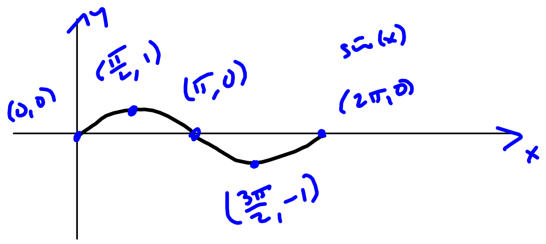
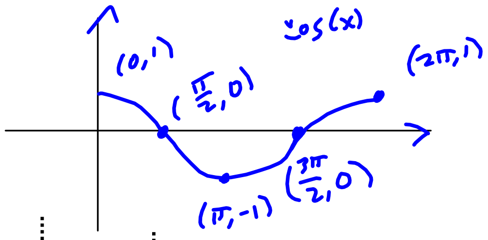
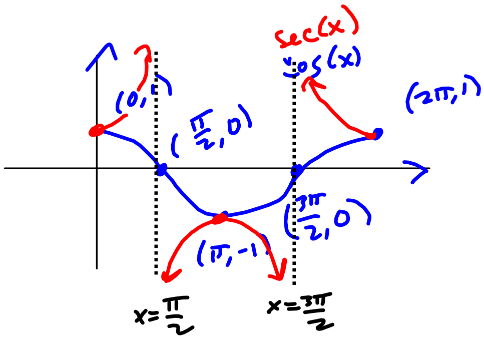
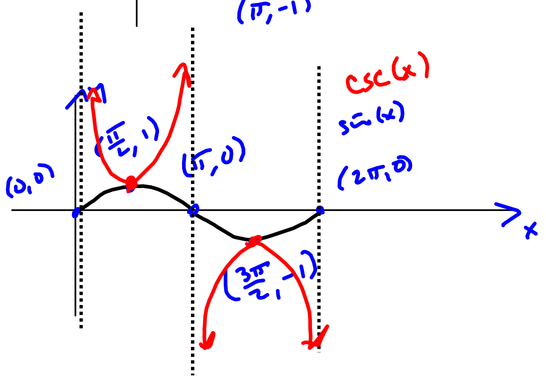
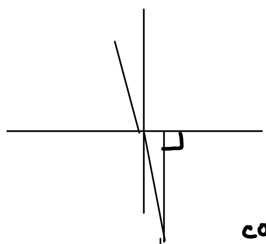
Please remind me to hit 'record.'

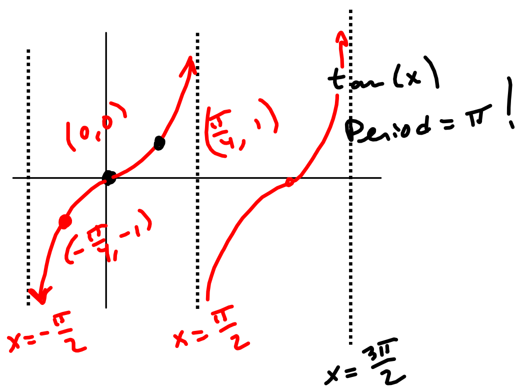


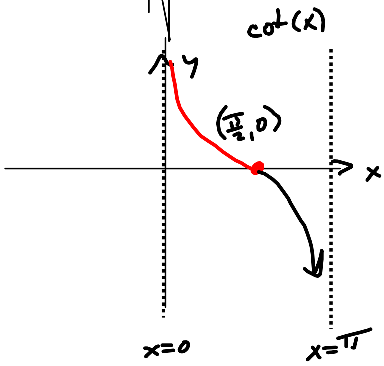




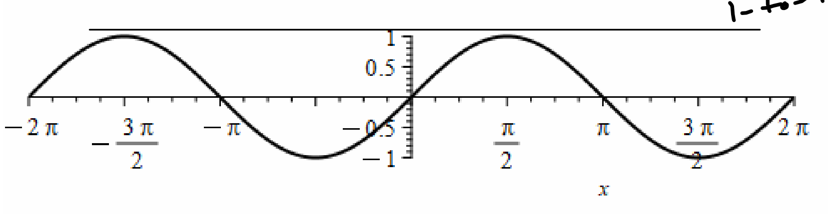






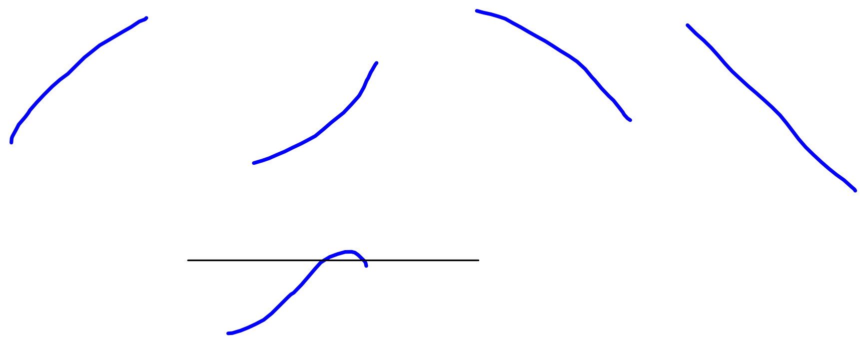


sin(x) isn't 1-to-1

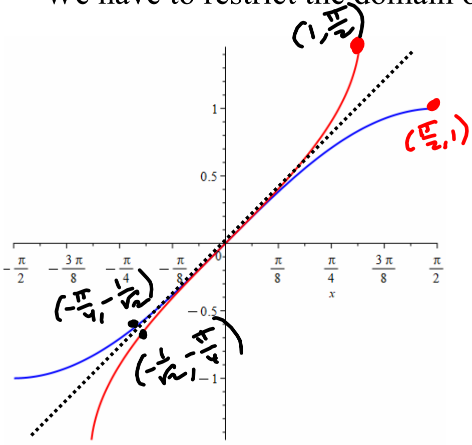


A function f(x) is a rule that assigns to each x in the domain of f a unique y-value.

A function f(x) is 1-to-1, if for every y there is only one x.



We have to restrict the domain of sine to make it 1-to-1. Likewise, cosine and tangent.



sin(x) in blue

arcsin(x) in red.

$\arcsin(x) = \sin^{-1}(x)$ is the inverse of $\sin(x)$

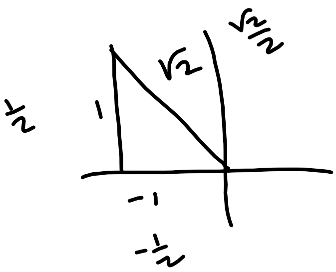
Inverse takes you back to x from $\sin(x)$.

$\arcsin(\sin(\frac{\pi}{4})) = \frac{\pi}{4}$ is good.

$\arcsin(\sin(\frac{3\pi}{4})) = ?$

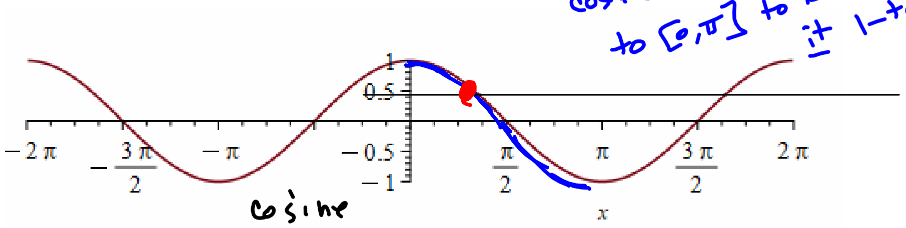
$\arcsin(-\frac{1}{\sqrt{2}}) = -\frac{\pi}{4}$

To keep $\sin(x)$ 1-to-1, we restrict x to $[-\frac{\pi}{2}, \frac{\pi}{2}]$



We obtain the graph of the inverse by reflecting about the line y = x. This swaps x and y!

cosine restricted to $[0,\pi]$ to keep it 1-to-1.



$y = -\frac{1}{\sqrt{2}}$

cosine

$\mathcal{D} = [0,\pi]$

$\mathcal{R} = [-1,1]$

Restricted.

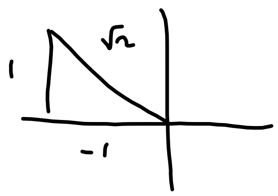
arccosine

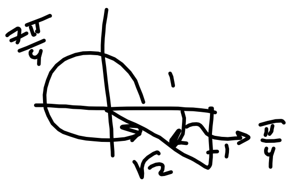
$\mathcal{D} = [-1,1]$

$\mathcal{R} = [0,\pi]$

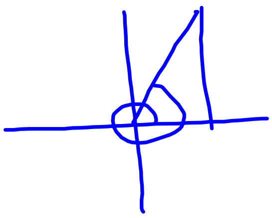
$$\arccos\left(\cos\left(\frac{3\pi}{4}\right)\right) = \arccos\left(-\frac{1}{\sqrt{2}}\right) = \frac{3\pi}{4}$$

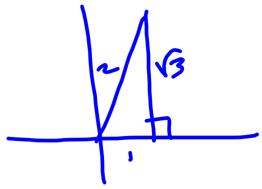
$$\arccos\left(\cos\left(\frac{7\pi}{4}\right)\right) = \arccos\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$$





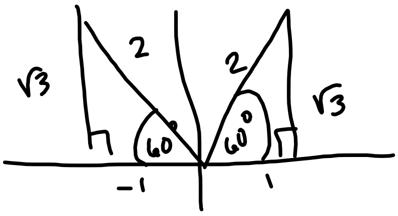
$\arcsin\left(\sin\left(\frac{7\pi}{3}\right)\right) = \arcsin\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$

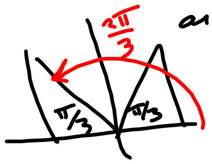




$\frac{7\pi}{3} = \frac{6\pi}{3} + \frac{\pi}{3} \to \frac{\pi}{3}$

Solve $\sin(x) = \frac{\sqrt{3}}{2}$





NOTE
$\arcsin(\sin(x)) = \arcsin\left(\frac{\sqrt{3}}{2}\right)$
$= \frac{\pi}{3} = 60^\circ$

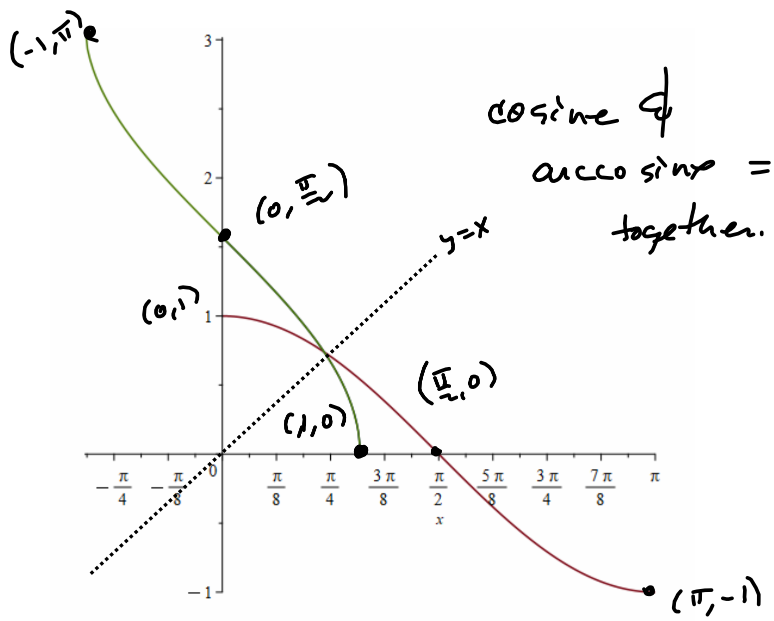
Two solutions in $[0, 2\pi]$

Your calculator can only spit out one of them

Restricting sine's domain to $[-\frac{\pi}{2}, \frac{\pi}{2}]$

means arcsine will have a range

that's restricted to $[-\frac{\pi}{2}, \frac{\pi}{2}]$



cosine &

arccosine $= \cos^{-1}$

together.

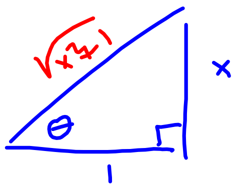
Trig substitution in Calculus II

Write an algebraic expression that is equivalent to the given expression.

$\sin(\arctan(x)) = \sin(\theta)$

$\int \frac{x}{x^2+1}\,dx$

$= \int \sin\theta \sim$



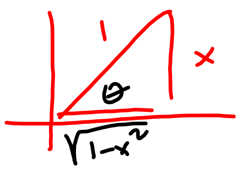
arctan(x) picture

Pythagorus!

$\sqrt{x^2+1}$

$$\boxed{\sin(\theta) = \frac{x}{\sqrt{x^2+1}}}$$

$$\sec(\arcsin(x)) = \sec(\theta) = \frac{1}{\sqrt{1-x^2}}$$

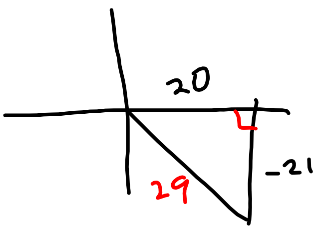


$\sqrt{1^2 - x^2}$

Find the exact value of the expression, if possible. (If not possible, enter IMPOSSIBLE.)

$$\csc\left[\arctan\left(-\frac{21}{20}\right)\right] = \csc\theta$$

$$\arctan\left(-\frac{21}{20}\right)$$



$$\sqrt{20^2+21^2}$$

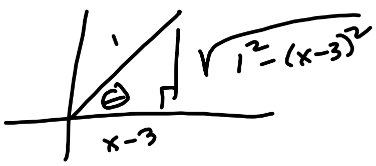
cosecant :

$$\csc\left(\arctan\left(-\frac{21}{20}\right)\right)$$

$$= \frac{1}{\sin\left(\arctan(-21/20)\right)}$$

$$= \frac{1}{-\frac{21}{29}} = -\frac{29}{21} = \csc\theta$$

$$\csc(\arccos(x-3)) = \csc\theta = \frac{1}{\sqrt{1-(x-3)^2}}$$

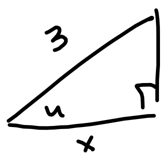


$$\sin\theta = \sqrt{1-(x-3)^2}$$

$$\Rightarrow \csc\theta = \frac{1}{\sqrt{1-(x-3)^2}}$$

$$\tan\left(\arccos\left(\frac{x}{3}\right)\right)$$

$$= \tan(u)$$



$$\sqrt{3^2 - x^2} = \sqrt{9-x^2}$$

$$\tan(u) = \frac{\sqrt{9-x^2}}{x}$$