$$\cos\theta = \frac{\bar{u}\cdot\bar{v}}{\|\bar{u}\|\|\bar{v}\|}$$

Section 4.1 - Complex #s

Section 4.2 - Complex Zeros of Polynomials

$\sqrt{-1} = i$

$i^2 = -1$

$a+bi \quad , a,b \in \mathbb{R}$

$3+2i \qquad 7.7-8i \qquad \tilde{z}$

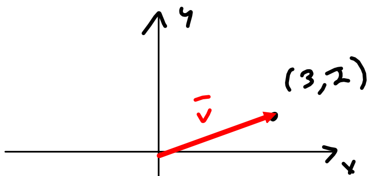
$z = a+bi \longrightarrow \bar{z} = a-bi =$ complex conjugate.

$z\bar{z} = (a+bi)(a-bi) = a^2 - abi + abi - (bi)^2$

$= a^2 - b^2i^2 = a^2+b^2$

$|z| = \sqrt{a^2+b^2}$

$=$ modulus / absolute value of z.

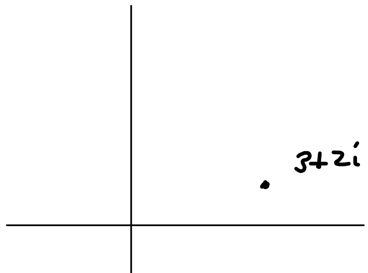


$\bar{v} = \langle 3, 2\rangle$

$= 3\bar{i} + 2\bar{j}$

$\|\bar{v}\| = \sqrt{3^2+2^2}$

$= \sqrt{\langle 3,2\rangle \cdot \langle 3,2\rangle} = \sqrt{\bar{v}\cdot\bar{v}}$



$|3+2i| =$

$\sqrt{(3+2i)\overline{(3+2i)}}$

$= \sqrt{(3+2i)(3-2i)}$

$= \sqrt{3^2+2^2}$

Write $\frac{1}{2+3i}$ in the form $a+bi$

$$\left(\frac{1}{2+3i}\right)\left(\frac{2-3i}{2-3i}\right) = \frac{2-3i}{2^2+3^2} = \frac{2}{13} - \frac{3}{13}i = \frac{2}{13} + \left(-\frac{3}{13}\right)i$$

$\underbrace{\qquad}_{b}$ ok ✓

$$\sqrt{-7} = i\sqrt{7} = \sqrt{7}\,i$$

$$x^2+2x+11 = 0$$

$$x^2+2x+1^2-1+11 = (x+1)^2+10 = 0 \Longrightarrow$$

$$(x+1)^2 = -10 \Longrightarrow$$

$$x+1 = \pm\sqrt{-10} = \pm\sqrt{10}\,i$$

$$x = -1 \pm \sqrt{10}\,i$$

Complex #s $\mathbb{C}$ is a field:

Additive identity 0

Multiplicative identity 1

Additive inverse $-z = -(a+bi) = -a-bi$

Multiplicative Inverse $\frac{1}{z} = \left(\frac{1}{a+bi}\right)\left(\frac{a-bi}{a-bi}\right) = \frac{a-bi}{a^2+b^2}$

$= \frac{a}{a^2+b^2} - \frac{b}{a^2+b^2} i = \frac{1}{z}$

$z\left(\frac{1}{z}\right) = (a+bi)\left(\frac{a-bi}{a^2+b^2}\right) = \frac{a^2+b^2}{a^2+b^2} = 1$

commutativity & Associativity of addition & multiplication

$z_1 \pm z_2 = z_2 \pm z_1$

$z_1 + (z_2 + z_3) = (z_1 + z_2) + z_3$

$z_1 z_2 = z_2 z_1$

$z_1 (z_2 z_3) = (z_1 z_2) z_3$

Closure $z_1, z_2 \in \mathbb{C}$

$z_1 \pm z_2,\ z_1 z_2,\ \frac{z_1}{z_2} \in \mathbb{C}$

provided $z_2 \neq 0$.

Find a cubic polynomial function *f* with real coefficients that has the given complex zeros and *x*-intercept. (There are many correct answers.)

Complex Zeros	x-Intercept
$x = -1 \pm \sqrt{2}i$	$(-5, 0)$

Recall from College Algebra

$(x-(-1+\sqrt{2}i))(x-(-1-\sqrt{2}i))(x+5)$

Factor Theorem

If c is a zero of $P(x)$, then $(x-c)$ is a factor of $P(x)$ of degree n.

& $P(x) = (x-c)Q(x)$, where $Q(x)$ has degree $n-1$.

$Q(x)$ is the Depressed Polynomial.

Fundamental Theorem of Algebra.

Every polynomial of degree n has at least one complex zero.

Linear Factorization Theorem

You can apply FTA n times to obtain

$P(x) = a_n(x-c_1)(x-c_2)\cdots(x-c_n)$

We can split any polynomial into linear factors.

$a_n(x-c_1)^{m_1}(x-c_2)^{m_2}\cdots(x-c_k)^{m_k}$,

where $m_1+m_2+\cdots+m_k = n$

$(x-1)^2(x+2)(x-3i)^3(x+3i)^3$

$m_1=2,\ m_2=1,\ m_3=m_4=3$

$m_1+m_2+m_3+m_4 = 9 = \text{degree of } P(x)$

$-1\pm\sqrt{2}i,\ -5$

$(x-(-1+\sqrt{2}i))(x-(-1-\sqrt{2}i))(x+5)$

$= (x+1-\sqrt{2}i)(x+1+\sqrt{2}i)(x+5)$

$= (x+5)\left(x^2+x+\sqrt{2}ix + x+1+\sqrt{2}i - \sqrt{2}ix - \sqrt{2}i - (\sqrt{2}i)^2\right)$

$(\sqrt{2}i)^2 = \sqrt{2}^2 i^2 = 2(-1) = -2$

$= (x+5)(x^2+2x+2) = x^3+2x^2+2x$ (2 → 3, 2x → 3x)

$\quad 5x^2+10x+10$ (10 → 15)

$x^3+7x^2+12x+10$ (12 → 13, 10 → 15)

Use the given zero to find all the zeros of the function. (Enter your answers as a comma-separated list. Include the given zero in your answer.)

Function	Zero
$g(x) = 3x^3 + 29x^2 + 68x - 26$	$-5 + i$

Dividing by $(x-(-5+i))$

$-5+i$	3	29	68	-26
		$-15+3i$	$-73-i$	26
$-5-i$	3	$14+3i$	$-5-i$	0 sweet!
		$-15-3i$	$5+i$	
	3	-1	0 sweet!	
	x	c	r	

$g(x) = (x-(-5+i))(x-(-5-i))(3x-1)$

zeros: $-5+i, -5-i, \frac{1}{3}$

$5^2+1^2=26$

$\cancel{(-5+i)(13+3i) = -65 - 15i + 13i - 3 = -68 - 2i}$

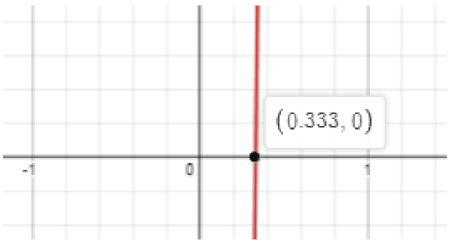
$(-5+i)(14+3i) = -70 - 15i + 14i - 3$

$= -73 - i$

Use the given zero to find all the zeros of the function. (Enter your answers as a comma-separated list. Include the given zero in your answer.)

Function	Zero
$g(x) = 3x^3 + 29x^2 + 68x - 26$	$-5 + i$

The (a) cheat: Graphing calculator will find the real zero.



Desmos suggests x = 1/3 is a zero.

$$\begin{array}{r|rrrr} \frac{1}{3} & 3 & 29 & 68 & -26 \\ & & 1 & 10 & 26 \\ \hline & 3 & 30 & 78 & 0 \end{array}$$

$$\frac{78}{3} = 26$$

This says $f(x) = \underbrace{(3x^2+30x+78)}_{\text{Hit w/ Quadratic Formula}}(x - \frac{1}{3})$

$(x^2+4)(x^2+7)$

$3x^2+30x+78=0 \Rightarrow$

$x^2+10x+26=0 \Rightarrow$

$b^2-4ac = 10^2 - 4(1)(26)$

$= 100 - 104 = -4$

$=$

$x = \dfrac{-10 \pm \sqrt{-4}}{2} = \dfrac{-10 \pm 2i}{2}$

$= -5 \pm i$