Use the trigonometric substitution to write the algebraic equation as a trigonometric equation of θ, where $-\pi/2 < \theta < \pi/2$.

2.1 II $4 = \sqrt{16 - x^2}$, $x = 4\sin\theta$

#14 Find $\sin\theta$ and $\cos\theta$. (Enter your answers as a comma-separated list.)

$$4 = \sqrt{16 - (4\sin\theta)^2} = \sqrt{16 - 16\sin^2\theta} = \sqrt{16}\sqrt{1-\sin^2\theta}$$

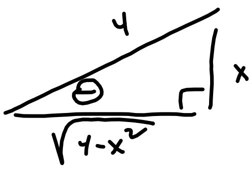
(a)

$$= 4\sqrt{\cos^2\theta} = 4|\cos\theta| = \boxed{4\cos\theta} \text{ b/c } -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

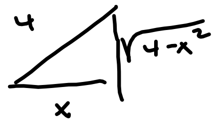
puts us in QI & QIV, where $\cos\theta > 0$.

(b) $x = 4\sin\theta$

$\frac{x}{4} = \sin\theta$



OR

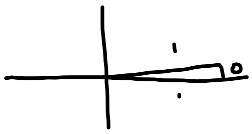


$x = 4\cos\theta$

Telling us $x = 4\sin\theta$ says that isn't the triangle

$4\cos\theta = 4$

$\cos\theta = 1 \Rightarrow$



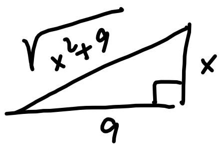
$\Rightarrow \sin\theta = 0!$

$\mu W\cos\theta = W\sin\theta$

$$\Rightarrow \mu = \frac{W\sin\theta}{W\cos\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

Other Trig Substitution Structures / Pythagorean Expressions

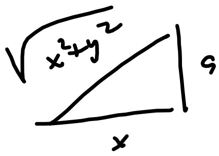
$$\sqrt{x^2+a^2} = \sqrt{x^2+9}$$



most common

$\frac{x}{9} = \tan\theta \Rightarrow$

$x = 9\tan\theta$



Less common but legit (careful with domains in CALC II)

$$\Rightarrow \sqrt{x^2+9} = \sqrt{(9\tan\theta)^2+9} = 3\sqrt{\tan^2\theta+1} = 3\sqrt{\sec^2\theta}$$

$$= 3|\sec\theta| = 3\sec\theta, \text{ if } -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

2.1 Part II

Rewrite the expression so that it is not in fractional form. There is more than one correct form of the answer.

10 $\dfrac{\sin^2 y}{1 - \cos y}$

11 Rewrite the expression so that it is not in fractional form. There is more than one correct form of the answer.

$$\frac{4}{\tan x + \sec x} =$$

$$\frac{4}{\tan(x)+\sec(x)} \cdot \frac{\tan(x)-\sec(x)}{\tan(x)-\sec(x)} = \frac{4(\tan(x)-\sec(x))}{\tan^2(x)-\sec^2(x)}$$

$$= \frac{4(\tan(x)-\sec(x))}{\sec^2(x)-1-\sec^2(x)} = \frac{4(\tan(x)-\sec(x))}{-1} = 4(\sec(x)-\tan(x))$$

See CALC I & ALGEBRA $\left(\dfrac{1}{\sqrt{x+h}+\sqrt{x}}\right)\left(\dfrac{\sqrt{x+h}-\sqrt{x}}{\sqrt{x+h}-\sqrt{x}}\right) = \dfrac{\sqrt{x+h}-\sqrt{x}}{x+h-x} = \dfrac{\sqrt{x+h}-\sqrt{x}}{h}$

"Conjugate Trick"

ALTERNATE

$$\frac{4}{\tan(x)+\sec(x)} = \frac{4}{\frac{\sin(x)}{\cos(x)} + \frac{1}{\cos(x)}} = \frac{4}{\frac{\sin(x)+1}{\cos(x)}} =$$

$$= \left(\frac{4\cos(x)}{\sin(x)+1}\right)\left(\frac{\sin(x)-1}{\sin(x)-1}\right) = \frac{4\cos(x)(\sin(x)-1)}{\sin^2(x)-1}$$

$$= \frac{4\cos(x)(\sin(x)-1)}{-\cos^2(x)} = \frac{4(\sin(x)-1)}{-\cos(x)} = 4\frac{\sin(x)}{-\cos(x)} - \frac{4}{-\cos(x)}$$

$$4(\tan(x)-\sec(x)) = 4(\sec(x)-\tan(x))$$

Section 2.3

Fill in the blank.

The equation $2\tan^2(x) - 3\tan(x) + 1 = 0$ is a trigonometric equation of [ ] type.

Quadratic

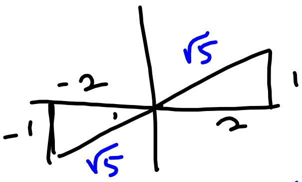
$2u^2 - 3u + 1$

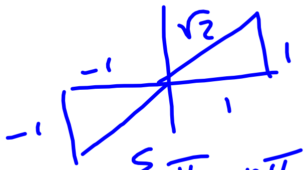
$= (2u - 1)(u - 1) = 0$

$\Rightarrow u = \frac{1}{2}$ $u = 1$

$\tan(x) = \frac{1}{2}$ $\tan(x) = 1$

OR





$x = \arctan\left(\frac{1}{2}\right), \pi + \arctan\left(\frac{1}{2}\right)$

solves it on $[0, 2\pi)$

$\left\{ \frac{\pi}{4} + n\pi \mid n \in \mathbb{Z} \right\}$

WebAssign $\frac{\pi}{4} + n\pi$

$\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

Now find all sol'ns

sol'n set:

$\left\{ \arctan\left(\frac{1}{2}\right) + n\pi \mid n \in \mathbb{Z} \right\}$

WebAssign:

$\arctan\left(\frac{1}{2}\right) + n\pi$

$\boxed{\text{combine for WebAssign: } \arctan\left(\tfrac{1}{2}\right) + n\pi, \ \tfrac{\pi}{4} + n\pi}$

Sam