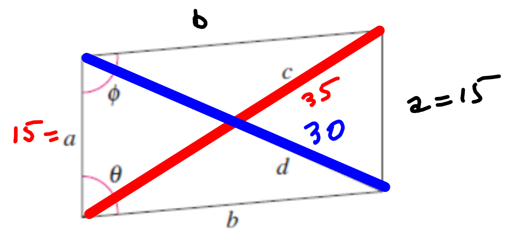
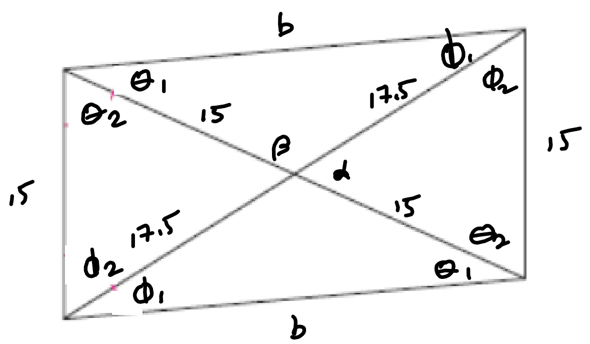
Find the missing values by solving the parallelogram shown in the figure. (The lengths of the diagonals are given by *c* and *d*. Round your answers to two decimal places.)

3.2 #8

a	*b*	*c*	*d*	*θ*	*φ*
15	✗ 28.94	35	30	✗ 79.21 °	✗ 100.79 °





$15^2 = 17.5^2 + 15^2 - 2(15)(17.5)\cos\phi_2$

$17.5^2 = 15^2 + 15^2 - 2(15)(15)\cos\theta_2$

$\alpha = 180^\circ - \phi_2 - \theta_2$

$\beta = 180^\circ - \alpha$

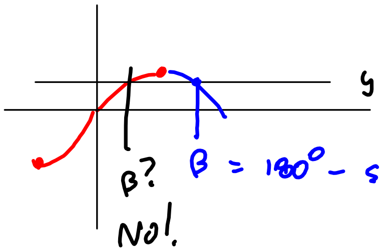
$b^2 = 15^2 + 17.5^2 - 2(15)(17.5)\cos\beta$

$\frac{\sin\phi_1}{15} = \frac{\sin\beta}{b}$

$\frac{\sin\theta_1}{17.5} = \frac{\sin\beta}{b}$

β is obtuse, so law of sines & $\sin^{-1}$ on your calculator would give you its supplement (α)

Finally, $\theta = \theta_1 + \theta_2$ & $\phi = \phi_1 + \phi_2$

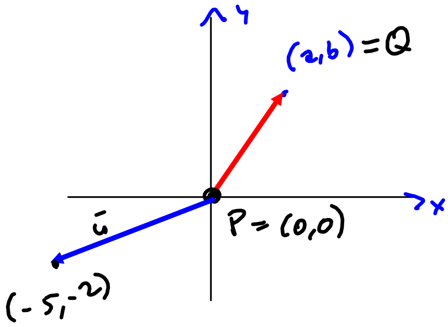


$y = \sin\beta = \omega$

β? $\beta = 180^\circ - \sin^{-1}(\omega)$

No!

§3.3 Vectors in the plane



$\bar{u} = \langle -5, -2 \rangle$

consider the directed line segment $\overrightarrow{PQ}$ emanating from the origin.

we call $\overrightarrow{PQ}$ "$\bar{v}$"

$\bar{v} = \langle a, b \rangle$

$\bar{v}$ has length, determined by Pythagorus to be

$\|\bar{v}\|$ = magnitude or modulus of $\bar{v} = \sqrt{a^2+b^2}$

Vector addition:

$\bar{u} + \bar{v} = \langle u_1+v_1, u_2+v_2 \rangle$, where

$\bar{u} = \langle u_1, u_2 \rangle$, $\bar{v} = \langle v_1, v_2 \rangle$

Subtraction:

$\bar{u} - \bar{v} = \bar{u} + (-\bar{v})$

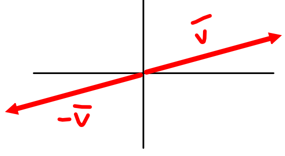
Scalar multiplication:

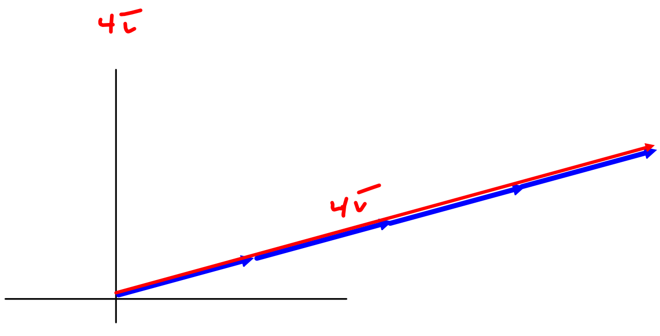
$a\bar{v} = a\langle v_1, v_2 \rangle = \langle av_1, av_2 \rangle$

a times $\bar{v}$ in the direction of $\bar{v}$.

if $a<0$, then it's in the opposite direction of $\bar{v}$.

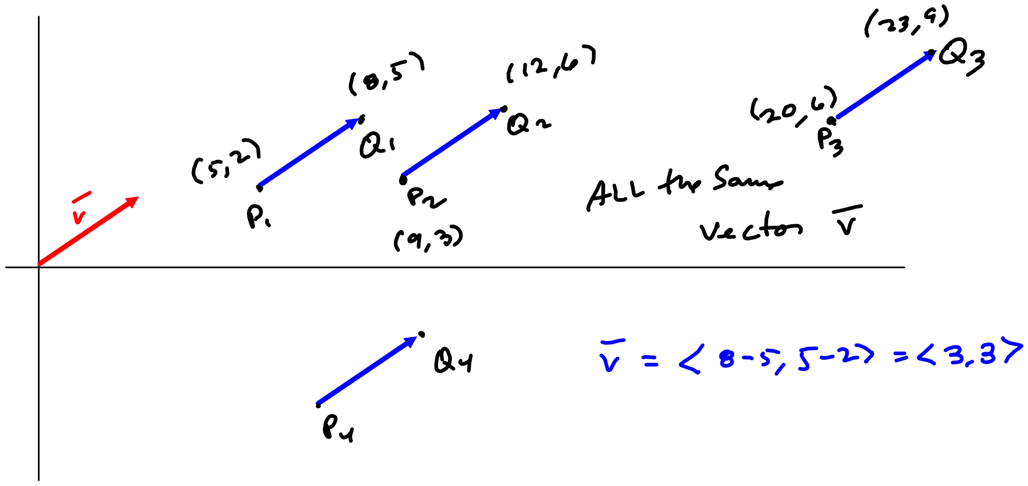
$-\bar{v} = \langle -v_1, -v_2 \rangle$





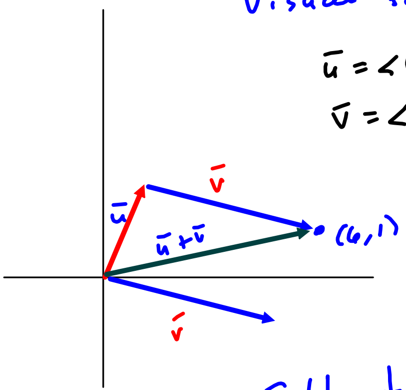
3.3 exercises are awesome.

See harryzaims.com listing for a more salubrious experience.

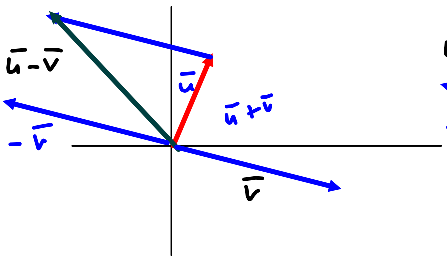


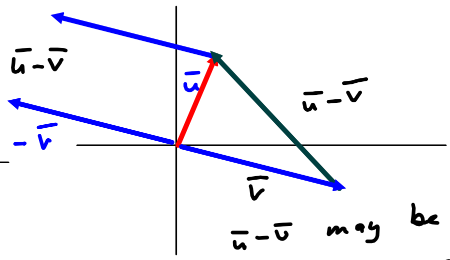
Visual for $\bar{u} + \bar{v}$

$\bar{u} = \langle 1,3 \rangle$
$\bar{v} = \langle 5,-2 \rangle$ } Then $\bar{u}+\bar{v} = \langle 6,1 \rangle$



Subtraction:
$\bar{u}-\bar{v}$ is obtained by adding
$\bar{u}+(-\bar{v})$





$\bar{u}-\bar{v}$ may be thought of as the vector from the terminal point of $\bar{v}$ to the terminal point of $\bar{u}$

Remember this visual.

We'll need it in the 3.4 sequel!

The canonical basis for R^2

$\bar{i} = \langle 1, 0 \rangle$
$\bar{j} = \langle 0, 1 \rangle$

Any vector $\bar{v} = \langle v_1, v_2 \rangle$ may be expressed as a linear combo of $\bar{i}$ & $\bar{j}$

$\bar{v} = v_1 \bar{i} + v_2 \bar{j}$
$= \langle v_1, 0 \rangle + \langle 0, v_2 \rangle = \langle v_1, v_2 \rangle = \bar{v}$

Any 2 non-parallel vectors can be used as a basis.

$\bar{b}_1 = \langle 1, 2 \rangle$, $\bar{b}_2 = \langle -1, 1 \rangle$

Let $\bar{u} = \langle 5, 6 \rangle$
Find constants c_1 & c_2 such that
$\bar{u} = c_1 \bar{b}_1 + c_2 \bar{b}_2$

$$\begin{array}{r} 1c_1 - 1c_2 = 5 \\ + \quad 2c_1 + 1c_2 = 6 \\ \hline 3c_1 = 11 \end{array}$$

$c_1 = \frac{11}{3} \implies c_1 - c_2 = \frac{11}{3} - c_2 = 5$
$-c_2 = \frac{15}{3} - \frac{11}{3} = \frac{4}{3} = .$
$\implies c_2 = -\frac{4}{3}$

$\bar{u}$ as a linear combo on $\bar{b}_1$ & $\bar{b}_2$

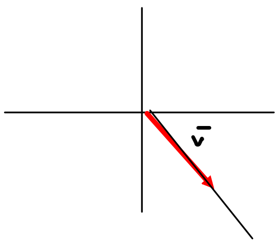
$\bar{u} = c_1 \bar{b}_1 + c_2 \bar{b}_2 = \frac{11}{3}\langle 1, 2 \rangle - \frac{4}{3}\langle -1, 1 \rangle$

$= \langle \frac{11}{3} + \frac{4}{3}, \frac{22}{3} - \frac{4}{3} \rangle = \langle \frac{15}{3}, \frac{18}{3} \rangle = \langle 5, 6 \rangle$

$\{b_1, b_2\}$ spans all of $\mathbb{R}^2$

Direction angle of $\bar{v}$ is the angle it makes with the positive x-axis (counter-clockwise)

$\bar{v} = \langle 3, -3 \rangle$



$$\tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{-3}{3}\right) = \tan^{-1}(-1)$$

$= -\frac{\pi}{4} = -45^\circ$, which we have to interpret as

$$2\pi + \left(-\frac{\pi}{4}\right) = \frac{7\pi}{4} = 315^\circ$$

Magnitude of $\bar{v}$ is

$$\|\bar{v}\| = \sqrt{3^2 + 3^2} = \sqrt{2 \cdot 9} = 3\sqrt{2}$$

Alternate representation of a vector:

Magnitude times (unit vector in the proper direction).

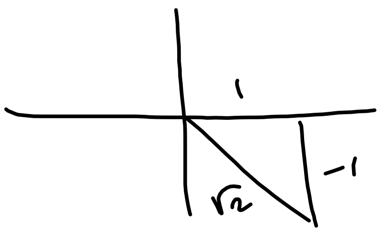
$\bar{u}$ is unit vector if $\|\bar{u}\| = 1$

Any vector of the form $\langle \cos\theta, \sin\theta \rangle$ is a unit vector!

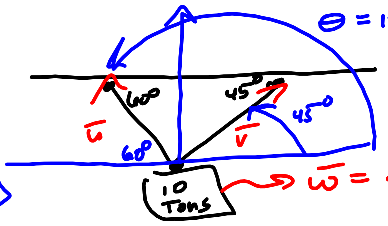
All points $(\cos\theta, \sin\theta) = (x, y)$ lie on the unit circle!

So, $\bar{v} = \langle 3, -3 \rangle = 3\sqrt{2} \langle \cos 315^\circ, \sin 315^\circ \rangle$

$$= 3\sqrt{2} \left\langle \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \right\rangle = \langle 3, -3 \rangle$$



Crucial for applications (Hanging weight, 2 tugboats, 3-way tug-o-war, ...)



$\bar{w} = \langle 0, -20{,}000 \rangle$ = resultant = sum of $\bar{u}$ & $\bar{v}$

$\bar{u} = \|\bar{u}\| \langle \cos 120^\circ, \sin 120^\circ \rangle$

$\bar{v} = \|\bar{v}\| \langle \cos 45^\circ, \sin 45^\circ \rangle$

Find the tension in the 2 cables.

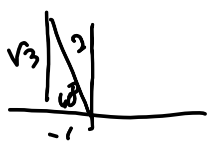
All we know is

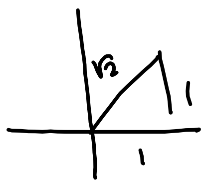
$\bar{u} + \bar{v} = \langle 0, 20{,}000 \rangle$

$\|\bar{u}\| \cos 120^\circ + \|\bar{v}\| \cos 45^\circ = 0$

$\|\bar{u}\| \sin 120^\circ + \|\bar{v}\| \sin 45^\circ = 20000$

Let $x = \|\bar{u}\|$, $y = \|\bar{v}\|$





This gives

$$-\frac{1}{2}x + \frac{1}{\sqrt{2}}y = 0$$

$$\frac{\sqrt{3}}{2}x + \frac{1}{\sqrt{2}}y = 20000$$

SOLVE!