Fundamental Trigonometric Identities

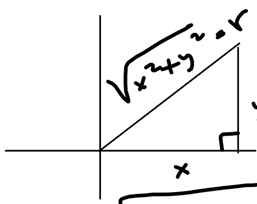
Reciprocal Identities

$$\sin u = \frac{1}{\csc u} \qquad \cos u = \frac{1}{\sec u} \qquad \tan u = \frac{1}{\cot u}$$

$$\csc u = \frac{1}{\sin u} \qquad \sec u = \frac{1}{\cos u} \qquad \cot u = \frac{1}{\tan u}$$

Quotient Identities

$$\tan u = \frac{\sin u}{\cos u} \qquad \cot u = \frac{\cos u}{\sin u}$$



$$\cot(u) = \frac{x}{y} = \frac{\frac{x}{\sqrt{x^2+y^2}}}{\frac{y}{\sqrt{x^2+y^2}}} = \frac{\cos(u)}{\sin(u)}$$

Pythagorean Identities

$$\sin^2 u + \cos^2 u = 1 \qquad \underline{1 + \tan^2 u = \sec^2 u} \qquad \boxed{1 + \cot^2 u = \csc^2(u)}$$

$$\cot^2(u) = \csc^2(u) - 1$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - u\right) = \cos u \qquad \cos\left(\frac{\pi}{2} - u\right) = \sin u$$

$$\tan\left(\frac{\pi}{2} - u\right) = \cot u \qquad \cot\left(\frac{\pi}{2} - u\right) = \tan u$$

$$\sec\left(\frac{\pi}{2} - u\right) = \csc u \qquad \csc\left(\frac{\pi}{2} - u\right) = \sec u$$

Even/Odd Identities

EVEN

$$\sin(-u) = -\sin u \qquad \boxed{\cos(-u) = \cos u} \qquad \tan(-u) = -\tan u$$

$$\csc(-u) = -\csc u \qquad \boxed{\sec(-u) = \sec u} \qquad \cot(-u) = -\cot u$$

Use the given values to find the values (if possible) of all six trigonometric functions. (If an answer is undefined, enter UNDEFINED.)

$\sin x = -\frac{1}{2}$, $\cos x = \frac{\sqrt{3}}{2}$

$\sin x = -\frac{1}{2}$ $\qquad \csc(x) = -2$

$\cos x = \frac{\sqrt{3}}{2}$ $\qquad \sec(x) = \frac{2}{\sqrt{3}}$

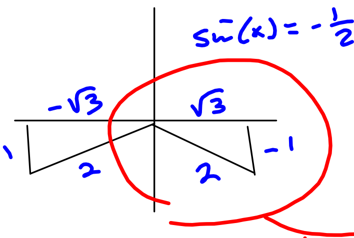
$\tan x = -\frac{1}{\sqrt{3}}$ $\qquad \cot(x) = -\sqrt{3}$

$\csc x =$

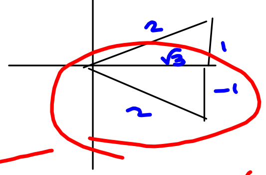
$\sec x =$

$\cot x =$

$\sin(x) = -\frac{1}{2}$



$\cos(x) = \frac{\sqrt{3}}{2}$



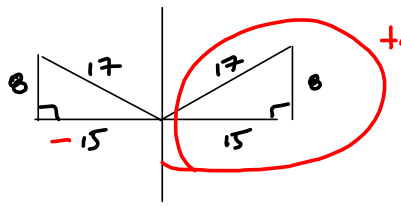
if $\sin(x) = -\frac{1}{2}$ and $\cos(x) = \frac{\sqrt{3}}{2}$

On tests, be sure to submit your answers before moving on!

$\csc\theta = \frac{17}{8}$, $\tan\theta = \frac{8}{15}$

$\sin\theta = \frac{8}{17}$

$\tan\theta > 0$

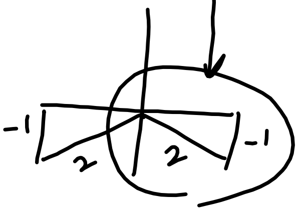


$\sin\theta = \frac{8}{17}$ $\qquad \csc\theta = \frac{17}{8}$

$\cos\theta = \frac{15}{17}$ $\qquad \sec\theta = \frac{17}{15}$

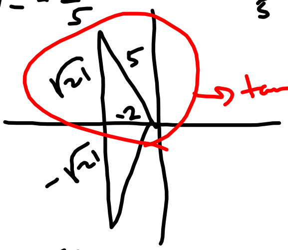
$\tan\theta = \frac{8}{15}$ $\qquad \cot\theta = \frac{15}{8}$

$\sin x = -\frac{1}{2}, \cos x = \frac{\sqrt{3}}{2}$



$\sec(x) = -\frac{5}{2}, \quad \tan(x) < 0$

$\cos(x) = -\frac{2}{5}$ $\quad 5^2 - (-2)^2 = 25 - 4 = 21$



$\rightarrow \tan(x) < 0$

etc
(Find 6 trigs)

Simplify

$\sec(x)\cos(x) = \frac{1}{\cos(x)}\cos(x) = 1$

$\cot^2\theta - \csc^2\theta$

$\csc^2\theta - 1 - \csc^2\theta = -1$

$\cos(x)(\cot^2(x)+1)$

$= \cos(x)\csc^2(x)$

$= \cos(x)\left(\frac{1}{\sin^2(x)}\right)$

$= \cot(x)\left(\frac{1}{\sin(x)}\right) = \cot(x)\csc(x)$

○ $\sec^2(x) + \tan^2(x)$
○ $\csc(x)$
○ $\cos(x) + \sin(x)$
○ $\sec(x)$
○ $\sec^2(x)$
○ $\sin(x)\tan(x)$

$\cos(x) + \cos(x)\tan^2(x)$

$\cos(x) + \cos(x)\left(\frac{\sin^2(x)}{\cos^2(x)}\right)$

$= \cos(x) + \frac{\sin^2(x)}{\cos(x)}$

$= \frac{\cos^2(x)}{\cos(x)} + \frac{\sin^2(x)}{\cos(x)} = \frac{1}{\cos(x)} = \sec(x)$

Factor the expression and use the fundamental identities to simplify. There is more than one correct form of the answer.

$7\tan^2 x - 7\tan^2 x \sin^2 x$

$= 7\tan^2(x)\left[1-\sin^2(x)\right] = 7\tan^2(x)\cos^2(x)$

$= 7\left(\frac{\sin^2(x)}{\cos^2(x)}\right)\cos^2(x) = \boxed{7\sin^2(x)} = 7 - 7\cos^2(x)$

Factor the expression. Use the fundamental identities to simplify, if necessary. (There is more than one correct form of each answer.)

$\csc^4(x) - \cot^4(x)$

$= (\csc^2(x) - \cot^2(x))(\csc^2(x) + \cot^2(x)) = (\cot^2(x)+1-\cot^2(x))(\cot^2(x)+1+\cot^2(x))$

$= (\csc(x)-\cot(x))(\csc(x)+\cot(x))(\csc^2(x)+\cot^2(x))$

$= 1(2\cot^2(x)+1)$

$= \boxed{2\cot^2(x)+1}$

$(\csc^2(x))^2 - (\cot^2(x))^2$

$(\cot^2(x)+1)^2 - \cot^4(x)$

$\cot^2(x) + 2\cot(x) + 1 - \cot^4(x)$

$= \cot^2(x) - \cot^4(x) + 2\cot(x) + 1$

$= \cot^2(x)\underbrace{\left[1-\cot^2(x)\right]}_{-\csc^2(x)} + 2\cot(x) + 1$

Factor the expression. Use the fundamental identities to simplify, if necessary. (There is more than one correct form of each answer.)

$6\cos^2(x) + 7\cos(x) - 3$

Let $u = \cos(x)$

$6u^2 + 7u - 3$

$= 6u^2 + 9u - 2u - 3$

$= 3u(2u+3) - 1(2u+3)$

$= (2u+3)(3u-1)$

$= (2\cos(x) + 3)(3\cos(x) - 1)$

24
3

CHEAT!

Suck @ Factoring?

Q.F. $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$a=6, b=7, c=-3$

$b^2-4ac = 7^2-4(6)(-3)$

$= 49+72 = 121 = 11^2$

$x = \dfrac{-7 \pm 11}{2(6)} = \dfrac{4}{12} = \dfrac{1}{3}$, $\dfrac{-18}{12} = -\dfrac{3}{2}$

$6u^2 + 7u - 3$

$= 6\left(u - \frac{1}{3}\right)\left(u + \frac{3}{2}\right)$

$= 3 \cdot 2 \left(u - \frac{1}{3}\right)\left(u + \frac{3}{2}\right)$

$= 3\left(u - \frac{1}{3}\right)(2)\left(u + \frac{3}{2}\right)$

$= (3u-1)(2u+3)$

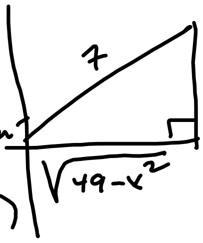
Trigonometric Substitution

Use the trigonometric substitution to write the algebraic expression as a trigonometric function of θ, where $0 < \theta < \pi/2$.

$\sqrt{49 - x^2}$, $x = 7 \sin \theta$

Recognize the Pythagorus in $\sqrt{49-x^2}$

$\sqrt{49-x^2}$



$\frac{x}{7} = \sin\theta$

$\Rightarrow x = 7\sin\theta$

$(\sqrt{x})^2 = x$ (It's understood $x \geq 0$.)

$\sqrt{x^2} = |x|$

$\sqrt{(-3)^2} = \sqrt{9} = 3 = -(-3)$

$\sqrt{49-x^2} = \sqrt{49-(7\sin\theta)^2}$

$= \sqrt{49-49\sin^2\theta}$

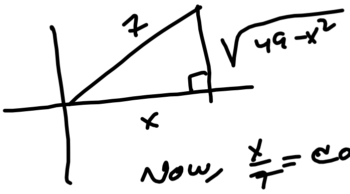
$= \sqrt{49}\sqrt{1-\sin^2\theta}$

$= 7\sqrt{\cos^2\theta} = 7|\cos\theta|$

$= 7\cos\theta$, but only because they said $0<\theta<\frac{\pi}{2}$ at the top

If there were no restriction on theta, then we couldn't simplify | cos(theta) |

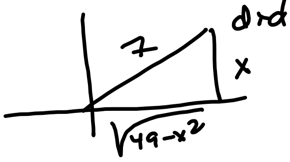
A legitimate sub for this $\sqrt{49-x^2}$ would be



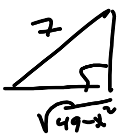
Now, $\frac{x}{7} = \cos\theta$

$x = 7\cos\theta$

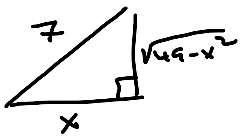
What we did



Using cosine instead of sine, here, is fine, but the standard way is to use sine instead of cosine.



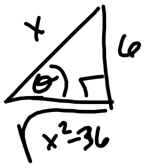
or



are both fine.

Use the trigonometric substitution to write the algebraic expression as a trigonometric function of θ, where $0 < \theta < \pi/2$.

$\sqrt{x^2 - 36}$, $x = 6 \sec \theta$

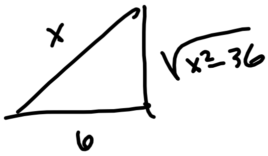


$\frac{6}{x} = \sin\theta$ (one way.)

$6 = x \sin\theta$

$\frac{6}{\sin\theta} = x = 6\csc\theta$

The $x = 6\sec\theta$ suggests this triangle:



$\Rightarrow \frac{6}{x} = \cos\theta$

$6\sec\theta = x$

$\Rightarrow \sqrt{(6\sec\theta)^2 - 36}$

$\sqrt{36\sec^2\theta - 36}$

$= \sqrt{36}\sqrt{\sec^2\theta - 1}$

$= 6\sqrt{\tan^2\theta} = 6|\tan\theta|$

& $0 < \theta < \frac{\pi}{2} \Rightarrow = 6\tan\theta.$

$ax^2 + bx + c = 0$

$x^2 + \frac{b}{a}x = -\frac{c}{a}$

$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}\cdot\frac{4a}{4a} = \frac{b^2-4ac}{4a^2}$

$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2-4ac}{4a^2}$

$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \sqrt{\frac{b^2-4ac}{4a^2}} = \frac{\sqrt{b^2-4ac}}{2|a|}$

$\left|x + \frac{b}{2a}\right| = \frac{\sqrt{b^2-4ac}}{2|a|}$

$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2-4ac}}{2|a|} = \pm\frac{\sqrt{b^2-4ac}}{2a}$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$