Product-to-Sum Formulas

$$\sin u \sin v = \frac{1}{2}[\cos(u - v) - \cos(u + v)]$$

$$\cos u \cos v = \frac{1}{2}[\cos(u - v) + \cos(u + v)]$$

$$\sin u \cos v = \frac{1}{2}[\sin(u + v) + \sin(u - v)]$$

$$\cos u \sin v = \frac{1}{2}[\sin(u + v) - \sin(u - v)]$$

$\sin(6\theta)\sin(2\theta)$

$= \frac{1}{2}\left[\cos(4\theta) - \cos(8\theta)\right]$

$u = 6\theta, \ v = 2\theta$

Find all solutions of the equation in the interval $[0, 2\pi)$. (Enter your answers as a comma-separated list.)

2.5 #13

$4 \sin \frac{x}{2} + 4 \cos x = 0$

$x = 2\theta \Rightarrow$

$\cos(2\theta) = 1 - 2\sin^2\theta = \quad \frac{x}{2} = \theta$

$$\cos(2u) = \cos^2(u) - \sin^2(u) = 2\cos^2(u) - 1 = 1 - 2\sin^2(u)$$

From the Cheat Sheet

$4\sin\left(\frac{x}{2}\right) + 4\left[1 - 2\sin^2\left(\frac{x}{2}\right)\right] = 0$

$4\sin\left(\frac{x}{2}\right) + 4 - 8\sin^2\left(\frac{x}{2}\right) = 0$

$-8\sin^2(u) + 4\sin(u) + 4 = 0$, where $\boxed{u = \frac{x}{2}}$

$8\sin^2(u) - 4\sin(u) - 4 = 0$

$2\sin^2(u) - \sin(u) - 1 = 0$

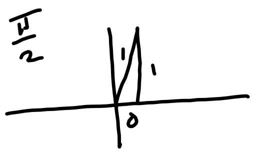
$2\sin^2 u - 2\sin u + \sin(u) - 1 = 0$

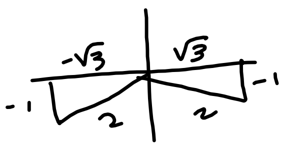
$2\sin(u)\left[\sin(u) - 1\right] + 1\left[\sin(u) - 1\right] = 0$

$\left[\sin(u) - 1\right]\left(2\sin(u) + 1\right) = 0$

$\sin(u) = 1$ or $2\sin(u) + 1 = 0$

$\sin(u) = -1/2$





$u = \frac{x}{2} = \frac{\pi}{2}$

$u = \frac{7\pi}{6}, \frac{11\pi}{6}$ $\rightarrow$ 2 b/s

Domain:

want $x \in [0, 2\pi)$

$\Rightarrow 0 \le x < 2\pi \rightarrow$

$0 \le \frac{x}{2} < \pi$

$u = \frac{x}{2} < \pi$

$\frac{x}{2} = \frac{7\pi}{6}$

$x = \frac{7\pi}{3} > 2\pi$

$\frac{x}{2} = \frac{11\pi}{6}$

$x = \frac{11\pi}{3} > 2\pi$

Solve the equation. (Find all solutions of the equation in the interval $[0, 2\pi)$. Enter your answers as a comma-separated list.)

$3 \sin(2x) \sin(x) = 3 \cos(x)$

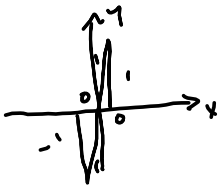
$$3(2\sin(x)\cos(x))\sin(x) - 3\cos(x) = 0$$

$$6\sin^2(x)\cos(x) - 3\cos(x) = 0$$

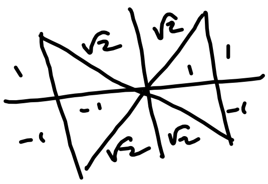
$$3\cos(x)\left[2\sin^2(x) - 1\right] = 0$$

$\cos(x) = 0$ OR $\sin^2(x) = \frac{1}{2}$

$$\sin(x) = \pm\frac{1}{\sqrt{2}}$$



$\frac{\pi}{2}, \frac{3\pi}{2}$



$\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

$\cos(u) = -\frac{4}{5}$ $\qquad \frac{\pi}{2} < u < \pi$

$\pi < 2u < 2\pi \implies$

we're in Q III or Q IV

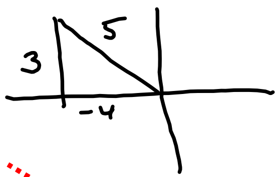
$\sin(2u) = 2\sin u \cos u$

can we narrow that down to 1 quadrant?

$\cos(2u) = \cos^2(u) - \sin^2(u)$

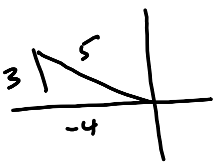
$\tan(2u) = \frac{2\tan(u)}{1-\tan^2(u)}$

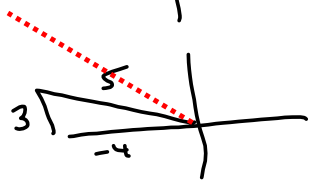
$\cos(u) = -\frac{4}{5}$



$\cos(u) = -\frac{4}{5}$, $\sin(u) = \frac{3}{5}$

$\tan(u) = -\frac{3}{4}$





Not only is $\frac{\pi}{2} < u < \pi$,

but $\frac{3\pi}{4} < u < \pi \implies$

$\frac{3\pi}{2} < 2u < 2\pi$

$2u \in$ Q IV

$$\tan\left(\frac{u}{2}\right) = \frac{1-\cos(u)}{\sin(u)}$$

$\sin(15^\circ)$

$\cos(15^\circ)$

$\tan(15^\circ)$

$u = 30^\circ$

$\cos(u) = \frac{\sqrt{3}}{2}$

$$\sin(15^\circ) = \pm\sqrt{\frac{1-\cos 30^\circ}{2}} = \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{\frac{2-\sqrt{3}}{2}}{2}}$$

QI

$$= \sqrt{\frac{2-\sqrt{3}}{4}} = \boxed{\frac{\sqrt{2-\sqrt{3}}}{2} = \sin 15^\circ}$$

$$\boxed{\cos 15^\circ = \frac{\sqrt{2+\sqrt{3}}}{2}}$$

$$\tan(15^\circ) = \frac{1-\frac{\sqrt{3}}{2}}{\frac{\sqrt{3}}{2}} = \frac{\frac{2-\sqrt{3}}{2}}{\frac{\sqrt{3}}{2}} = \frac{2-\sqrt{3}}{2}\cdot\frac{2}{\sqrt{3}}$$

$$= \frac{4-2\sqrt{3}}{2\sqrt{3}} = \frac{2-\sqrt{3}}{\sqrt{3}} = \boxed{\frac{2\sqrt{3}-3}{3} = \tan 15^\circ}$$

Fill in the blank to complete the trigonometric formula.

$\cos u - \cos v =$ (No Response) ✗

From cheat sheet

$$\cos u - \cos v = -2\sin\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$

$$\sqrt{\frac{1+\cos(u)}{2}} = \cos\left(\frac{u}{2}\right)$$

$$\cos^2\left(\frac{u}{2}\right) = \frac{1+\cos(u)}{2}$$

where does this come from?

It comes from

$\cos(2u) = 2\cos^2(u) - 1$

$\Rightarrow 2\cos^2(u) = \cos(2u)+1$

$\Rightarrow \cos^2(u) = \frac{\cos(2u)+1}{2}$

$\cos(u) = \pm\sqrt{\frac{\cos(2u)+1}{2}}$

$\cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{\cos(x)+1}{2}}$

$$\sqrt{} = \sqrt{}$$

$$\left|\cos\left(\frac{u}{2}\right)\right| = \sqrt{\frac{1+\cos(u)}{2}}$$

$$\cos\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1+\cos(u)}{2}}$$

Use the power-reducing formulas to rewrite the expression in terms of first powers of the cosines of multiple angles

$\cos^4(2x)$

$$\cos^2(u) = \frac{\cos(2u)+1}{2} \qquad \sin^2(u) = \frac{1-\cos(2u)}{2}$$

$$\left(\cos^2(2x)\right)^2 = \left(\frac{\cos(4x)+1}{2}\right)^2$$

$$= \frac{1}{4}\left(\cos^2(4x) + 2\cos(4x) + 1\right)$$

$$= \frac{1}{4}\left[\frac{\cos(8x)+1}{2} + 2\cos(4x) + 1\right]$$

$$= \frac{1}{8}\cos(8x) + \frac{1}{8} + \frac{1}{2}\cos(4x) + \frac{1}{4}$$

$$= \frac{1}{8}\cos(8x) + \frac{1}{2}\cos(4x) + \frac{3}{8}$$