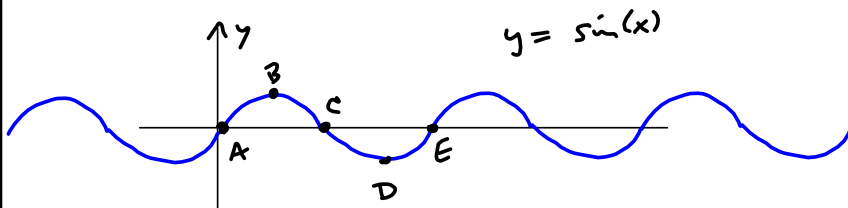
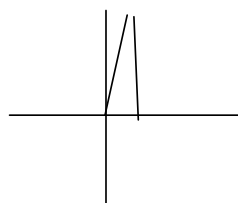
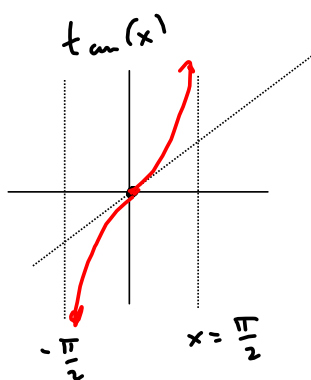
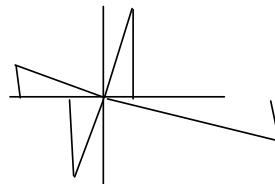
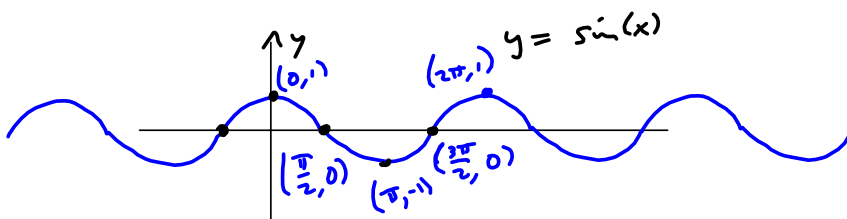


Questions from Chapter 1?

Graphs of trig functions....



- $A = (0, 0)$
- $B = (\frac{\pi}{2}, 1)$
- $C = (\pi, 0)$
- $D = (\frac{3\pi}{2}, -1)$
- $E = (2\pi, 0)$



Inverse Trig Functions

$$\sin^{-1}(y) = \arcsin(y)$$

$$x = \arcsin(y) \text{ means}$$

$$\sin(x) = y$$

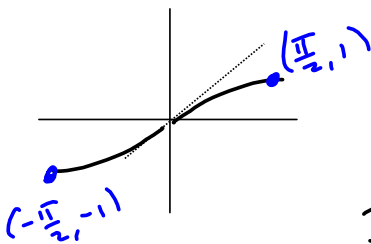
$$\arcsin\left(\frac{1}{2}\right) = 30^\circ$$

$$\text{Note } \sin(150^\circ) = \frac{1}{2}$$

So there's more than one way to obtain

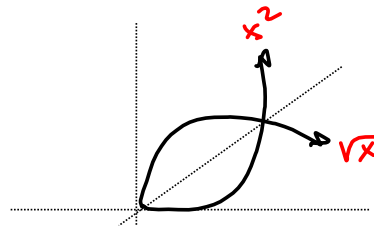
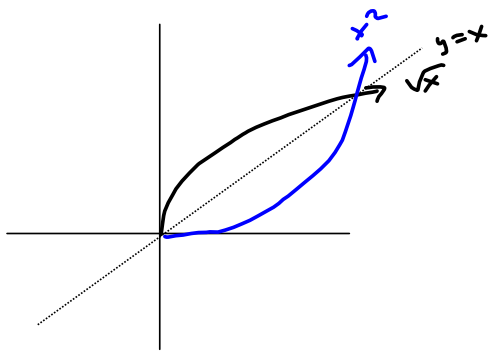
$$\sin(x) = \frac{1}{2}$$

This means we have to restrict  $x$  to make arcsine a function.



If we restrict  
Domain to  $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$   
Then arcsine is a function.

Then we can talk about an  
inverse sine FUNCTION.



Recall:

$$y = \sqrt{x} \iff$$

$$x = \sqrt{y} = x$$

$$\implies (\sqrt{y})^2 = x^2$$

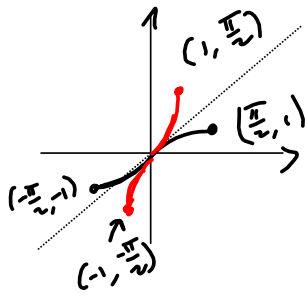
$$y = x^2 = \text{Inverse}$$

$$f(x) = \sqrt{x} \quad \mathcal{R} = [0, \infty)$$

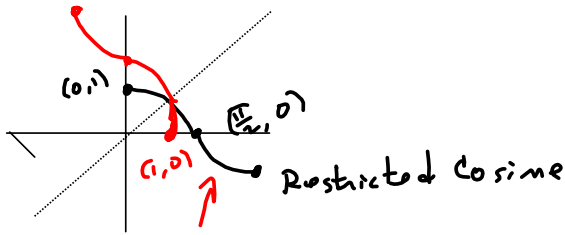
$$\mathcal{D} = [0, \infty)$$

$$\implies f^{-1}(x) = x^2 \quad (\mathcal{D} = [0, \infty))$$

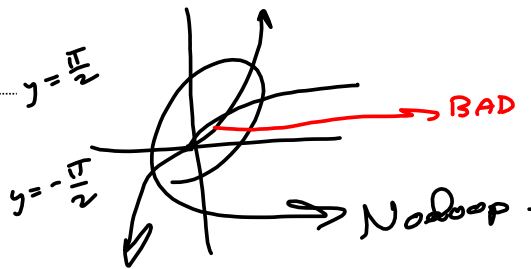
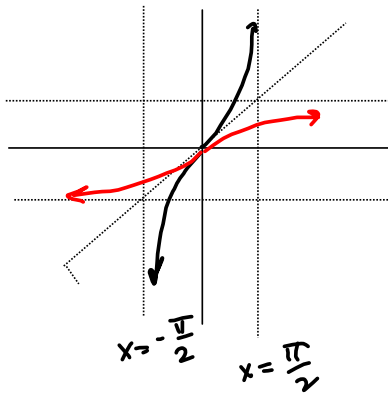
$$\mathcal{R} = [0, \infty)$$



$\sin(x)$  —  
 $\arcsin(x)$  —



Restricted cosine



Restricted tangent:

$\mathcal{D} = (-\frac{\pi}{2}, \frac{\pi}{2})$

$\mathcal{R} = (-\infty, \infty)$

$\text{TAN}^{-1}$ , arctangent

$\mathcal{D} = (-\infty, \infty)$

$\mathcal{R} = (-\frac{\pi}{2}, \frac{\pi}{2})$

Same for sine

$\arcsin, \sin^{-1}$

$\mathcal{D} = [-\frac{\pi}{2}, \frac{\pi}{2}]$

$\mathcal{D} = [-1, 1]$

$\mathcal{R} = [-1, 1]$

$\mathcal{R} = [-\frac{\pi}{2}, \frac{\pi}{2}]$

Same for cosine

$\text{Arccosine}, \cos^{-1}$

$\mathcal{D} = [0, \pi]$

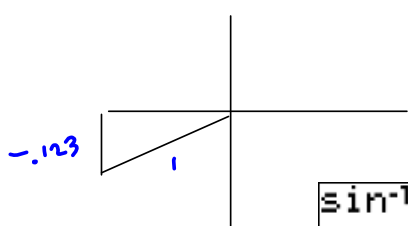
$\mathcal{D} = [-1, 1]$

$\mathcal{R} = [-1, 1]$

$\mathcal{D} = [0, \pi]$

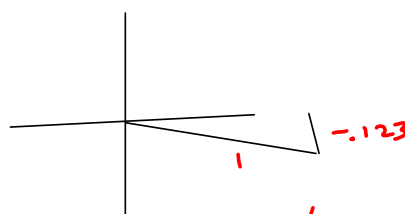
Find the angle in  $Q_{III}$  whose sine is  $-0.123$ .

$$\arcsin(-.123) \approx -7.065272931^\circ$$



```

sin-1(-.123)
-7.065272931
mode: Degrees
  
```



Wrong quadrant!  
 Reference angle  $-7.06^\circ$   
 So in  $Q_{III}$ , that's  $180^\circ + 7.06^\circ$

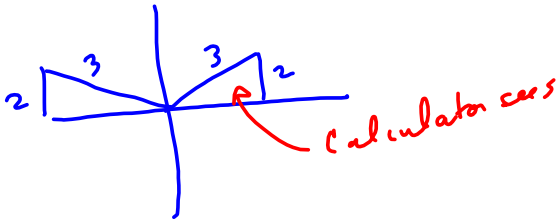
$$\text{So angle} = 187.06^\circ$$

Solving angles with a calculator requires you to interpret and put things in the right quadrant.

Generally, inverse trig functions give you a reference angle, but not THE angle, unless it's from the restricted domain to start with.

Almost EVERY trig equation has two solutions between 0 and 2Pi or between 0 and 360°

Solve  $\sin \theta = \frac{2}{3}$



$x = \arcsin(\frac{2}{3}), 180^\circ - \arcsin(\frac{2}{3})$  Degrees  
 OR  
 $\arcsin(\frac{2}{3}), \pi - \arcsin(\frac{2}{3})$  Radians

<https://harryzaims.com/122/122-fall-20/tests-u-took/122-test-1-fall-20-solutions.pdf>

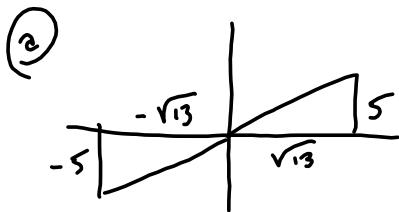


3. Basic concept: Draw the doggone pictures!

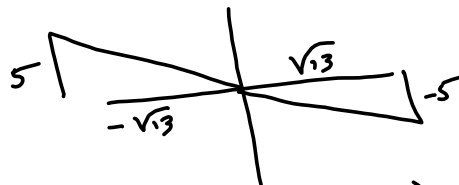
a. (5 points) Sketch two triangles that satisfy  $\tan(\theta) = \frac{5}{\sqrt{13}}$ .

b. (5 pts) Assume the terminal side of the angle  $\theta$  lies in the 3<sup>rd</sup> quadrant. Find the other five trigonometric functions of  $\theta$ .

c. (5 pts) Again, assuming  $\theta$ 's terminal side lies in Q III, and  $0 \leq \theta < 2\pi$ , find  $\theta$ , in radians *and* degrees, rounded to 3 decimal places.



$\tan(\theta) = \frac{5}{\sqrt{13}}$



```

sin^-1(-.123)
-7.065272931
tan^-1(-5/sqrt(13))
-54.20424009
tan^-1(-5/sqrt(13))
-.946042458
    
```

degrees  
 Radians

SOLVE for  $\theta \in [0, 2\pi)$ . In Radians  
 $\arctan(-\frac{5}{\sqrt{13}}) \approx -.946042458 \notin [0, 2\pi)!$

$\theta \approx 2\pi - .9460\dots, \pi - .9460\dots$   
 $\approx 2\pi + \arctan(-\frac{5}{\sqrt{13}}), \pi + \arctan(-\frac{5}{\sqrt{13}})$

$\approx$

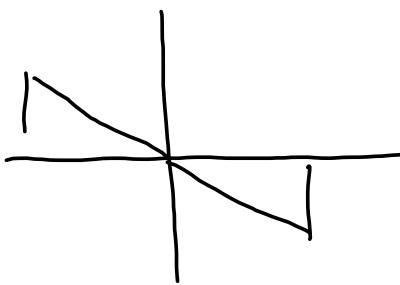
$\tan(\theta) = \frac{5}{\sqrt{13}}$

$\theta = \arctan(\frac{5}{\sqrt{13}}), \pi + \arctan(\frac{5}{\sqrt{13}})$  EXACT

```

-.946042458
Ans+2pi
5.337142849
tan^-1(-5/sqrt(13))
-.946042458
pi+Ans
2.195550196
    
```

FIND ALL SOLUTIONS TO  $\tan \theta = -\frac{5}{\sqrt{13}}$



	-.946042458	
Ans+2 $\pi$	5.337142849	$\approx 5.337$
$\tan^{-1}(-5/\sqrt{13})$	-.946042458	
$\pi$ +Ans	2.195550196	$\approx 2.196$

$$\theta \approx 5.337 + 2n\pi \quad \forall n \in \mathbb{Z}$$

$$\text{OR } \theta \approx 2.196 + 2n\pi \quad \forall n \in \mathbb{Z}$$

$$\text{SLICK: } \boxed{2.196 + n\pi, n \in \mathbb{Z}}$$