frac{3}{5}\right)^2$
$= \frac{16-9}{25} = \boxed{\frac{7}{25} = \cos(2u)}$

$\tan(2u) = \frac{\sin(2u)}{\cos(2u)}$
$= -\frac{24}{25} \cdot \frac{25}{7} = \boxed{-\frac{24}{7}} = \tan(2u)$

From Cheat Sheet

$\frac{2\tan u}{1-\tan^2 u} = \frac{2\left(-\frac{3}{4}\right)}{1-\left(-\frac{3}{4}\right)^2} = \frac{-\frac{3}{2}}{1-\frac{9}{16}} = \frac{-\frac{3}{2}}{\frac{16-9}{16}} = \frac{-\frac{3}{2}}{\frac{7}{16}}$

$= -\frac{3}{2} \cdot \frac{16}{7} = \frac{-3(4)}{7} = \boxed{-\frac{24}{7} = \tan(2u)}$

Power-Reducing Formulas

$$\sin^2 u = \frac{1 - \cos 2u}{2}$$

$$\cos^2 u = \frac{1 + \cos 2u}{2}$$

$$\tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}$$

Half-Angle Formulas

$$\sin\frac{u}{2} = \pm\sqrt{\frac{1-\cos u}{2}} \qquad \cos\frac{u}{2} = \pm\sqrt{\frac{1+\cos u}{2}}$$

$$\tan\frac{u}{2} = \frac{1-\cos u}{\sin u} = \frac{\sin u}{1+\cos u}$$

The signs of $\sin\frac{u}{2}$ and $\cos\frac{u}{2}$ depend on the quadrant in which $\frac{u}{2}$ lies.

I'll show you how to figure it out, analytically, and how to cheat it with a calculator.

Using Half-Angle Formulas In Exercises 35–40, use the half-angle formulas to determine the exact values of the sine, cosine, and tangent of the angle.

35. 75° **36.** 165°

37. $112^\circ\ 30'$ **38.** $67^\circ\ 30'$

39. $\pi/8$ **40.** $7\pi/12$

2. (20 pts) Find $\sin\left(\frac{u}{2}\right)$, $\cos\left(\frac{u}{2}\right)$, and $\tan\left(\frac{u}{2}\right)$, given that $\cos(u) = \frac{3}{4}$ and $\frac{3\pi}{2} \le u < 2\pi$

Product-to-Sum Formulas

$$\sin u \sin v = \frac{1}{2}[\cos(u - v) - \cos(u + v)]$$

$$\cos u \cos v = \frac{1}{2}[\cos(u - v) + \cos(u + v)]$$

$$\sin u \cos v = \frac{1}{2}[\sin(u + v) + \sin(u - v)]$$

$$\cos u \sin v = \frac{1}{2}[\sin(u + v) - \sin(u - v)]$$

Sum-to-Product Formulas

$$\sin u + \sin v = 2\sin\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$

$$\sin u - \sin v = 2\cos\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$

$$\cos u + \cos v = 2\cos\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$

$$\cos u - \cos v = -2\sin\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$