9. Consider the equation $\tan(2x)=\sqrt{3}$.

   a. Find all solutions $x \in [-2\pi, 2\pi]$. Answers must be exact.

   b. Find all solutions $x \in (-\infty, \infty)$. Report your solutions as a *set*.

2nd version of the solution is much better.

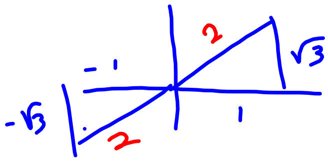
I kind of floundered a bit, at first.

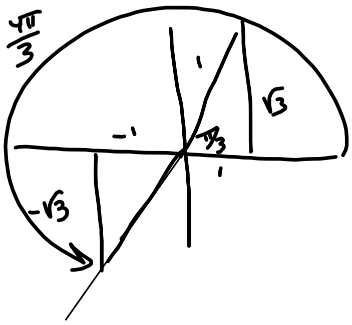
Want all $x \in [-2\pi, 2\pi]$

$-2\pi \le x \le 2\pi$

$\Rightarrow -4\pi \le 2x \le 4\pi$

$\tan(2x) = \sqrt{3}$





$[0, 2\pi]$ sol'n:

$2x = 60^\circ$ OR $\frac{\pi}{3}$

$\Rightarrow x = \frac{\pi}{6}$

$2x = \frac{4\pi}{3}$

$x = \frac{2\pi}{3}$

We need all 2x's between -2π & 2π (inclusive)

No. You're working with 2x's!

$\frac{\pi}{6} + 2\pi = \frac{\pi}{6} + \frac{12\pi}{6} = \frac{13\pi}{6} = 2\pi + \frac{\pi}{6} < 2\pi$

$\frac{13\pi}{6} + \frac{12\pi}{6} = \frac{25\pi}{6} = 4\pi + \frac{\pi}{6}$ Too far.

$\frac{\pi}{6} - 2\pi = \frac{\pi}{6} - \frac{12\pi}{6} = -\frac{11\pi}{6}$

$-\frac{11\pi}{6} - \frac{12\pi}{6} = -\frac{23\pi}{6}$

So all the $\frac{\pi}{6}$'s: $\frac{\pi}{6}, \frac{13\pi}{6}, -\frac{11\pi}{6}, -\frac{23\pi}{6}$

$\frac{2\pi}{3} + 2\pi = \frac{2\pi + 6\pi}{3} = \frac{8\pi}{3} = 2\pi + \frac{2\pi}{3}$

No. $\frac{4\pi}{3}$

$\frac{8\pi}{3} + \frac{6\pi}{3} = \frac{14\pi}{3}$ Too far.

4.

$2x = \frac{\pi}{3}, \frac{4\pi}{3}$ are the $[0, 2\pi]$ answers

Take it to 4π:

$\frac{\pi}{3} + \frac{6\pi}{3} = \frac{7\pi}{3}$

$\frac{7\pi}{3} + \frac{6\pi}{3} = \frac{13\pi}{3}$ Too far

$\frac{\pi}{3} - \frac{6\pi}{3} = -\frac{5\pi}{3}$

$-\frac{5\pi}{3} - \frac{6\pi}{3} = -\frac{11\pi}{3}$

Now $\frac{4\pi}{3}$:

$\frac{4\pi + 6\pi}{3} = \frac{10\pi}{3}$

$\frac{4\pi - 6\pi}{3} = -\frac{2\pi}{3}$

$\frac{-2\pi - 6\pi}{3} = -\frac{8\pi}{3}$

$\frac{(-8-6)\pi}{3} = \frac{-14\pi}{3}$ Too far

$\frac{\pi}{3}, \frac{7\pi}{3}, -\frac{5\pi}{3}, -\frac{11\pi}{3}$

$2x = -\frac{11\pi}{3}, -\frac{8\pi}{3}, -\frac{5\pi}{3}, -\frac{2\pi}{3}, \frac{\pi}{3}, \frac{4\pi}{3}, \frac{7\pi}{3}, \frac{10\pi}{3}$

$x = -\frac{11\pi}{6}, -\frac{4\pi}{3}, -\frac{5\pi}{6}, -\frac{\pi}{3}, \frac{\pi}{6}, \frac{2\pi}{3}, \frac{7\pi}{6}, \frac{5\pi}{3}$

$x \in \left\{ -\frac{11\pi}{6}, -\frac{4\pi}{3}, -\frac{5\pi}{6}, -\frac{\pi}{3}, \frac{\pi}{6}, \frac{2\pi}{3}, \frac{7\pi}{6}, \frac{5\pi}{3} \right\}$

$\frac{4\pi}{3}, \frac{10\pi}{3}, -\frac{2\pi}{3}, -\frac{8\pi}{3}$

$2x = \frac{\pi}{3}, \frac{4\pi}{3}, \frac{7\pi}{3}, \cancel{\frac{13\pi}{3}}, \frac{10\pi}{3},$ captures every $2x$ in $[0, 2\pi]$

$\frac{\pi + 6\pi}{3} = \frac{7\pi}{3}$ $\quad$ $\frac{4\pi}{3} + \frac{6\pi}{3} = \frac{10\pi}{3}$

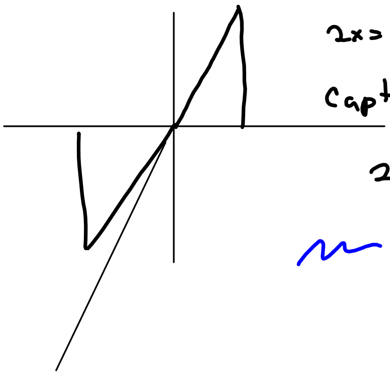
$\frac{7\pi}{3} + \frac{6\pi}{3}$

So $x = \frac{\pi}{6}, \frac{2\pi}{3}, \frac{7\pi}{6}, \frac{10\pi}{3}$

All of these $+ 2n\pi$ will do it.

GOTTA Do the $2x$'s. Add $2n\pi$'s.

THEN divide by 2.



$2x = \frac{\pi}{3}$

capture ALL of THEM:

$2x = \frac{\pi}{3} + 2n\pi, \ n \in \mathbb{Z}.$

$\rightsquigarrow x = \frac{\pi}{6} + n\pi, \ n \in \mathbb{Z}.$

$2x = \frac{4\pi}{3} + 2n\pi,\ n \in \mathbb{Z}.$

$\rightsquigarrow\ x = \frac{2\pi}{3} + n\pi,\ n \in \mathbb{Z}.$

$\frac{2\pi}{3} + \frac{6\pi}{3} = \frac{8\pi}{3}$

$2x = \frac{10\pi}{3} + 2n\pi$

$\rightsquigarrow\ x = \frac{5\pi}{6} + n\pi$

$2x = \frac{7\pi}{3} + 2n\pi$

$\rightsquigarrow\ x = \frac{7\pi}{6} + n\pi$

So, we captured all 2x's in $[0, 4\pi]$. (That gave all the x's in $[0, 2\pi]$.)

Add $2n\pi$ to each

Divide by 2.

$x = \frac{\pi}{6} + n\pi,\ \frac{2\pi}{3} + n\pi,\ \frac{5\pi}{6} + n\pi,\ \frac{7\pi}{6} + n\pi$

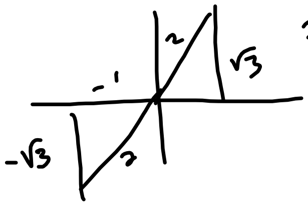
← BOOK ANSWER

A set answer: $\{x + n\pi \mid x = \frac{\pi}{6}, \frac{2\pi}{3}, \frac{5\pi}{6}, \frac{7\pi}{6},\ n \in \mathbb{Z}\}$

$A = \{\frac{\pi}{6}, \frac{2\pi}{3}, \frac{5\pi}{6}, \frac{7\pi}{6}\}$

$S = \{x + n\pi \mid x \in A,\ n \in \mathbb{Z}\}$

$\tan(2x) = \sqrt{3}$



$2x = \frac{\pi}{3}, \frac{4\pi}{3}, \frac{7\pi}{3}, \frac{10\pi}{3}$

$\frac{\pi + 6\pi}{3} = \frac{7\pi}{3}$

$\frac{4\pi + 6\pi}{3} = \frac{10\pi}{3}$

Now, just capture <u>all</u> by adding $2n\pi$ to each:

$2x = \frac{\pi}{3} + 2n\pi, \frac{4\pi}{3} + 2n\pi, \frac{7\pi}{3} + 2n\pi, \frac{10\pi}{3} + 2n\pi$

$x = \frac{\pi}{6} + n\pi, \frac{2\pi}{3} + n\pi, \frac{7\pi}{6} + n\pi, \frac{5\pi}{3} + n\pi$

$x \in \left\{ x + n\pi \;\middle|\; x \in \left\{\frac{\pi}{6}, \frac{2\pi}{3}, \frac{7\pi}{6}, \frac{5\pi}{3}\right\}, n \in \mathbb{Z} \right\}$

NOTE $2x = \frac{\pi}{3}$ & $\frac{4\pi}{3}$ are π apart!

More efficient answer for $2x$ sol'ns:

$2x = \frac{\pi}{3} + n\pi \longrightarrow x \in \left\{ \frac{\pi}{6} + \frac{n\pi}{2} \;\middle|\; n \in \mathbb{Z} \right\}$

$x = \frac{\pi}{6} + \frac{n\pi}{2}$!

14. 0/3 points LarTrig10 2.1.059. [3882149]

Use the trigonometric substitution to write the algebraic equation as a trigonometric equation of θ, where $-\pi/2 < \theta < \pi/2$.

$5 = \sqrt{25 - x^2}$, $x = 5 \sin \theta$

$5 =$ ✗ $\boxed{5\cos(\theta)}$

Find $\sin \theta$ and $\cos \theta$. (Enter your answers as a comma-separated list.)

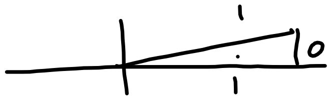
$\sin \theta =$ ✗ $\boxed{0}$

$\cos \theta =$ ✗ $\boxed{1}$

$$\sqrt{25-x^2} = \sqrt{25-(5\sin\theta)^2}$$
$$= \sqrt{25-25\sin^2\theta}$$
$$= \sqrt{25(1-\sin^2\theta)}$$
$$= 5\sqrt{1-\sin^2\theta}$$
$$= 5\sqrt{\cos^2\theta}$$
$$= 5|\cos(\theta)|$$
$$= 5\cos(\theta), \text{ b/c}$$

2nd Part:

$$\sqrt{25-x^2} = 5$$
$$5\cos\theta = 5$$
$$\cos\theta = 1$$



$$\rightarrow \sin\theta = 0$$

Reciprocal Identities

$\sin u = \dfrac{1}{\csc u}$ $\cos u = \dfrac{1}{\sec u}$ $\tan u = \dfrac{1}{\cot u}$

$\csc u = \dfrac{1}{\sin u}$ $\sec u = \dfrac{1}{\cos u}$ $\cot u = \dfrac{1}{\tan u}$

Quotient Identities

$\tan u = \dfrac{\sin u}{\cos u}$ $\cot u = \dfrac{\cos u}{\sin u}$

$\tan\theta = \dfrac{y \cdot \frac{1}{r}}{x \cdot \frac{1}{r}} = \dfrac{\frac{y}{r}}{\frac{x}{r}} = \dfrac{\sin\theta}{\cos\theta}$

Pythagorean Identities

$\sin^2 u + \cos^2 u = 1$ $1 + \tan^2 u = \sec^2 u$ $1 + \cot^2 u = \csc^2 u$

$1+\tan^2\theta = 1 + \dfrac{\sin^2\theta}{\cos^2\theta} = \dfrac{\cos^2\theta}{\cos^2\theta} + \dfrac{\sin^2\theta}{\cos^2\theta} = \dfrac{1}{\cos^2\theta}$

$= \sec^2\theta$

Cofunction Identities

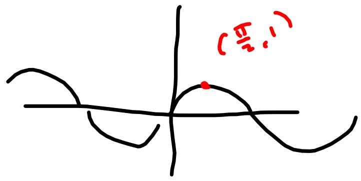
$$\sin\left(\frac{\pi}{2} - u\right) = \cos u \qquad \cos\left(\frac{\pi}{2} - u\right) = \sin u$$

$$\tan\left(\frac{\pi}{2} - u\right) = \cot u \qquad \cot\left(\frac{\pi}{2} - u\right) = \tan u$$

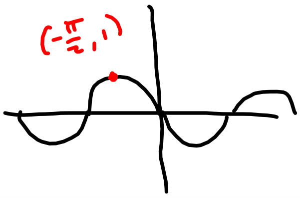
$$\sec\left(\frac{\pi}{2} - u\right) = \csc u \qquad \csc\left(\frac{\pi}{2} - u\right) = \sec u$$

$\sin\left(\frac{\pi}{2} - x\right) = \sin\left(-\left(x - \frac{\pi}{2}\right)\right)$

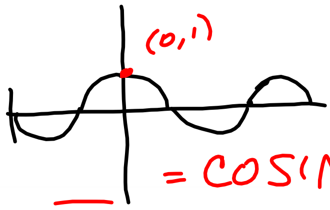
$\sin(x)$



$\sin(-x)$



$\sin\left(-\left(x - \frac{\pi}{2}\right)\right)$



= COSINE!

Even/Odd Identities

$\sin(-u) = -\sin u$

$\csc(-u) = -\csc u$

ODD

EVEN

$\cos(-u) = \cos u$

$\sec(-u) = \sec u$

$\tan(-u) = -\tan u$

$\cot(-u) = -\cot u$

ODD

Using Identities to Evaluate a Function

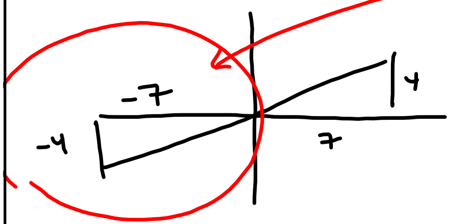
In Exercises 7–12, use the given conditions to find the values of all six trigonometric functions.

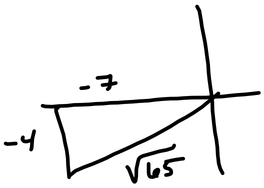
7. $\sec x = -\frac{5}{2}$, $\tan x < 0$
8. $\csc x = -\frac{7}{6}$, $\tan x > 0$
9. $\sin \theta = -\frac{3}{4}$, $\cos \theta > 0$
10. $\cos \theta = \frac{2}{3}$, $\sin \theta < 0$
11. $\tan x = \frac{2}{3}$, $\cos x > 0$
12. $\cot x = \frac{7}{4}$, $\sin x < 0$

$\cot(x) = \frac{7}{4}$

$\tan(x) = \frac{4}{7}$

& $\sin(x) < 0$!





$4^2 + 7^2 = 16 + 49 = 65$

$\sin\theta = -\frac{4}{\sqrt{65}}$ $\csc\theta = \frac{-\sqrt{65}}{4}$

$\cos\theta = \frac{-7}{\sqrt{65}}$ $\sec\theta = -\frac{\sqrt{65}}{7}$

$\tan\theta = \frac{4}{7}$ $\cot\theta = \frac{7}{4}$

Simplifying a Trigonometric Expression

In Exercises 19–22, use the fundamental identities to simplify the expression. (There is more than one correct form of each answer).

19. $\frac{\tan\theta \cot\theta}{\sec\theta} = \frac{1}{\sec\theta} = \cos\theta$

20. $\cos\left(\frac{\pi}{2} - x\right)\sec x = \sin(x)\sec(x) = \frac{\sin(x)}{\cos(x)} = \tan(x)$

21. $\tan^2 x - \tan^2 x \sin^2 x$

22. $\sin^2 x \sec^2 x - \sin^2 x$

$\tan^2(x)\left[1 - \sin^2(x)\right]$

$= \tan^2(x)\left[\cos^2(x)\right]$

$= \frac{\sin^2(x)}{\cos^2(x)} \cdot \cos^2(x) = \sin^2(x)$

Factoring a Trigonometric Expression

In Exercises 23–32, factor the expression. Use the fundamental identities to simplify, if necessary. (There is more than one correct form of each answer.)

23. $\dfrac{\sec^2 x - 1}{\sec x - 1}$ **24.** $\dfrac{\cos x - 2}{\cos^2 x - 4}$

25. $1 - 2\cos^2 x + \cos^4 x$ **26.** $\sec^4 x - \tan^4 x$

27. $\cot^3 x + \cot^2 x + \cot x + 1$

28. $\sec^3 x - \sec^2 x - \sec x + 1$

FACTOR BY GROUPING.

29. $3\sin^2 x - 5\sin x - 2$ **30.** $6\cos^2 x + 5\cos x - 6$

31. $\cot^2 x + \csc x - 1$ **32.** $\sin^2 x + 3\cos x + 3$

23 $\dfrac{\cancel{(\sec(x)-1)}(\sec(x)+1)}{\cancel{\sec(x)-1}}$

$= \boxed{\sec(x)+1}$

25 $1 - 2\cos^2(x) + \cos^4(x)$

$= 1 - 2u^2 + u^4$, where $u = \cos(x)$

$= 1 - 2v + v^2$ where $v = u^2$

$= v^2 - 2v + 1$

$= (v-1)^2 = (u^2-1)^2 = ((u-1)(u+1))^2$

$= ((\cos(x)-1)(\cos(x)+1))^2$

$\sec^4\theta - \tan^4\theta = (\sec^2\theta - \tan^2\theta)(\sec^2\theta + \tan^2\theta)$

$= (\sec\theta - \tan\theta)(\sec\theta + \tan\theta)(\sec^2\theta + \tan^2\theta)$

Simplifying a Trigonometric Expression

In Exercises 33–40, use the fundamental identities to simplify the expression. (There is more than one correct form of each answer.)

33. $\tan\theta\csc\theta$
34. $\tan(-x)\cos x$
35. $\sin\phi(\csc\phi - \sin\phi)$
36. $\cos x(\sec x - \cos x)$
37. $\sin\beta\tan\beta + \cos\beta$
38. $\cot u\sin u + \tan u\cos u$
39. $\dfrac{1 - \sin^2 x}{\csc^2 x - 1}$
40. $\dfrac{\cos^2 y}{1 - \sin y}$

Multiplying Trigonometric Expressions

In Exercises 41 and 42, perform the multiplication and use the fundamental identities to simplify. (There is more than one correct form of each answer.)

41. $(\sin x + \cos x)^2$
42. $(2\csc x + 2)(2\csc x - 2)$

Adding or Subtracting Trigonometric Expressions In Exercises 43–48, perform the addition or subtraction and use the fundamental identities to simplify. (There is more than one correct form of each answer.)

43. $\dfrac{1}{1 + \cos x} + \dfrac{1}{1 - \cos x}$

44. $\dfrac{1}{\sec x + 1} - \dfrac{1}{\sec x - 1}$

45. $\dfrac{\cos x}{1 + \sin x} - \dfrac{\cos x}{1 - \sin x}$

46. $\dfrac{\sin x}{1 + \cos x} + \dfrac{\sin x}{1 - \cos x}$

47. $\tan x - \dfrac{\sec^2 x}{\tan x}$

48. $\dfrac{\cos x}{1 + \sin x} + \dfrac{1 + \sin x}{\cos x}$

Rewriting a Trigonometric Expression **In Exercises 49 and 50, rewrite the expression so that it is *not* in fractional form. (There is more than one correct form of each answer.)**

49. $\dfrac{\sin^2 y}{1 - \cos y}$ **50.** $\dfrac{5}{\tan x + \sec x}$