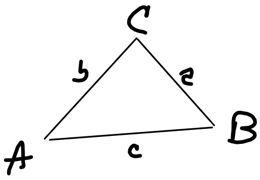
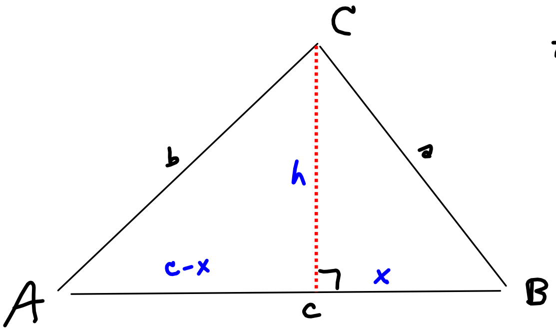
Section 3.2 - Law of Cosines

HUGE

$$a^2=b^2+c^2-2bc\cos(A)$$





$\frac{x}{a}=\cos B$

$x=a\cos B$

Get rid of h:

$\frac{h}{b}=\sin A$

$h=b\sin A$

Proof:

$a^2=x^2+h^2=(a\cos B)^2+(b\sin A)^2$

$\cos A=\frac{c-x}{b} \rightarrow c-x=b\cos A$

Scratch

$-x=b\cos A-c$

$x=-b\cos A+c=c-b\cos A$

$=(c-b\cos A)^2+(b\sin A)^2$

$=c^2-2bc\cos A+b^2\cos^2 A+b^2\sin^2 A$

$=c^2-2bc\cos A+b^2(\cos^2 A+\sin^2 A)$

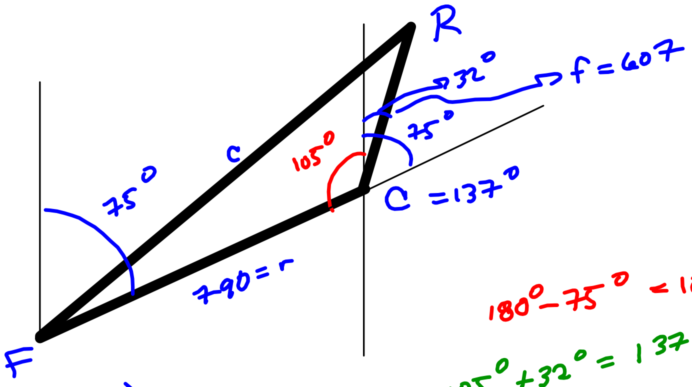
$=c^2-2bc\cos A+b^2$

$=b^2+c^2-2bc\cos A$ ■

A plane flies 790 miles from Franklin to Centerville with a bearing of 75°. Then it flies 607 miles from Centerville to Rosemount with a bearing of 32°. Draw a diagram that gives a visual representation of the problem. Then find the straight-line distance and bearing from Franklin to Rosemount. (Round your answers to one decimal place.)

[ ] ✗ 1301.5 mi

N [ ] ✗ 56.5 ° E



$a^2 = b^2 + c^2 - 2bc\cos A$

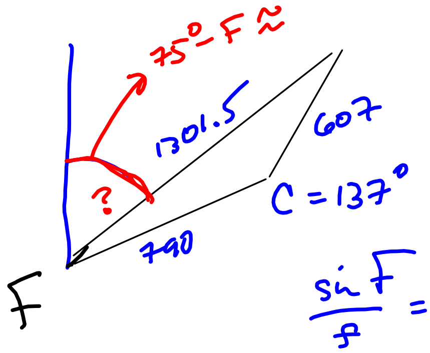
$c^2 = r^2 + f^2 - 2rf\cos F$

$180° - 75° = 105°$

$105° + 32° = 137°$

$137°$

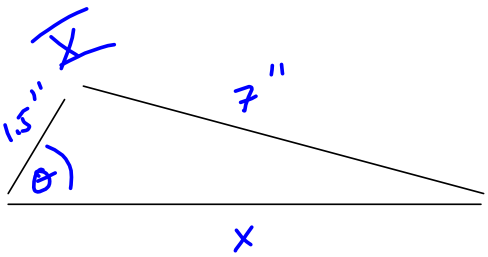
$= 790^2 + 607^2 - 2(790)(607)\cos(137°)$

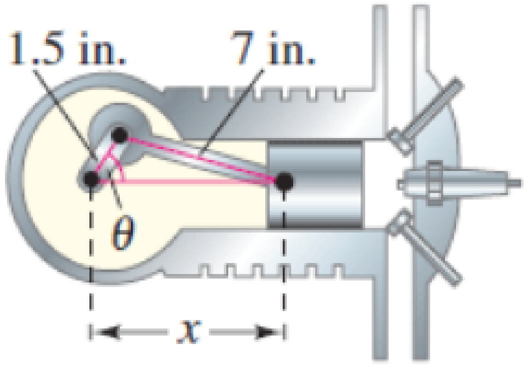


$\frac{\sin F}{f} = \frac{\sin C}{c}$

$\sin F = \frac{f \sin C}{c} = \frac{607 \sin 137°}{1301.5}$

$\approx 18.22396782°$





(a) Use the Law of Cosines to write an equation giving the relationship between x and θ.

$$7^2 = 1.5^2 + x^2 - 2(1.5)x\cos\theta$$

$$a^2 = b^2 + c^2 - 2bc\cos\theta$$

$$7^2 = 1.5^2 + x^2 - 2(1.5)(x)\cos\theta$$

$$x^2 - 3x\cos\theta + 2.25 - 49 = 0$$

$$x^2 - (3\cos\theta)x - 46.75 = 0$$

$$a = 1,\ b = -3\cos\theta,\ c = -46.75$$

$$b^2 - 4ac = 9\cos^2\theta - 4(1)(-46.75)$$

$$= 9\cos^2\theta + 187$$ ← Student fixed this.

$$x = \frac{3\cos\theta \pm \sqrt{9\cos^2\theta + 187}}{2}$$

Take the "+"
The "−" can be negative.
That's un-possible

$\frac{h}{b} = \sin A$

$h = b\sin A$

$\frac{x}{a} = \cos B$

$x = a\cos B$

$a^2 = x^2 + h^2 = (a\cos B)^2 + (b\sin A)^2$

Need to get rid of cos B, somehow.

Hmmmmm, we have another equation involving x:

$\cos A = \frac{c-x}{b}$ Solve for x:

$c - x = b\cos A$

$-x = -c + b\cos A$

$\boxed{x = c - b\cos A}$

Now, $\cos B = \frac{x}{a} = \frac{c - b\cos A}{a}$ ⟶

$a^2 = (a\cos B)^2 + (b\sin A)^2$

$= a^2\cos^2 B + b^2\sin^2 A$

$= a^2\left(\frac{c - b\cos A}{a}\right)^2 + b^2\sin^2 A$

$= a^2\left(\frac{c^2 - 2bc\cos A + b^2\cos^2 A}{a^2}\right) + b^2\sin^2 A$

$= c^2 - 2bc\cos A + b^2\cos^2 A + b^2\sin^2 A$

$= c^2 - 2bc\cos A + b^2(\sin^2 A + \cos^2 A)$

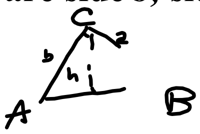
$= c^2 - 2bc\cos A + b^2$, i.e.,)

$a^2 = b^2 - c^2 - 2bc\cos A$ ■

Tutors, teachers and textbooks will give you a big list of procedures and rules for when Law of Sines or Law of Cosines applies.

Here's my procedure:

Try Law of Sines. You'll know in seconds if you have enough information to solve the triangle with it. Just make sure to compare side *b,* side *a*, and the height *h* from my proof, if you run into an ASS.



Tips/Notes:

There are (at least) two formulations for the Law of Cosines. I just remember one. My algebra's good enough to get the cos(A) all by itself, if I have to (SSS case).

Honestly, I don't think much when I'm working these. I just remember to try Law of Sines, first, and when I do, that *a* has to be big enough ($a > h$) and there are two solutions when it's bigger than *h* and smaller than *b.* Otherwise, if it's big enough ($a > h$), there's one solution.

$\frac{\sin A}{a} = \frac{\sin B}{b}$

You'll know right away if you have 3 out of 4 quantities and can use Law of Sines to solve for the 4th. If you don't, use Law of Cosines.

Note: The Law of Cosines is the linchpin for finding angles between vectors, calculating orthogonal projections, and if you ever take Linear Algebra, you'll be glad you learned what orthogonal projections are, and how to compute them. Law of Cosines is the foundation on which "orthogonalization" rests.

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