4. Consider the equation $3\csc^3(x) - 6\csc^2(x) - 4\csc(x) + 8 = 0$.

$3\csc^2(x)[\csc(x) - 2] - 4[\csc(x) - 2]$

a. (10 pts) Find all solutions x, in radians, to the equation, above, in the interval $[0, 2\pi)$. Give *exact* answers, here. (Hint: Factor by grouping.)

$= [\csc(x) - 2](3\csc^2(x) - 4) \overset{SET}{=} 0$

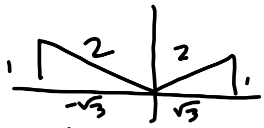
b. (10 pts) Find *all* real solutions x, in radians.

$\Rightarrow \csc(x) = 2$ OR $3\csc^2(x) = 4$

$\sin(x) = \frac{1}{2}$ $\qquad$ $\csc^2(x) = \frac{4}{3}$

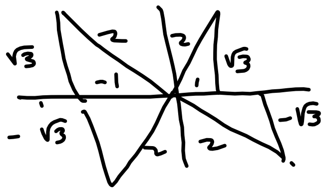
$\csc(x) = \pm\sqrt{\frac{4}{3}} = \frac{\pm 2}{\sqrt{3}}$

$\Rightarrow \sin(x) = \pm\frac{\sqrt{3}}{2}$



$30° \rightsquigarrow \frac{\pi}{6}$

$180° - 30° = 150° = \frac{5\pi}{6}$



$\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

$y \in \left\{ \frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3} \right\} = A$

$x \in \{ y + 2n\pi \mid y \in A, n \in \mathbb{Z} \}$

Alternate observe the Δ's on the right are π apart

so we capture all of them

$A = \{ y + n\pi \mid y \in \{ \frac{\pi}{3}, \frac{2\pi}{3} \}, n \in \mathbb{Z} \}$

$B = \{ y + 2n\pi \mid y \in \{ \frac{\pi}{6}, \frac{5\pi}{6} \}, n \in \mathbb{Z} \}$

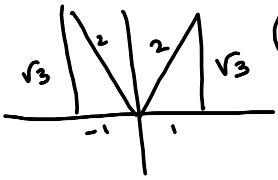
$x \in A \cup B$

6. (10 pts) Solve $\csc^2(x) - 4\csc(x) = -4$. Find all solutions in $[0, 2\pi)$. You may certainly use degrees to "see" things, better, but I expect (require) an exact answer, in radians, as your final answer.

Find all $x \in [-2\pi, 2\pi] \ni \sin(2x) = \frac{\sqrt{3}}{2}$

Find all $2x \in [-4\pi, 4\pi]$

$\sin(2x) = \frac{\sqrt{3}}{2}$



$2x = \frac{\pi}{3}, \frac{2\pi}{3}$. ALSO

$\frac{\pi}{3} + 2\pi = \frac{\pi}{3} + \frac{6\pi}{3} = \frac{7\pi}{3}$

~~$\frac{7\pi}{3} + \frac{6\pi}{3} = \frac{13\pi}{3} > 4\pi$~~

$\frac{\pi}{3} - \frac{6\pi}{3} = -\frac{5\pi}{3}$

$-\frac{5\pi}{3} - \frac{6\pi}{3} = -\frac{11\pi}{3}$

$\frac{2\pi}{3} + \frac{6\pi}{3} = \frac{8\pi}{3}$

$\frac{8\pi}{3} + \frac{6\pi}{3} = \frac{14\pi}{3} > 4\pi$

$\frac{2\pi}{3} - \frac{6\pi}{3} = -\frac{4\pi}{3}$

$-\frac{4\pi}{3} - \frac{6\pi}{3} = -\frac{10\pi}{3}$

$-\frac{10\pi}{3} - \frac{6\pi}{3} = -\frac{16\pi}{3} < 4\pi$

Tristan ez

$2x = \frac{2\pi}{3}, \frac{\pi}{3}, -\frac{5\pi}{3}, -\frac{11\pi}{3}, \frac{8\pi}{3}, \frac{7\pi}{3}$

$\cancel{-\frac{5\pi}{3}}, \cancel{-\frac{11\pi}{3}}, -\frac{4\pi}{3}, -\frac{10\pi}{3}$

$x = -\frac{11\pi}{6}, -\frac{10\pi}{6}, -\frac{5\pi}{6}, -\frac{4\pi}{6},$

$\frac{\pi}{6}, \frac{2\pi}{6}, \frac{7\pi}{6}$

$x \in \{ \dots \quad \}$

FIND ALL SOLUTIONS:

Find all $2x \in [0, 4\pi]$ (we have)

Then add $2n\pi$ to all of those.

Finally, divide by 2.

$2x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{7\pi}{3}, \frac{8\pi}{3} + 2n\pi$ Scratch.

$x = \frac{\pi}{6}, \frac{\pi}{3}, \frac{7\pi}{6}, \frac{4\pi}{3} + n\pi$ Scratch.

$x = \frac{\pi}{6} + n\pi, \frac{\pi}{3} + n\pi, \frac{7\pi}{6} + n\pi, \frac{4\pi}{3} + n\pi$ WebAssign-ish

Set answer: $A = \{\frac{\pi}{6}, \frac{\pi}{3}, \frac{7\pi}{6}, \frac{4\pi}{3}\}$

$x \in \{x + n\pi \mid x \in A, n \in \mathbb{Z}\}$

Use the trigonometric substitution to write the algebraic equation as a trigonometric equation of θ, where $-\pi/2 < \theta < \pi/2$.

$2 = \sqrt{4 - x^2}, \quad x = 2 \sin \theta$

$2 =$ [ ] ✗ $\boxed{2\cos(\theta)}$

See Monday 2/20 notes.

Find $\sin \theta$ and $\cos \theta$. (Enter your answers as a comma-separated list.)

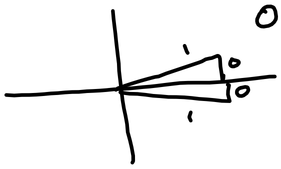
$\sin \theta =$ [ ] ✗ $\boxed{0}$

$\cos \theta =$ [ ] ✗ $\boxed{1}$

$\sqrt{4-x^2} = 2$

$2\cos\theta = 2$

$\cos\theta = 1$



0 or 2π

$-\frac{\pi}{2} < \theta < \frac{\pi}{2} \Rightarrow$

$\theta = 0.$

$\cos\theta = 1, \sin\theta = 0$

$= 10x^2 + 36x - 70$ "Teacher can't f.o.i.l," sez Instan

Factor $10\sin^2(x) + 16\sin(x) - 70$

It's "quadratic in $\sin(x)$."

$10u^2 + 16u - 70$ -700 Magic!

$16 = 17 - 1$ -17

$= 18 - 2$ -36 TOO SMALL

$= 100 - 84$ -8400 TOO BIG

$= 50 - 34$ -1700 HIGHER → -1700 Lower!

$= 75 - 59$ -4425 Lower DUMMY

$\begin{array}{r} 75 \\ 16 \\ \hline 59 \end{array}$

$= 60 - 44$ -2640 Lower

UGH!

$= 25 - 9$ -225 HIGHER

$= 30 - 14$ -420 HIGHER

$= 40 - 24$ -960 Lower!

$= 36 - 20$ -780 Lower!

$= 35 - 19$ -665 HIGHER?!

Need to be very meticulous, Steve.

$10x^2+36x-70$ $(5x-7)(2x+10)$

SLEDGE HAMMER

simplify:

$2(5x^2+18x-35) \overset{SET}{=} 0 \Rightarrow$

$5x^2+18x-35=0$

$a=5, b=18, c=-35$

b^2-4ac = discriminant

$= 18^2-4(5)(-35) = 324 + 700 = 1024$

$= 2^{10} \Rightarrow$

$\sqrt{b^2-4ac} = \sqrt{2^{10}} = 2^5 = 32$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-18 \pm 32}{2(5)} = \frac{-18 \pm 32}{10}$

$\frac{-18+32}{10} = \frac{14}{10} = \frac{7}{5}$

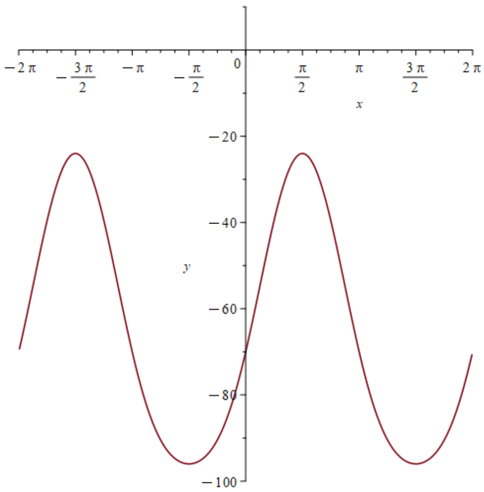
$\frac{-18-32}{10} = \frac{-50}{10} = -5$

$5\left(x-\frac{7}{5}\right)(x-(-5))$

$= (5x-7)(x+5)$

$\Rightarrow f(x) = (2)(5x-7)(x+5)$

2 | 1024
2 | 512
2 | 256
2 | 128
2 | 64
2 | 32
2 | 16
2 | 8
2 | 4
2



$f(\sin(x)) = 2(5\sin(x)-7)(\sin(x)+5)$

If I set $=0$, what's the solution?

4. 0/1 points LarTrig10 2.1.042. [3882417]

Perform the multiplication and use the fundamental identities to simplify. There is more than one correct form of the answer.

$(3 \csc x + 3)(3 \csc x - 3)$

$(a+b)(a-b) = a^2 - b^2$

$= 9\csc^2(x) - 9 =$ Pythagorean Identities.

$= 9(\csc^2(x) - 1)$

$= 9\cot^2(x)$

Angle Sums $\quad \sin(x-y) = \sin(x)\cos(-y) + \sin(-y)\cos(x)$

$= \sin(x)\cos(y) - \sin(y)\cos(x)$

$\sin(x+y) = \sin(x)\cos(y) + \sin(y)\cos(x)$

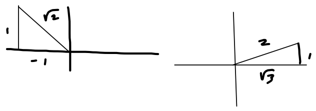
App: Find the exact value of $\sin\left(\frac{11\pi}{12}\right)$

$11 = 10+1 = 9+2$

$\sin\left(\frac{9\pi}{12} + \frac{2\pi}{12}\right)$

$= \sin\left(\frac{3\pi}{4} + \frac{\pi}{6}\right)$

$= \sin\left(\frac{3\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) + \sin\left(\frac{\pi}{6}\right)\cos\left(\frac{3\pi}{4}\right)$



$= \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2} + \frac{1}{2}\cdot\left(-\frac{1}{\sqrt{2}}\right)$

$= \frac{\sqrt{3}-1}{2\sqrt{2}}$ Not simplified radical form, but O.K., unless I specify "simplified radical form"

To simplify:

$\frac{\sqrt{3}-1}{2\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}-\sqrt{2}}{4}$ is simplified.

$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$

$= \sin\left(\frac{\pi}{2} - (x+y)\right)$ by co-function identity

$= \sin\left(\frac{\pi}{2} - x - y\right)$

$= \sin\left(\left(\frac{\pi}{2} - x\right) - y\right)$

$= \sin\left(\frac{\pi}{2} - x\right)\cos(-y) + \sin(-y)\cos\left(\frac{\pi}{2} - x\right)$

$= \cos(x)\cos(y) - \sin(y)\sin(x)$

Double ANGLE

$\sin(2x) = \sin(x+x) = \sin(x)\cos(x) + \sin(x)\cos(x)$

$= 2\sin(x)\cos(x)$

$\cos(2x) = 2\cos^2(x) - 1$

$$\boxed{\sin(2u) = 2\sin(u)\cos(u),\ \cos(2u) = \cos^2(u) - \sin^2(u) = 2\cos^2(u) - 1 = 1 - 2\sin^2(u)}$$

$\cos(2x) = 2\cos^2(x) - 1 \Rightarrow$

$2\cos^2(x) = \cos(2x) + 1$

$\cos^2(x) = \frac{\cos(2x)+1}{2}$

$|\cos(x)| = \sqrt{\frac{\cos(2x)+1}{2}}$

$\cos(x) = \pm\sqrt{\frac{1+\cos(2x)}{2}}$

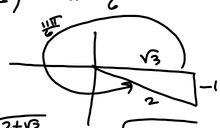
Depends on the quadrant

$\frac{1}{2}$-angle:

$\cos\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1+\cos(u)}{2}}$

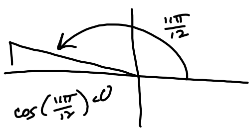
$\cos\left(\frac{11\pi}{12}\right) = \cos\left(\frac{\frac{11\pi}{6}}{2}\right) \quad u = \frac{11\pi}{6}$

$= \pm\sqrt{\frac{1+\cos\left(\frac{11\pi}{6}\right)}{2}}$



$= \pm\sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}} = \pm\sqrt{\frac{\frac{2+\sqrt{3}}{2}}{2}} = \pm\sqrt{\frac{2+\sqrt{3}}{4}} = \pm\frac{\sqrt{2+\sqrt{2}}}{2}$

Now, $\frac{11\pi}{12} \in Q\,II \Rightarrow \cos\left(\frac{11\pi}{12}\right) = -\frac{\sqrt{2+\sqrt{3}}}{2}$



$\therefore \cos\left(\frac{11\pi}{12}\right) < 0$

$\cos\left(\frac{9\pi}{12} + \frac{2\pi}{12}\right) = \cos\left(\frac{3\pi}{4} + \frac{\pi}{6}\right)$

$= \cos\left(\frac{3\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) - \sin\left(\frac{3\pi}{4}\right)\sin\left(\frac{\pi}{6}\right)$

$= -\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}\cdot\frac{1}{2} = \frac{-\sqrt{3}-1}{2\sqrt{2}}$