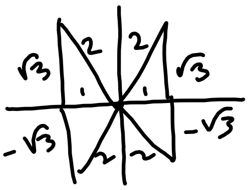
$\sin^2(x) = 3\cos^2(x) = 3(1-\sin^2(x)) = 3-3\sin^2(x)$

$4\sin^2(x) = 3$

$\sin^2(x) = \frac{3}{4}$

$\sin(x) = \pm\sqrt{\frac{3}{4}} = \pm\frac{\sqrt{3}}{2} = \sin(x)$



$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

MAT 122 Cheat Sheet

Double-Angle Formulas: $\sin(2u) = 2\sin(u)\cos(u)$, $\cos(2u) = \cos^2(u) - \sin^2(u) = 2\cos^2(u) - 1 = 1 - 2\sin^2(u)$,

$\tan(2u) = \dfrac{2\tan(u)}{1-\tan^2(u)} = \dfrac{\sin(2u)}{\cos(2u)}$!

Half-Angle Formulas: $\sin\left(\dfrac{u}{2}\right) = \pm\sqrt{\dfrac{1-\cos(u)}{2}}$, $\cos\left(\dfrac{u}{2}\right) = \pm\sqrt{\dfrac{1+\cos(u)}{2}}$, $\tan\left(\dfrac{u}{2}\right) = \dfrac{1-\cos(u)}{\sin(u)} = \dfrac{\sin\left(\frac{u}{2}\right)}{\cos\left(\frac{u}{2}\right)}$!!!

You have to determine the "$\pm$" deal by determing the quadrant in which $\dfrac{u}{2}$ resides.

Power-Reducing Formulas: $\sin^2(u) = \dfrac{1-\cos(2u)}{2}$, $\cos^2(u) = \dfrac{1+\cos(2u)}{2}$, $\tan^2(u) = \dfrac{1-\cos(2u)}{1+\cos(2u)} = \dfrac{\sin^2(u)}{\cos^2(u)}$

Product-to-Sum Formulas

$\sin u \sin v = \dfrac{1}{2}[\cos(u-v) - \cos(u+v)]$

$\cos u \cos v = \dfrac{1}{2}[\cos(u-v) + \cos(u+v)]$

$\sin u \cos v = \dfrac{1}{2}[\sin(u+v) + \sin(u-v)]$

Sum-to-Product Formulas

$\sin u + \sin v = 2\sin\left(\dfrac{u+v}{2}\right)\cos\left(\dfrac{u-v}{2}\right)$

$\cos u + \cos v = 2\cos\left(\dfrac{u+v}{2}\right)\cos\left(\dfrac{u-v}{2}\right)$

$\cos u - \cos v = -2\sin\left(\dfrac{u+v}{2}\right)\sin\left(\dfrac{u-v}{2}\right)$

Pythagorean Identities

$\tan^2(x) + 1 = \sec^2(x)$

$\cot^2(x) + 1 = \csc c^2(x)$

Angle Sum Formulas

$\sin(u+v) = \sin(u)\cos(v) + \sin(v)\cos(u)$

$\cos(u+v) = \cos(u)\cos(v) - \sin(u)\sin(v)$

Radians without π

1.570796327

3.141592654 — 6.283185308

4.712388981

Law of Sines $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$ **Law of Cosines** $a^2 = b^2 + c^2 - 2bc\cos A$

Heron's $Area = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \dfrac{a+b+c}{2}$. **Magnitude** $\bar{u} = \langle a, b\rangle \Rightarrow \|\bar{u}\| = \sqrt{a^2+b^2}$

Arc Length: $s = r\theta$, **Area of a Sector:** $A = \dfrac{1}{2}r^2\theta$

$7\sin(2x)\sin(x) = 7\cos(x)$

$2\sin(x)\cos(x)\sin(x) = \cos(x)$

$2\sin^2(x)\cos(x) - \cos(x) = \cos(x)\left(\overbrace{2\sin^2(x) - 1}^{-\cos(2x)}\right) = 0$

$\Longrightarrow \cos(x) = 0$ OR $2\sin^2(x) - 1 = 0$ (or $\cos(2x) = 0$)

$2\sin^2(x) = 1$

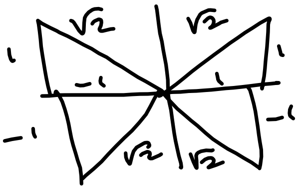
$\sin^2(x) = \frac{1}{2}$

$\sin(x) = \pm\sqrt{\frac{1}{2}} = \pm\frac{1}{\sqrt{2}}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

$\cos(x) = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$



Using Half-Angle Formulas In Exercises 35–40, use the half-angle formulas to determine the exact values of the sine, cosine, and tangent of the angle.

35. $75°$	**36.** $165°$
37. $112° 30'$	**38.** $67° 30'$
39. $\pi/8$	**40.** $7\pi/12$

$\sin\frac{\pi}{8} = \pm\sqrt{\frac{1-\cos\frac{\pi}{4}}{2}} = \sqrt{\frac{1-\frac{\sqrt2}{2}}{2}} = \sqrt{\frac{2-\sqrt2}{4}} = \boxed{\frac{\sqrt{2-\sqrt2}}{2} = \sin\frac{\pi}{8}}$

$+$, b/c $\frac{\pi}{8} \in QI$

$\cos\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1+\cos u}{2}}$

$\cos\left(\frac{\pi}{8}\right) = \sqrt{\frac{1+\cos\frac{\pi}{4}}{2}} = \frac{\sqrt{2+\sqrt2}}{2}$ $\cos\left(\frac{\pi}{8}\right)$

$\tan\left(\frac{\pi}{8}\right) = \frac{\sin\left(\frac{\pi}{8}\right)}{\cos\left(\frac{\pi}{8}\right)} = \frac{\frac{\sqrt{2-\sqrt2}}{2}}{\frac{\sqrt{2+\sqrt2}}{2}} = \sqrt{\frac{2-\sqrt2}{2+\sqrt2}} = \tan\frac{\pi}{8}$

?!

$\tan\frac{\pi}{8} = \frac{1-\cos(u)}{\sin(u)} = \frac{1-\frac{1}{\sqrt2}}{\frac{1}{\sqrt2}} = \frac{\sqrt2}{1}\left(1 - \frac{1}{\sqrt2}\right) = \sqrt2 - 1 = \tan\frac{\pi}{8}$

$\frac{u}{2} = \frac{\pi}{8} \Longrightarrow u = \frac{\pi}{4}$

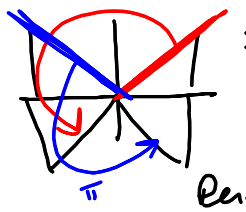
Find the *x*-intercepts of the graph. (Enter your answers as a comma-separated list. Use *n* as an integer constant.)

$y = \tan^2\left(\frac{\pi x}{4}\right) - 1$

$y = \tan^2\left(\frac{\pi x}{4}\right) - 1 \overset{\text{SET}}{=} 0$

$\tan^2\left(\frac{\pi}{4}x\right) = 1$

$\tan\left(\frac{\pi}{4}x\right) = \pm 1$



$\frac{\pi}{4}x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

captures all solns in $[0, 2\pi)$

Period of tangent is π

Almost

$\frac{\pi}{4} + n\pi, \frac{3\pi}{4} + n\pi$

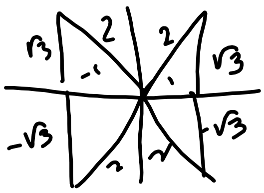
$\frac{\pi}{4} + 2n\pi, \frac{3\pi}{4} + 2n\pi, \frac{5\pi}{4} + 2n\pi, \frac{7\pi}{4} + 2n\pi$

$\frac{n\pi}{\frac{\pi}{4}} = 4n \qquad \frac{\pi}{4}x = \frac{\pi}{4} + n\pi, \frac{3\pi}{4} + n\pi \qquad 1 + \frac{2n\pi}{\frac{\pi}{4}} = 1 + 8n$

$x = 1 + 4n, 3 + 4n \qquad 3 + 8n, 5 + 8n, 7 + 8n$

$\tan^2\left(\frac{\pi}{6}x\right) = 3$

$\tan\left(\frac{\pi}{6}x\right) = \pm\sqrt{3}$



$\frac{\pi}{6}x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

$x = \frac{\pi}{3}\cdot\frac{6}{\pi}, \frac{2\pi}{3}\cdot\frac{6}{\pi}, \frac{4\pi}{3}\cdot\frac{6}{\pi}, \frac{5\pi}{7}\cdot\frac{6}{\pi}$

$= 2, 4, 8, 0$

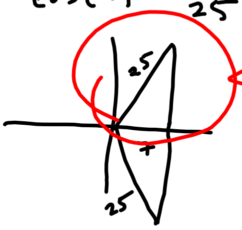
$\left(\frac{\pi}{3} + n\pi\right)\frac{6}{\pi} = 2 + 6n$

$\left(\frac{2\pi}{3} + n\pi\right)\frac{6}{\pi} = 4 + 6n$

Using Half-Angle Formulas In Exercises 41–44, use the given conditions to (a) determine the quadrant in which $u/2$ lies, and (b) find the exact values of $\sin(u/2)$, $\cos(u/2)$, and $\tan(u/2)$ using the half-angle formulas.

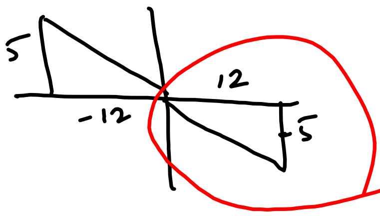
41. $\cos u = 7/25, \quad 0 < u < \pi/2$
42. $\sin u = 5/13, \quad \pi/2 < u < \pi$
43. $\tan u = -5/12, \quad 3\pi/2 < u < 2\pi$
44. $\cot u = 3, \quad \pi < u < 3\pi/2$

$\cos(u) = \frac{7}{25}$ $\qquad 0 < u < \frac{\pi}{2}$



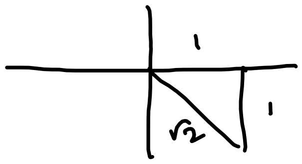
$\Rightarrow \frac{u}{2} \in Q\mathrm{I}$

$\tan(u) = -\frac{5}{12}$ $\qquad \frac{3\pi}{2} < u < 2\pi$

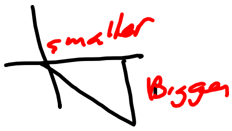


$\frac{3\pi}{2} + \frac{\pi}{4} = \frac{7\pi}{4}$

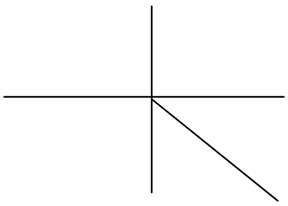
Narrow things down by observing



$\frac{7\pi}{4} < u < 2\pi$



$\frac{3\pi}{2} < u < \frac{7\pi}{4}$

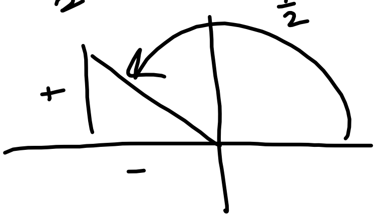


$\frac{1}{2}\left[\frac{3\pi}{2} < u < 2\pi\right]$

$\frac{3\pi}{4} < \frac{u}{2} < \pi$

$\frac{u}{2} \in Q\mathrm{II}$

$\frac{u}{2} \in Q\mathrm{II}$



$\cancel{\cos(u) = \frac{7}{25}}$

$\cancel{\sin\left(\frac{u}{2}\right) = +\sqrt{\frac{1-\frac{7}{25}}{2}}}$

$\cancel{\cos\left(\frac{u}{2}\right) = -\sqrt{\frac{1+\frac{7}{25}}{2}}}$

$\cancel{\tan\left(\frac{u}{2}\right) = \frac{1-\cos(u)}{\sin(u)} = \frac{1-\frac{7}{25}}{}}$

No, This cosine $= \frac{7}{25}$ is from previous problem.

$\tan(u) = -\frac{5}{12}$ $\frac{3\pi}{2} < u < 2\pi$

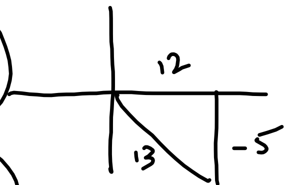
$u \in Q\,\overline{IV}$

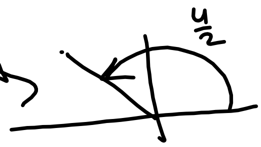
we determined

$\frac{u}{2} \in Q\,\overline{II}$

$\sin(u) = -\frac{5}{13}$

$\cos(u) = \frac{12}{13}$





$\sin\left(\frac{u}{2}\right)$ +
$\cos\left(\frac{u}{2}\right)$ −
$\tan\left(\frac{u}{2}\right)$ −

$5^2 + 12^2 = 25 + 144 = 169 \rightsquigarrow \sqrt{169} = 13$

$$\sin\left(\frac{u}{2}\right) = \sqrt{\frac{1-\cos(u)}{2}} = \sqrt{\frac{1-\frac{12}{13}}{2}} = \sqrt{\frac{\frac{1}{13}}{2}} = \sqrt{\frac{1}{26}} = \boxed{\frac{1}{\sqrt{26}} = \sin\left(\frac{u}{2}\right)}$$

$$\cos\left(\frac{u}{2}\right) = -\sqrt{\frac{1+\cos(u)}{2}} = -\sqrt{\frac{25}{26}} = \boxed{-\frac{5}{\sqrt{26}} = \cos\left(\frac{u}{2}\right)}$$

$$\tan\left(\frac{u}{2}\right) = \frac{1-\cos(u)}{\sin(u)} = \frac{1-\frac{12}{13}}{-\frac{5}{13}} = \frac{\frac{1}{13}}{-\frac{5}{13}} = \boxed{-\frac{1}{5} = \tan\left(\frac{u}{2}\right)}$$

Use the power-reducing formulas to rewrite the expression in terms of first powers of the cosines of multiple angles.

$3\cos^4(x)$

Power-Reducing Formulas: $\sin^2(u)=\dfrac{1-\cos(2u)}{2}, \cos^2(u)=\dfrac{1+\cos(2u)}{2}, \tan^2(u)=\dfrac{1-\cos(2u)}{1+\cos(2u)}=\dfrac{\sin^2(u)}{\cos^2(u)}$

It all flows from Double-Angle for cosine which all flows from Angle-Addition formulas

$$3\cos^4(x) = 3(\cos^2(x))^2 = \left(\left(\frac{1+\cos(2x)}{2}\right)^2\right)(3)$$

$$=3\left(\frac{\cos(2x)+1}{2}\right)^2 = \left(\frac{\cos^2(2x)+2\cos(2x)+1}{4}\right)(3)$$

$$=\left(\frac{\cos^2(2x)}{4} + \frac{1}{2}\cos(2x) + \frac{1}{4}\right)(3)$$

$$=\left(\frac{\frac{\cos(4x)+1}{2}}{4}\right)(3) + \sim \quad = \left(\frac{\cos(4x)}{8} + \frac{1}{8} + \frac{1}{2}\cos(2x) + \frac{1}{4}\right)(3)$$

$$=\left(\frac{1}{8}\cos(4x) + \frac{1}{2}\cos(2x) + \frac{3}{8}\right)(3)$$