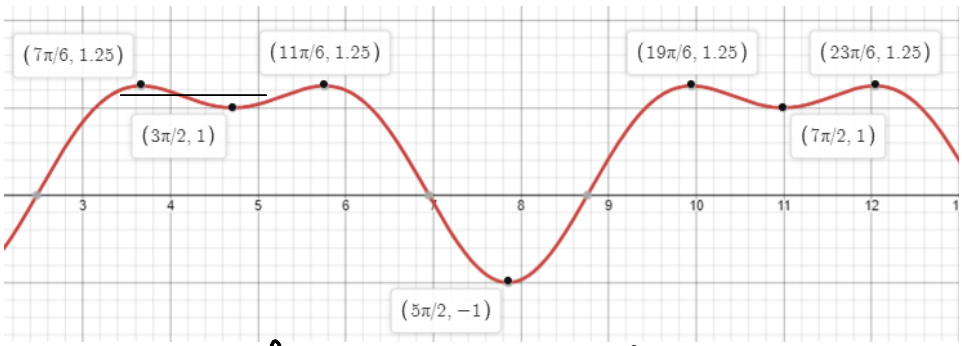
Section 2.3 #42

WolframAlpha computational intelligence.

extrema of 2cos(x)^2-sin(x)



Calculus: Find slope function, $f'(x)$ & SET $= 0$

$f(x) = 2\cos^2(x) - \sin(x) \implies$

$f'(x) = 4\cos(x)(-\sin(x)) - \cos(x)$

$= \cos(x)\left(-4\sin(x) - 1\right)$

$\cos(x) = 0$ or $4\sin(x) + 1 = 0$

$\sin(x) = -\frac{1}{4}$

etc.

Find the exact value of

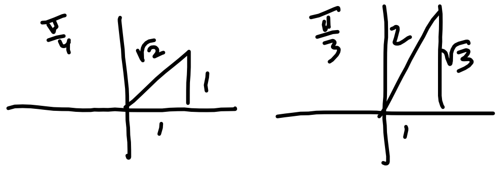
$\sin\left(\frac{\pi}{12}\right)$

$\frac{\pi}{12} = \frac{\pi}{3 \cdot 4} =$

$\frac{\pi}{3} - \frac{\pi}{4} = \frac{4\pi - 3\pi}{12} = \frac{\pi}{12}$

$\sin\left(\frac{\pi}{12}\right) = \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \sin\left(\frac{\pi}{3}\right)\cos\left(-\frac{\pi}{4}\right) + \sin\left(-\frac{\pi}{4}\right)\cos\left(\frac{\pi}{3}\right)$

$= \sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{3}\right)$



$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$

$= \frac{\sqrt{3}-1}{2\sqrt{2}}$

$= \frac{\sqrt{3}-1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

S2.4

Angle Sum Formulas

$\sin(u+v) = \sin(u)\cos(v) + \sin(v)\cos(u)$

$\cos(u+v) = \cos(u)\cos(v) - \sin(u)\sin(v)$

Double-Angle

$\cos(2u) = \cos(u)\cos(u) - \sin(u)(\sin(u))$

$= \cos^2(u) - \sin^2(u)$

$= \cos^2(u) - (1 - \cos^2(u))$

$= 2\cos^2(u) - 1 = \cos(2u)$

Let $v = 2u$

Then $2\cos^2\left(\frac{v}{2}\right) - 1 = \cos(v)$

$2\cos^2\left(\frac{v}{2}\right) = \cos(v) + 1 = 1 + \cos(x)$

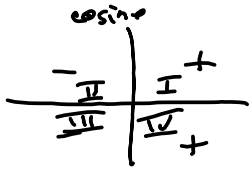
$\Longrightarrow \cos^2\left(\frac{v}{2}\right) = \frac{1+\cos(v)}{2}$

$\cos\left(\frac{v}{2}\right) = \pm\sqrt{\frac{1+\cos(v)}{2}}$

HALF-ANGLE FORMULA

It will be + if $\frac{v}{2} \in$ QI or QIV

cosine



what about

$\sin(2\theta) = 2\sin\theta\cos\theta$

$\tan(2\theta) = \frac{\sin(2\theta)}{\cos(2\theta)}$ is the cheap way:

$= \frac{2\tan(u)}{1-\tan^2(u)}$ is the book way & probably BETTER, b/c it's easier to simplify.

$$\tan(2u) = \frac{2\tan(u)}{1-\tan^2(u)} = \frac{\sin(2u)}{\cos(2u)}!$$

Half-Angle Formulas: $\sin\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1-\cos(u)}{2}}$, $\cos\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1+\cos(u)}{2}}$, $\tan\left(\frac{u}{2}\right) = \frac{1-\cos(u)}{\sin(u)} = \frac{\sin\left(\frac{u}{2}\right)}{\cos\left(\frac{u}{2}\right)}$!!!

You have to determine the "$\pm$" deal by determing the quadrant in which $\frac{u}{2}$ resides.

Don't sweat separate formulas for cos(*u* - *v*):

$\cos(u-v) = \cos(u+(-v))$

$$\sin\left(\frac{11\pi}{12}\right) = \sin\left(\frac{3\pi}{4}+\frac{\pi}{6}\right) = \sin\left(\frac{3\pi}{4}\right)\cos\left(\frac{\pi}{6}\right)+\sin\left(\frac{\pi}{6}\right)\cos\left(\frac{3\pi}{4}\right)$$

Scratch

$$\frac{11}{12}=\frac{10+1}{12}=\frac{9+2}{12}=\frac{9}{12}+\frac{2}{12}=\frac{3}{4}+\frac{1}{6} \text{ !}$$

$$= \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2} + \frac{1}{2}\left(-\frac{1}{\sqrt{2}}\right)$$

$$= \frac{\sqrt{3}-1}{2\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}-\sqrt{2}}{4}$$

$$\cos\left(\frac{11\pi}{12}\right) = \cos\left(\frac{3\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) - \sin\left(\frac{3\pi}{4}\right)\sin\left(\frac{\pi}{6}\right)$$

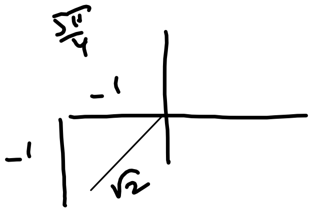
$165^\circ = \frac{11\pi}{12}$

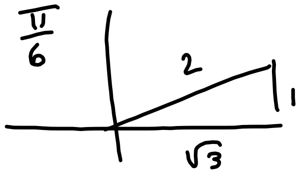
$$= \left(-\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) - \frac{1}{\sqrt{2}}\left(\frac{1}{2}\right) = \frac{-\sqrt{3}-1}{2\sqrt{2}} = \frac{-\sqrt{6}-\sqrt{2}}{4}$$

$$\frac{17\pi}{12} = \frac{5\pi}{4} + \frac{\pi}{6}$$

$$\frac{17}{12} = \frac{16+1}{12} = \frac{15+2}{12} = \frac{15}{12} + \frac{2}{12} = \frac{5}{4} + \frac{1}{6}$$

$$\sin\left(\frac{17\pi}{12}\right) = \sin\left(\frac{5\pi}{4} + \frac{\pi}{6}\right)$$





Write the expression as the sine, cosine, or tangent of an angle.

$\sin 2 \cos 1.4 - \cos 2 \sin 1.4$

$$= \sin(2)\cos(1.4) + \cos(2)\sin(-1.4)$$

$$= \sin(2 + (-1.4)) = \sin(.6)$$

Write the trigonometric expression as an algebraic expression.

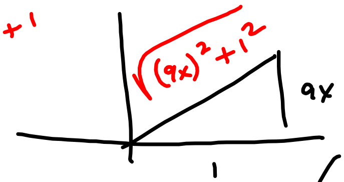
$\sin(\arctan 9x - \arccos x) = \sin(u - v)$

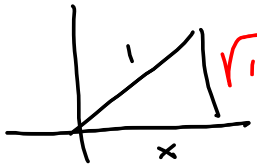
$= \sin(u)\cos(-v) + \sin(-v)\cos(u)$

$= \sin(u)\cos(v) - \sin(v)\cos(u)$

$u = \arctan(9x)$ $\qquad v = \arccos(x)$

$\sqrt{81x^2+1}$





$\sqrt{1^2-x^2} = \sqrt{1-x^2}$

$$\left(\frac{9x}{\sqrt{81x^2+1}}\right)(x) - \left(\sqrt{1-x^2}\right)\left(\frac{1}{\sqrt{81x^2+1}}\right)$$

$$= \frac{9x^2 - \sqrt{1-x^2}}{\sqrt{81x^2+1}}$$