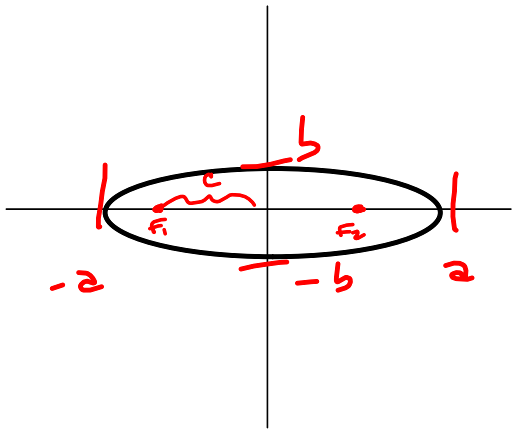
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Ellipse $a > b$

c = focal length

$= \sqrt{a^2 - b^2}$

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$



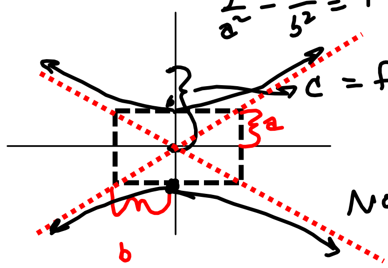
The polar equation of the hyperbola

6.9 #25 $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ is $r^2 = \dfrac{-b^2}{1 - e^2\cos^2\theta}$.

Use the result above to write the polar form of the equation of the conic.

$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$

$\dfrac{y^2}{a^2} - \dfrac{x^2}{b^2} = 1$



c = focal length.

Note $y=0$ is impossible

Hyperbola:

$c^2 = a^2 + b^2$

$=$ focal length

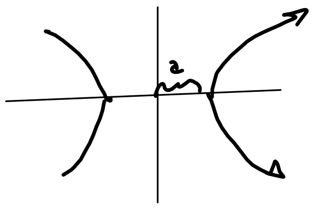
$e = \dfrac{c}{a} =$ eccentricity > 1

Ellipse:

$c^2 = a^2 - b^2$

$e = \dfrac{c}{a} < 1$

$\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$



$\dfrac{x^2}{16} - \dfrac{y^2}{9} = 1$ $a^2 = 16, b^2 = 9$

$r^2 = \dfrac{-9}{1 - e^2\cos^2\theta}$

$= \dfrac{-9}{1 - \frac{25}{16}\cos^2\theta}$

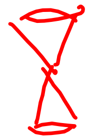
All we need is $e = \dfrac{c}{a}$

$c^2 = a^2 + b^2 = 16 + 9 = 25$

$c = 5$

$\dfrac{c}{a} = \dfrac{5}{4} = e$

Polar Equations of Conics



The graph of a polar equation of the form

1. $r = \dfrac{ep}{1 \pm e\cos\theta}$ or **2.** $r = \dfrac{ep}{1 \pm e\sin\theta}$

is a conic, where $e > 0$ is the eccentricity and $|p|$ is the distance between the focus (pole) and the directrix.

$r = \dfrac{ep}{1 \pm e\sin\theta}$ Horizontal directrix

$r = \dfrac{ep}{1 \pm e\cos\theta}$ Vertical directrix

1. Horizontal directrix above the pole: $r = \dfrac{ep}{1 + e\sin\theta}$
2. Horizontal directrix below the pole: $r = \dfrac{ep}{1 - e\sin\theta}$
3. Vertical directrix to the right of the pole: $r = \dfrac{ep}{1 + e\cos\theta}$
4. Vertical directrix to the left of the pole: $r = \dfrac{ep}{1 - e\cos\theta}$

17. 0/1 points LarTrig10 6.9.0

Find a polar equation of the indicated conic in terms of *r* with the given characteristics and focus at the pole.

Conic	Eccentricity	Directrix
Parabola	$e = 1$	$x = -1$

$r = \dfrac{1}{1 - \cos(\theta)}$

To the left

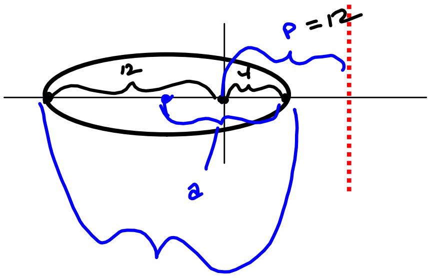
$e = 1,\ p = 1.$

$r = \dfrac{ep}{1 - e\cos\theta} = \dfrac{1 \cdot 1}{1 - \cos\theta}$

Find a polar equation of the conic in terms of r with its focus at the pole.

6.9 #20

Conic	Vertices
Ellipse	$(4, 0), (12, \pi)$ $= (r, \theta)$



Length of major axis is 12 + 4 = 16, so that 2a = 16 --> a = 8

p = distance from focus to directrix =

$r = \frac{ep}{1+e\cos\theta}$

$2a$ = major axis length

$e = c/a$, where $c = \sqrt{a^2 - b^2}$

$c = 4$, using 2 of the 2 points

$a = 8 \Rightarrow$

$e = \frac{c}{a} = \frac{4}{8} = \frac{1}{2}$

$r = \frac{\frac{1}{2}p}{1 - \frac{1}{2}\cos\theta} = \frac{\frac{1}{2}p}{1+\frac{1}{2}\cos\theta}$

$r(0) = 4 \Rightarrow +$

$\frac{\frac{1}{2}p}{1 - \frac{1}{2}(1)} = 4$ $+$

$\frac{\frac{1}{2}p}{\frac{1}{2}} = 4$ $\frac{3}{2}$

$\frac{\frac{1}{2}p}{\frac{3}{2}} = \left(\frac{1}{2}p\right)\left(\frac{2}{3}\right) = \frac{1}{3}p = 4$

$p = 12$

$\frac{1}{2}p = 4\left(\frac{3}{2}\right) = 6$

$\boxed{p = 12}$

$r = \frac{ep}{1+e\cos\theta} = \frac{\frac{1}{2}(12)}{1+\frac{1}{2}\cos\theta}$

$= \frac{6}{1+\frac{1}{2}\cos\theta}$

$= \frac{12}{2+\cos\theta}$

$r(0) = 4$, $r(\pi) = 12$

$\frac{ep}{1+e\cos(0)} = 4$

$\frac{ep}{1+e} = 4$

$ep = 4 + 4e$

$\frac{ep}{1+e\cos(\pi)} = 12$

$\frac{ep}{1-e} = 12$

$ep = 12 - 12e$

$4 + 4e = 12 - 12e$

$16e = 8$

$\boxed{e = \frac{1}{2}}$

Charlie, with his "r=" nonsense is very annoying.

$\frac{ep}{1+e} = 4$

$\frac{\frac{1}{2}p}{1+\frac{1}{2}} = 4$

$\frac{1}{2}p = 4\left(\frac{3}{2}\right)$

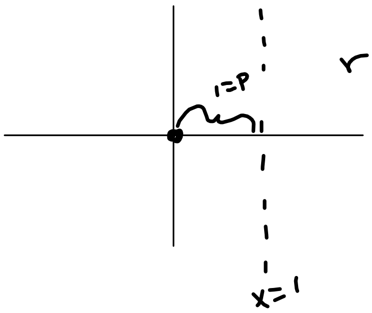
$p = 4(3) = 12$

Find a polar equation of the indicated conic in terms of r with the given characteristics and focus at the pole.

#19

Conic	Eccentricity	Directrix
Hyperbola	$e = 2$	$x = 1$

to the right



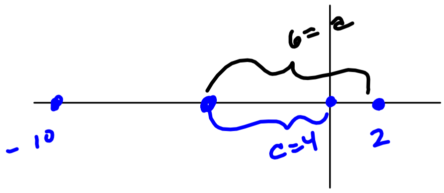
$$r = \frac{eD}{1+e\cos\theta} = \frac{2(1)}{1+2\cos\theta}$$

#16 - rotated hyperbola:

$$r = \frac{6}{2+6\sin(\theta + \frac{2\pi}{3})}$$

$$r(0) = \frac{6}{2+6\sin(\frac{2\pi}{3})} = \frac{6}{2+6(\frac{\sqrt{3}}{2})} = \frac{6}{2+3\sqrt{3}} \approx$$

Ellipse $(2,0)$, $(10,\pi)$ Another version



$a = 6$

$$e = \frac{c}{a} = \frac{4}{6} = \frac{2}{3}$$

$$r = \frac{ep}{1+e\cos\theta}$$

Directrix vertical to the right.

$$r = \frac{\frac{2}{3}p}{1+\frac{2}{3}\cos\theta}$$

$$r(0) = 2 = \frac{\frac{2}{3}p}{1+\frac{2}{3}} = \frac{\frac{2}{3}p}{\frac{5}{3}} = \frac{3}{5}\cdot\frac{2}{3}p = \frac{2}{5}p$$

& so $\frac{2}{5}p = 2$

$\Rightarrow p = 2(\frac{5}{2}) = 5$

$$\Rightarrow r = \frac{ep}{1+e\cos\theta} = \frac{(\frac{2}{3})(5)}{1+\frac{2}{3}\cos\theta}$$

The other way:

$$r(0) = \frac{ep}{1+e\cos(0)} = 2 = \frac{ep}{1+e} \Rightarrow ep = 2+2e$$

$$r(\pi) = \frac{ep}{1+e\cos(\pi)} = \frac{ep}{1-e} = 10 \Rightarrow ep = 10-10e$$

$ep = ep \Rightarrow$

$2+2e = 10-10e$

$12e = 8$

$$e = \frac{8}{12} = \boxed{\frac{2}{3} = e}$$

$$\Rightarrow r(0) = \frac{\frac{2}{3}p}{1+\frac{2}{3}} = 2$$

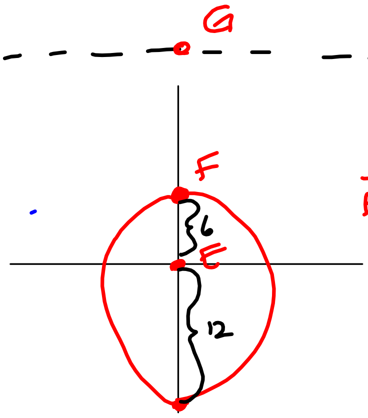
solve for p.

21. 0/1 points LarTrig10 6.9.048. [3884566]

Find a polar equation of the indicated conic in terms of r with the given characteristics and focus at the pole.

Conic	Vertices
Ellipse	$\left(6, \frac{\pi}{2}\right), \left(12, \frac{3\pi}{2}\right)$

$r = \dfrac{24}{\sin(\theta) + 3}$



$e = \frac{1}{3}$

$\frac{EF}{FG} = \frac{1}{3}$

Want $r\left(\frac{\pi}{2}\right) = 6$

$r\left(\frac{3\pi}{2}\right) = 12$

$$r = \frac{ep}{1 + e\sin\theta}$$

$$r\left(\frac{\pi}{2}\right) = \frac{ep}{1 + e\sin\frac{\pi}{2}} = 6$$

$$\frac{ep}{1+e} = 6$$

$$ep = 6(1+e)$$

$$e = \frac{6(1+e)}{p}$$

$$r\left(\frac{3\pi}{2}\right) = \frac{ep}{1 + e\sin\left(\frac{3\pi}{2}\right)} = 12$$

$$\frac{ep}{1-e} = 12$$

$$\frac{\frac{6(1+e)}{p}\, p}{1-e} = 12$$

$$\frac{6+6e}{1-e} = 12$$

$$6 + 6e = 12 - 12e$$

$$18e = 6$$

$$e = \frac{1}{3}$$

$$\frac{ep}{1+e} = \frac{\frac{1}{3}p}{1+\frac{1}{3}} = 6$$

$$\frac{\frac{1}{3}p}{\frac{4}{3}} = 6$$

$$\frac{1}{4}p = 6$$

$$p = 24$$

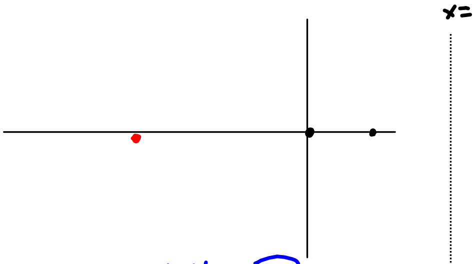
$$r = \frac{\left(\frac{1}{3}\right)(24)\,(3)}{\left(1 + \frac{1}{3}\sin\theta\right)(3)} = \frac{24}{3 + \sin\theta}$$

20. 0/1 points

Find a polar equation of the conic in terms of r with its focus at the pole.

Conic	*Vertices*
Ellipse	(4, 0), (12, π)

$r = \dfrac{12}{\cos(\theta) + 2}$



$$\frac{ep}{1+e\cos\theta}$$

$r(0) = 4 \Rightarrow$

$$\frac{ep}{1+e} = 4$$

$$r(\pi) = \frac{ep}{1-e} = 12$$

$$\left.\begin{aligned} ep &= 4+4e \\ ep &= 12-12e \end{aligned}\right\} \Rightarrow 4+4e = 12-12e$$

$16e = 8$

$\boxed{e = \frac{1}{2}}$

$\Rightarrow \frac{1}{2}p = 4 + 2 = 6$

$\Rightarrow \boxed{p = 12}$

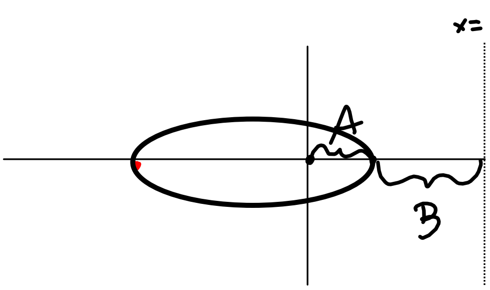
$$r = \frac{\frac{1}{2}(12)}{1+\frac{1}{2}\cos\theta} = \frac{12}{2+\cos\theta}$$

$$= \frac{6}{1+\frac{1}{2}\cos\theta}$$

Directrix for hand sketch?

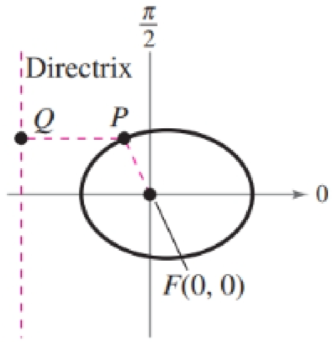
p = Distance from pole to directrix = 12

$e = \frac{1}{2}$



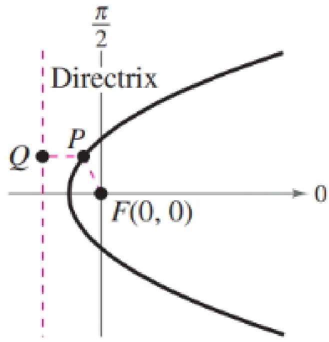
$\frac{A}{B} = e = \frac{1}{2}$?

$\left.\begin{aligned} A &= 4 \\ B &= 8 \end{aligned}\right\} \frac{4}{8} = \frac{1}{2}$ ✓



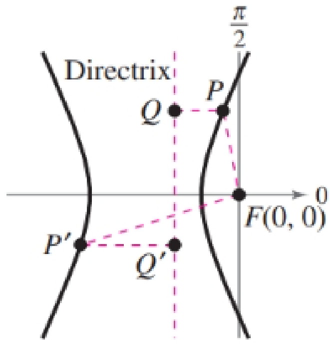
Ellipse: $0 < e < 1$

$\frac{PF}{PQ} < 1$



Parabola: $e = 1$

$\frac{PF}{PQ} = 1$

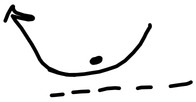


Hyperbola: $e > 1$

$\frac{PF}{PQ} = \frac{P'F}{P'Q'} > 1$

$r = \frac{1}{1-\sin\theta}$ -vs- $\frac{-1}{1-\sin\theta}$

parabola, H.D. BELOW POLE

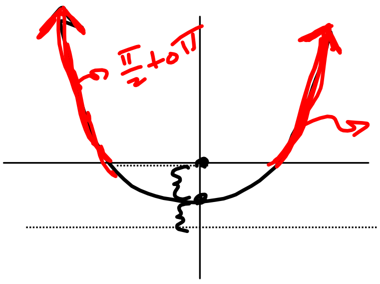


$\frac{1}{1 - 1\sin\theta}$ ← $ep = 1$, ↑ $e = 1$

$p = 1$ gives $y = -1$

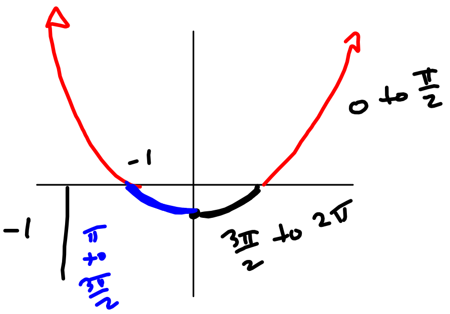
and

$e = 1$ means vertex @ $(x,y) = (0, -\frac{1}{2})$



$r(\frac{\pi}{2}) = \frac{1}{1-\sin\frac{\pi}{2}} = \frac{1}{0} = \infty$?!

$r(0) = \frac{1}{1-0} = 1$



$\frac{1}{1-\sin\frac{5\pi}{4}} = \frac{1}{1-(-\frac{\sqrt{2}}{2})}$

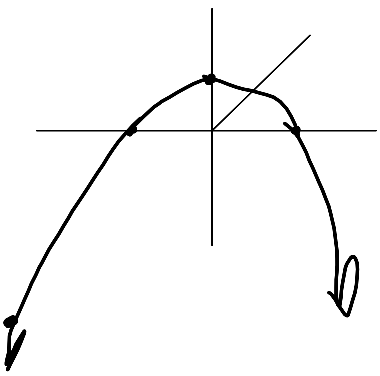
$\frac{1}{1-\sin\frac{3\pi}{2}} = \frac{1}{1-(-1)} = \frac{1}{2}$

$\frac{1}{1+\frac{\sqrt{2}}{2}} \approx \frac{1}{1+.4} = \frac{1}{2.4} = \frac{10}{24}$

$= \frac{5}{12}$ → .5857864376

.707...

$r = \frac{1}{1-\sin\theta}$ -vs- $\frac{-1}{1-\sin\theta}$



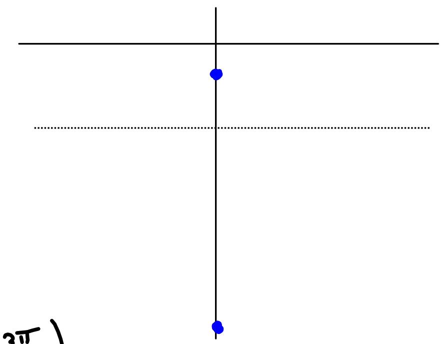
$r(\frac{\pi}{4}) = \frac{-1}{1-\sin\frac{\pi}{4}} = \frac{-1}{.3} = -1.\overline{3}$

Same parabola, up-side-down

50. Hyperbola (2, 0), (8, 0)
51. Hyperbola $(1, 3\pi/2)$, $(9, 3\pi/2)$
52. Hyperbola $(4, \pi/2)$, $(1, \pi/2)$

51 $\left(1, \frac{3\pi}{2}\right), \left(9, \frac{3\pi}{2}\right) = \left(-9, \frac{\pi}{2}\right)$?

$\left(1, \frac{3\pi}{2}\right) = \left(-1, \frac{\pi}{2}\right)$?



$$\frac{ep}{1 - e\sin\theta}$$

$\theta = \frac{3\pi}{2}$

$$\frac{ep}{1 - e\sin\left(\frac{3\pi}{2}\right)} = \frac{ep}{1+e} = 1 \to -1$$

$$\frac{ep}{1 - e\sin\frac{3\pi}{2}} = 9 ?$$

$\left(1, \frac{3\pi}{2}\right)$

OR IS IT

$\left(-1, \frac{\pi}{2}\right)$

$$\frac{ep}{1+e} = 9$$

$$\frac{ep}{1-e} = -1 \qquad \left(\text{from } \frac{ep}{1 - e\sin\frac{\pi}{2}} = -1\right)$$

$$\frac{ep}{1+e} = 9 \Rightarrow ep = 9 + 9e$$

$$\frac{ep}{1-e} = -1 \Rightarrow ep = -1 + e$$

$9 + 9e = -1 + e$

$8e = -10$

$e = \frac{-10}{8}$?!

$= -\frac{5}{4}$?

$e < 0$?

alternate

$\left(9, \frac{3\pi}{2}\right) = \left(-9, \frac{\pi}{2}\right)$

$r\left(\frac{\pi}{2}\right) = -9$

$\frac{ep}{1-e\sin\left(\frac{\pi}{2}\right)} = -9$

$\frac{ep}{1-e\cdot 1} = \frac{ep}{1-e} = -9$

$ep = -9+9e$

$-9+9e = 1+e$

$8e = 10$

$e = \frac{10}{8} = \frac{5}{4}$

$r\left(\frac{3\pi}{2}\right) = 1$

$\frac{ep}{1-e\sin\left(\frac{3\pi}{2}\right)} = 1$

$\frac{ep}{1+e} = 1$

$ep = 1+e$

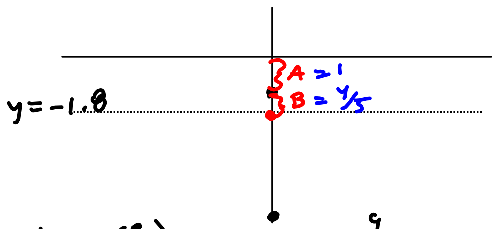
$\left(\frac{5}{4}\right)p = 1+\frac{5}{4} = \frac{9}{4}$

$p = \left(\frac{9}{4}\right)\left(\frac{4}{5}\right) = \boxed{\frac{9}{5} = p}$

$\frac{A}{B} = \frac{5}{4}$

$\frac{1}{B} = \frac{5}{4}$

$\frac{4}{5} = B$

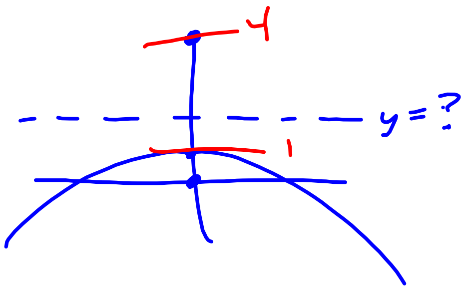


$r = \frac{ep}{1-e\sin\theta} = \frac{(5/4)\left(\frac{9}{5}\right)}{1-\frac{5}{4}\sin\theta} = \frac{\frac{9}{4}}{1-\frac{5}{4}\sin\theta} = \frac{9}{4-5\sin\theta}$

$r\left(\frac{\pi}{2}\right) = \frac{\frac{9}{4}}{1-\frac{5}{4}} = \frac{\frac{9}{4}}{-\frac{1}{4}} = -9$

$r\left(\frac{3\pi}{2}\right) = \frac{\frac{9}{4}}{1-\frac{5}{4}(-1)} = \frac{\frac{9}{4}}{1+\frac{5}{4}} = \frac{\frac{9}{4}}{\frac{9}{4}} = 1$

$\left(4, \frac{\pi}{2}\right), \left(1, \frac{\pi}{2}\right)$ OR $\left(-4, \frac{3\pi}{2}\right), \left(1, \frac{\pi}{2}\right)$



Above pole

$\frac{ep}{1+e\sin\theta}$

$r\left(\frac{3\pi}{2}\right) = -4$ $\qquad$ $r\left(\frac{\pi}{2}\right) = 1$

$\frac{ep}{1+e\sin\left(\frac{3\pi}{2}\right)} = -4$ $\qquad$ $\frac{ep}{1+e\sin\left(\frac{\pi}{2}\right)} = 1$

$\frac{ep}{1-e} = -4$ $\qquad$ $\frac{ep}{1+e} = 1$

$ep = -4+4e = 1+e$

$3e = 5$

$\boxed{e = \frac{5}{3}} \Longrightarrow \frac{\frac{5}{3}p}{1-\frac{5}{3}} = -4$

$\frac{\frac{5}{3}p}{1+\frac{5}{3}\sin\theta}$ $\qquad$ $\frac{\frac{5}{3}p}{-\frac{2}{3}} = -4$

$\frac{5}{3}p = \left(\frac{2}{3}\right)(4)$

$p = \frac{3}{5}\left(\frac{2}{3}\right)(4) = \boxed{\frac{8}{5} = p}$

$r = \frac{ep}{1+e\sin\theta} = \frac{\left(\frac{5}{3}\right)\left(\frac{8}{5}\right)}{1+\frac{5}{3}\sin\theta}$

$\frac{\frac{40}{5}}{3+5\sin\theta} = \frac{8}{3+5\sin\theta}$