12	Sec 4.4, 4.5 Test 4 Covers Chapters 1 - 4, Due Sunday, 4/16	11/6
13	Secs 6.6, 6.7	11/13
14	Secs 6.8, 6.9	11/20
15	Sec 6.9 Final Test Part 1, Covers Chapter 6, Due Sunday, 12/3	11/27
	Final Test Part II, Comprehensive Chapters 1 - 4	

Do Section 4.4	Mon, Nov 13, 2023, 10:59 PM PST
Practice Test 4	Mon, Nov 13, 2023, 10:59 PM PST
Test 4	Tue, Nov 14, 2023, 10:59 PM PST

Section 6.6 - Parametric Equations

Definition of Plane Curve

If f and g are continuous functions of t on an interval I, then the set of ordered pairs $(f(t), g(t))$ is a **plane curve** C. The equations

$$x = f(t) \quad \text{and} \quad y = g(t)$$

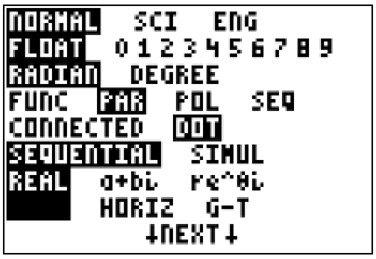
are **parametric equations** for C, and t is the **parameter.**

Consider the parametric equations $x = 4\cos^2\theta$ and $y = 2\sin\theta$.

(a) Create a table of x- and y-values using $\theta = -\pi/2, -\pi/4, 0, \pi/4$, and $\pi/2$.

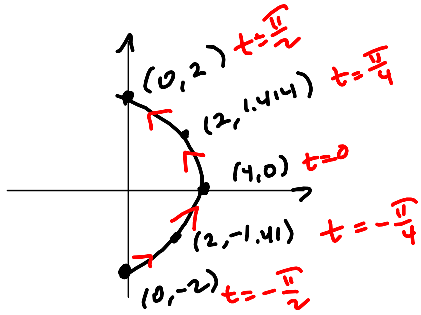
(b) Plot the points (x, y) generated in part (a), and sketch a graph of the parametric equations.

(c) Sketch the graph of $x = -y^2 + 4$. How do the graphs differ?



T	X_{1T}	Y_{1T}
-1.571	0	-2
-.7854	2	-1.414
0	4	0
.7854	2	1.4142
1.5708	0	2
2.3562	2	1.4142
3.1416	4	0

Press + for △Tbl



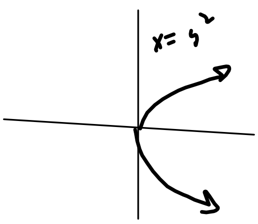
Eliminate the Parameter

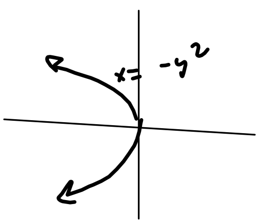
$x = 4\cos^2\theta,\ y = 2\sin\theta \longrightarrow$

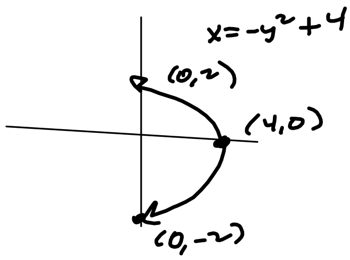
$x = 4(1-\sin^2\theta),\ \frac{y}{2} = \sin\theta \longrightarrow$

$x = 4\left(1-\left(\frac{y}{2}\right)^2\right)$

$= 4 - 4\left(\frac{y^2}{4}\right) = 4 - y^2 = x$



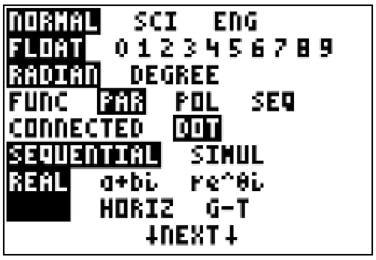




Example 1

Sketch and describe the orientation of the curve given by the parametric equations

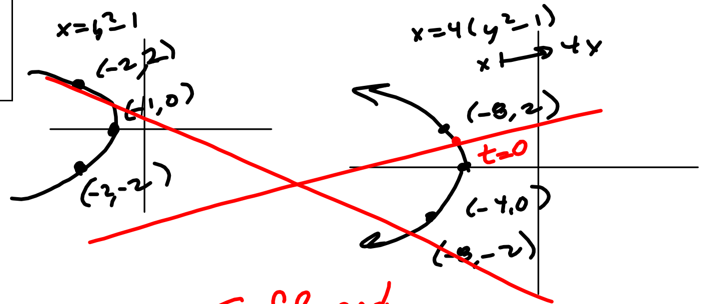
$$x = t^2 - 4 \quad \text{and} \quad y = \frac{t}{2}, \quad -2 \le t \le 3.$$



Eliminate Parameter:

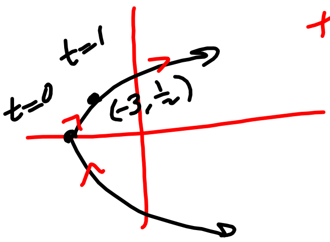
$t = 2y$

$x = t^2 - 4 = (2y)^2 - 4 = 4y^2 - 4 = 4(y^2 - 1)$



I flipped it left-to-right, like the previous!

t	x (t^2-4)	y ($\frac{t}{2}$)
-2	0	-1
-1	-3	$-\frac{1}{2}$
0	-4	0
1	-3	$\frac{1}{2}$
2	0	1
3	5	$\frac{3}{2}$



Sketching a Curve In Exercises 13–38, (a) sketch the curve represented by the parametric equations (indicate the orientation of the curve) and (b) eliminate the parameter and write the resulting rectangular equation whose graph represents the curve. Adjust the domain of the rectangular equation, if necessary.

$x = \sqrt{t}, \quad y = 1 - t$

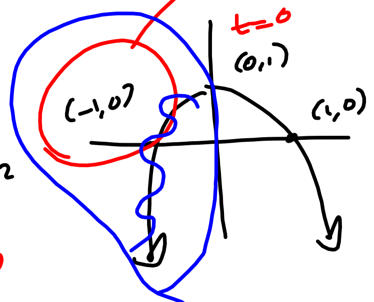
$\Rightarrow x^2 = t \Rightarrow y = 1 - x^2$

NOTE! $\sqrt{t} \Rightarrow t \geq 0$

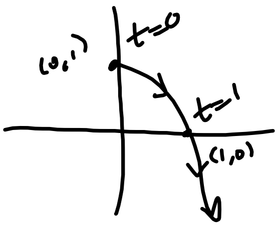
Nevah!

$x = \sqrt{t}$ &

$x^2 = t$



Nope. $t \geq 0$!



$y = \dfrac{t}{t+1}$ $\qquad$ $y = \dfrac{t}{t-1}$

$x = 1 + \cos\theta$

$y = 1 + 2\sin\theta$

"Obviously," this is an ellipse.

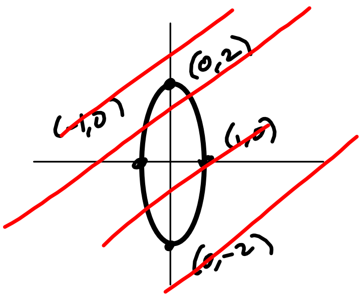
$\sin^2\theta + \cos^2\theta = 1$

$$\Longrightarrow \left.\begin{array}{l} x-1 = \cos\theta \\ \dfrac{y-1}{2} = \sin\theta \end{array}\right\} \Longrightarrow \cos^2\theta + \sin^2\theta = (x-1)^2 + \left(\frac{y-1}{2}\right)^2 = 1$$

$$\Longrightarrow \frac{(x-1)^2}{1^2} + \frac{(y-1)^2}{2^2} = 1$$

This is not centered @ (1,1), like

$(x-1)^2 + \dfrac{(y-1)^2}{2^2} = 1$



$\cos(0) = 1$

$x - 1 = 1$

$x = 0$

$x = 1 + \cos\theta$

$x = 1 + \cos(0)$

$= 2$

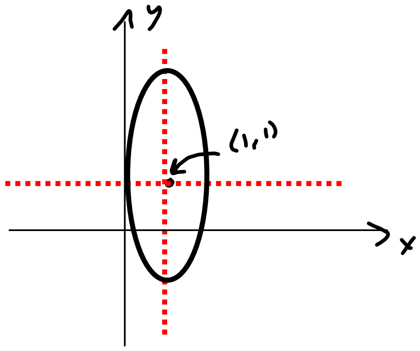
$1 + 2\sin(0) = y$

$= 1$

B.

$(h,k) = (1,1)$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)}{b^2} = 1$$



Eliminating the Parameter **In Exercises 51–54, eliminate the parameter and obtain the standard form of the rectangular equation.**

51. Line passing through (x_1, y_1) and (x_2, y_2):
$x = x_1 + t(x_2 - x_1), \quad y = y_1 + t(y_2 - y_1)$

52. Circle: $x = h + r\cos\theta, \quad y = k + r\sin\theta$

53. Ellipse with horizontal major axis:
$x = h + a\cos\theta, \quad y = k + b\sin\theta$

54. Hyperbola with horizontal transverse axis:
$x = h + a\sec\theta, \quad y = k + b\tan\theta$

$\sec^2\theta = \tan^2\theta + 1$

Parametric Equations of the line SEGMENT from (x_1,y_1) to (x_2,y_2)

$x = x_1 + t(x_2 - x_1)$

$y = y_1 + t(y_2 - y_1)$

for $0 \le t \le 1$

$t=0$: (x_1,y_1)

$t=1$: (x_2,y_2)

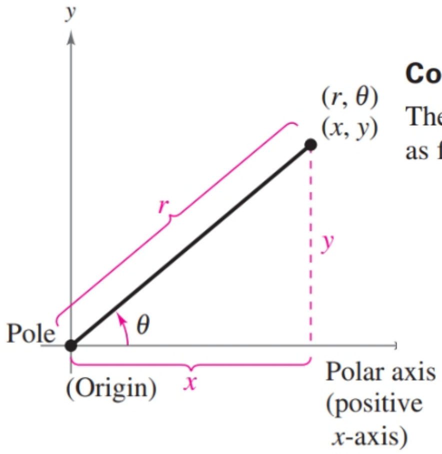
52 Circle:

$x-h = r\cos\theta \qquad y-k = r\sin\theta$

$\frac{x-h}{r} = \cos\theta \qquad \frac{y-k}{r} = \sin\theta$

$$\frac{(x-h)^2}{r^2} + \frac{(y-k)^2}{r^2} = 1$$

Section 6.7 - Polar Coordinates



Coordinate Conversion

The polar coordinates (r, θ) and the rectangular coordinates (x, y) are related as follows.

Polar-to-Rectangular	**Rectangular-to-Polar**
$x = r\cos\theta$	$\tan\theta = \dfrac{y}{x}$
$y = r\sin\theta$	$r^2 = x^2 + y^2$

$(r, \theta) = (r, \theta + 2n\pi)$ $n \in \mathbb{Z}$

So the representation is <u>NOT</u>

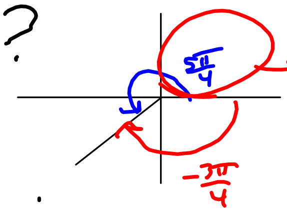
Find three additional polar representations of the point, using $-2\pi < \theta < 2\pi$. (Enter your answers in order from smallest to largest first by r-value, then by θ-value.)

$(r, \theta) = \left(-3 , -\frac{7\pi}{4} \right)$

$(r, \theta) = \left(-3 , \frac{\pi}{4} \right)$!?

$(r, \theta) = \left(3 , -\frac{3\pi}{4} \right)$

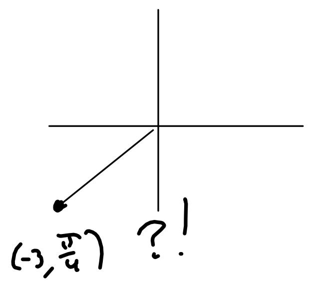
$(r, \theta) = (3, \frac{5\pi}{4})$

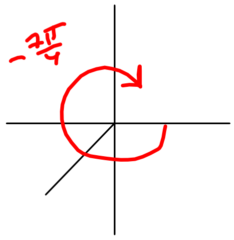


→ No. 3

ADDITIONAL

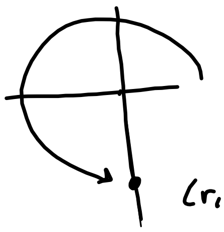
representations.





$(3\sqrt{2}, 4.71)$

$\frac{3\pi}{2} \approx 4.71$



$(r, \theta) = (3\sqrt{2}, 4.71)$

$(-3\sqrt{2}, \frac{\pi}{2}) \approx (-3\sqrt{2}, 4.71 - \pi)$

$(3\sqrt{2}, -\frac{\pi}{2}) \approx (3\sqrt{2},$

$(3\sqrt{2}, 4.71 - 2\pi)$

Converting a Rectangular Equation to Polar Form In Exercises 59–78, convert the rectangular equation to polar form. Assume $a > 0$.

59. $x^2 + y^2 = 9$	**60.** $x^2 + y^2 = 16$
61. $y = x$	**62.** $y = -x$
63. $x = 10$	**64.** $y = -2$
65. $3x - y + 2 = 0$	**66.** $3x + 5y - 2 = 0$
67. $xy = 16$	**68.** $2xy = 1$
69. $x = a$	**70.** $y = a$
71. $x^2 + y^2 = a^2$	**72.** $x^2 + y^2 = 9a^2$
73. $x^2 + y^2 - 2ax = 0$	**74.** $x^2 + y^2 - 2ay = 0$
75. $(x^2 + y^2)^2 = x^2 - y^2$	**76.** $(x^2 + y^2)^2 = 9(x^2 - y^2)$
77. $y^3 = x^2$	**78.** $y^2 = x^3$

$x^2+y^2=9$

$(r\cos\theta)^2 + (r\sin\theta)^2 = 9$

$r^2\cos^2\theta + r^2\sin^2\theta$

$= r^2[\cos^2\theta + \sin^2\theta] =$

$= \boxed{r^2 = 9}$

$y = x$

$r\sin\theta = r\cos\theta$

$\frac{r\sin\theta}{r\cos\theta} = 1$

$\tan\theta = 1$

$xy = 16$

$r^2\sin\theta\cos\theta = 16$

$\sin(2\theta) = 2\sin\theta\cos\theta$

$r^2(\sin 2\theta) = 16$

$r^2 = \frac{16}{\sin(2\theta)} = 16\csc(2\theta)$

Rectangular Form **In Exercises 79–100, convert the polar equation to rectangular form.**

79. $r = 5$

80. $r = -7$

81. $\theta = 2\pi/3$

82. $\theta = -5\pi/3$

83. $\theta = \pi/2$

84. $\theta = 3\pi/2$

85. $r = 4 \csc \theta$

86. $r = 2 \csc \theta$

87. $r = -3 \sec \theta$

88. $r = -\sec \theta$

89. $r = -2 \cos \theta$

90. $r = 4 \sin \theta$

91. $r^2 = \cos \theta$

92. $r^2 = 2 \sin \theta$

93. $r^2 = \sin 2\theta$

94. $r^2 = \cos 2\theta$

95. $r = 2 \sin 3\theta$

96. $r = 3 \cos 2\theta$

97. $r = \dfrac{2}{1 + \sin \theta}$

98. $r = \dfrac{1}{1 - \cos \theta}$