

**Double-Angle Formulas:**  $\sin(2u) = 2 \sin(u)\cos(u)$ ,  $\cos(2u) = \cos^2(u) - \sin^2(u) = 2 \cos^2(u) - 1 = 1 - 2 \sin^2(u)$ ,

$$\tan(2u) = \frac{2 \tan(u)}{1 - \tan^2(u)} = \frac{\sin(2u)}{\cos(2u)}$$

**Half-Angle Formulas:**  $\sin\left(\frac{u}{2}\right) = \pm \sqrt{\frac{1 - \cos(u)}{2}}$ ,  $\cos\left(\frac{u}{2}\right) = \pm \sqrt{\frac{1 + \cos(u)}{2}}$ ,  $\tan\left(\frac{u}{2}\right) = \frac{1 - \cos(u)}{\sin(u)} = \frac{\sin\left(\frac{u}{2}\right)}{\cos\left(\frac{u}{2}\right)}$ !!!

You have to determine the "±" by determining the quadrant in which  $\frac{u}{2}$  resides.

**Power-Reducing Formulas:**

**Product-to-Sum Formulas**

$$\sin u \sin v = \frac{1}{2} [\cos(u - v) - \cos(u + v)]$$

$$\cos u \cos v = \frac{1}{2} [\cos(u - v) + \cos(u + v)]$$

$$\sin u \cos v = \frac{1}{2} [\sin(u + v) + \sin(u - v)]$$

$$\sin^2(u) = \frac{1 - \cos(2u)}{2}, \quad \cos^2(u) = \frac{1 + \cos(2u)}{2}, \quad \tan^2(u) = \frac{1 - \cos(2u)}{1 + \cos(2u)} = \frac{\sin^2(u)}{\cos^2(u)}$$

**Sum-to-Product Formulas**

$$\sin u + \sin v = 2 \sin\left(\frac{u + v}{2}\right) \cos\left(\frac{u - v}{2}\right)$$

$$\cos u + \cos v = 2 \cos\left(\frac{u + v}{2}\right) \cos\left(\frac{u - v}{2}\right)$$

$$\cos u - \cos v = -2 \sin\left(\frac{u + v}{2}\right) \sin\left(\frac{u - v}{2}\right)$$

**Angle Sum Formulas**

**Pythagorean Identities**

$$\tan^2(x) + 1 = \sec^2(x)$$

$$\cot^2(x) + 1 = \csc^2(x)$$

$$\sin(u + v) = \sin(u)\cos(v) + \sin(v)\cos(u)$$

$$\cos(u + v) = \cos(u)\cos(v) - \sin(u)\sin(v)$$

$$\tan(u + v) = \frac{\tan(u) + \tan(v)}{1 - \tan(u)\tan(v)}$$

Radians without  $\pi$

1.570796327

3.141592654

4.712388981

6.283185308

**Law of Sines**  $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

**Law of Cosines**  $a^2 = b^2 + c^2 - 2bc \cos A$

**Heron's**  $Area = \sqrt{s(s-a)(s-b)(s-c)}$ , where  $s = \frac{a+b+c}{2}$ . **Arc Length:**  $s = r\theta$ , **Area of a Sector:**  $A = \frac{1}{2}r^2\theta$

**Vector magnitude, dot product and projections from Chapter 3:**

$$\vec{u} = \langle a, b \rangle \text{ and } \vec{v} = \langle c, d \rangle \Rightarrow \vec{u} \bullet \vec{v} = ac + bd, \quad \|\vec{u}\| = \sqrt{a^2 + b^2} = \sqrt{\vec{u} \bullet \vec{u}}, \quad \cos(\theta) = \frac{\vec{u} \bullet \vec{v}}{\|\vec{u}\| \|\vec{v}\|}$$

$$proj_{\vec{v}} \vec{u} = \left( \frac{\vec{u} \bullet \vec{v}}{\|\vec{v}\|^2} \right) \vec{v}$$