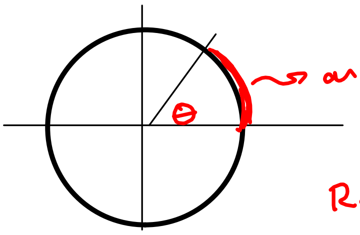
Recall

$\theta = \frac{s}{r}$ = radian measure of the angle θ

$\theta = 1 \Rightarrow \frac{s}{r} = 1 \Rightarrow s = r$



$\rightsquigarrow$ arc length = radius

$\theta = 1$ radian

Radians are the link between angle & arc length.

Arc Length of a circle of radius r:

$\theta = \frac{s}{r} \Rightarrow r\theta = s$! $\quad$ θ Must be in radians for this formula to hold!

Circumference of a circle of radius $r=5$.

$s = r\theta = 5 \cdot 2\pi = 10\pi$

How about arc length of angle subtended by an angle of $120°$?

$$s = r\theta = r \cdot \underbrace{(120°)\left(\frac{\pi}{180°}\right)}_{\text{Convert to radians!}} = \frac{2\pi}{3} r = \frac{2\pi r}{3}$$

CONVERT TO Miles/hr:

$$\frac{75 \text{ ft}}{\text{sec}} = \frac{75 \text{ ft}}{\text{sec}} \cdot \frac{1 \text{ MILE}}{5280 \text{ ft}} \cdot \frac{60 \text{ sec}}{1 \text{ MIN}} \cdot \frac{60 \text{ MIN}}{1 \text{ Hr}}$$

is now in $\frac{\text{mi}}{\text{hr}}$

```
75*60*60/5280
     51.13636364
75*60/88
     51.13636364
```

$$= \left(\frac{\frac{75 \text{ ft}}{\text{sec}}}{1}\right)\left(\frac{\frac{60 \text{ Miles}}{\text{hr}}}{\frac{88 \text{ ft}}{\text{sec}}}\right) \approx 51.13636364 \frac{\text{mi}}{\text{hr}}$$

Link Angle to arc length

1 revolution $\longrightarrow$ 2π radians.

I'm pedaling my bike @ 2 revolutions /sec

Convert that to arc length (length of chain)

in radians/sec

$$\left(\frac{2 \text{ revs}}{1 \text{ sec}}\right)\left(\frac{2\pi \text{ radians}}{1 \text{ rev}}\right) = \frac{4\pi \text{ radians}}{\text{sec.}}$$

If the front sprocket has a radius of 4 inches, how much chain per second?

$$\left(\frac{4\pi \text{ radians}}{\text{sec}}\right)(4 \text{ inches})$$

$$\left(\frac{\text{change in Angle}}{\text{sec}}\right)(\text{radius}) = \frac{\theta \cdot r}{\text{sec}} = \frac{\text{arc length}}{\text{sec}}$$

$= 16\pi$ inches/sec.

The rear sprocket has a radius of 2 inches.
How fast is the rear wheel turning? (Radians/sec)
which is angular speed of the rear wheel.

$$\frac{16\pi \text{ inches}}{\text{sec}} \Rightarrow \left(\frac{\text{change in arc length}}{\text{sec}}\right)\left(\frac{1}{r}\right)$$

$$= \text{Radians/Sec}$$

$$\theta = \frac{s}{r} \qquad = \text{change in angle/sec}$$

$$\frac{s}{\text{sec}} = r\frac{\theta}{\text{sec}} \qquad = \frac{16\pi}{2}\ \frac{\text{radians}}{\text{sec}}$$

Rear tire has 14-inch radius

Then multiply $\frac{\text{radians}}{\text{sec}} \cdot 14$ to get inches/sec along the ground

$(8\pi)(14)\ \frac{\text{inches}}{\text{sec}}$ along the ground.

$= 120\pi\ \frac{\text{inches}}{\text{sec}}$.

Dad's stride is 3 feet. Junior's stride is 2 feet.

Dad's striding at 120 steps per minute. What's Junior's step rate?

$$\left(\frac{120 \text{ steps by Dad}}{\text{minute}}\right) \frac{3 \text{ STEPS by Junior}}{2 \text{ STEPS}} = 180\ \frac{\text{steps}}{\text{minute}}$$

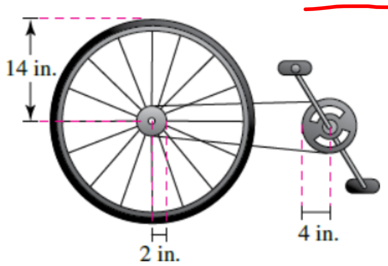
Front Sprocket is 4-inch radius &
rear Sprocket is 3-inch radius, then

$$(1 \text{ rev front sprocket})\left(\frac{4 \text{ revs rear sprocket}}{3 \text{ revs front sprocket}}\right)$$

$$= \frac{4}{3} \text{ rev rear sprocket}$$

4. 0/6 points LarTrig10 1.1.068. [388

The radii of the pedal sprocket, the wheel sprocket, and the wheel of the bicycle in the figure are 4 inches, 2 inches, and 14 inches, respectively. A cyclist pedals at a rate of 1 revolution per second.



START: $\frac{\text{Revs}}{\text{sec}}$ .... END SPEED of BIKE

(a) Find the speed of the bicycle in feet per second and miles per hour.

$\frac{14\pi}{3}$ feet per second

$\frac{35\pi}{11}$ mph

$\frac{\text{Radians}}{\text{sec}}$ Rear $\frac{\theta}{\text{sec}}$

$$\left(\frac{1 \text{ REV } \cancel{\text{FRONT SPROCKET}}}{\text{SEC}}\right)\left(\frac{4 \text{ REVS REAR SPROCKET}}{2 \text{ REVS } \cancel{\text{FRONT SPROCKET}}}\right) \cdot \left(\frac{2\pi \text{ radians}}{1 \text{ rev Rear}}\right) \cdot \left(\frac{14\text{-inch radius}}{\text{rear wheel}}\right)$$

radius rear: r

$$\cdot\left(\frac{1 \text{ ft}}{12 \text{ inches}}\right) = \frac{4\pi \cdot 14}{12} = \frac{14\pi}{3} \frac{\text{ft}}{\text{sec}}$$

$$\left(\frac{14\pi}{3} \frac{\text{ft}}{\text{sec}}\right)\left(\frac{60 \text{ mph}}{88 \frac{\text{ft}}{\text{sec}}}\right) = \left(\frac{14\pi}{3}\right)\left(\frac{15}{22}\right) = \left(\frac{7\pi}{1}\right)\left(\frac{5}{11}\right)$$

$$= \boxed{\frac{35\pi}{11} \frac{\text{mi}}{\text{hr}}}$$

Area of a circle of radius r is πr^2 = Area

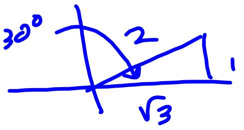
All the way around the circle is 2π radians.

$$\pi r^2 = (2\pi)\left(\tfrac{1}{2}\right) r^2$$

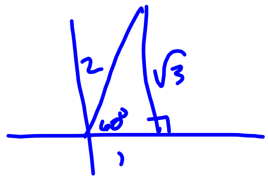
$$= \theta \cdot \tfrac{1}{2} r^2 = \boxed{\tfrac{1}{2} r^2 \theta}$$ = Area of the sector swept by angle on a circle of radius r.

$$\boxed{\begin{aligned} s &= r\theta \\ A &= \tfrac{1}{2} r^2 \theta \end{aligned}}$$

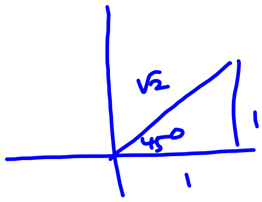
§1.2 Quickies:



$\frac{\pi}{6} = 30°$

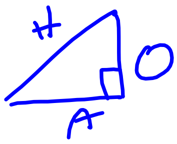


$\frac{\pi}{3} = 60°$



$\frac{\pi}{4} = 45°$

$\sin\theta = \frac{O}{H}$, $\cos\theta = \frac{A}{H}$, $\tan\theta = \frac{O}{A}$



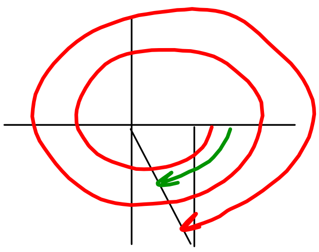
$\cos\left(-\frac{13\pi}{3}\right)$

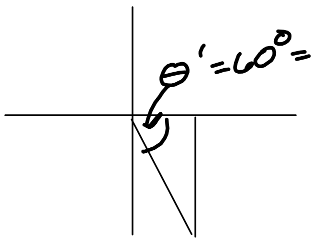
cosine repeats every 2π radians

$$-\frac{13\pi}{3} = -\frac{12\pi}{3} - \frac{\pi}{3} = -4\pi - \frac{\pi}{3}$$

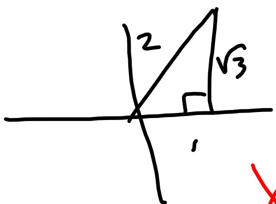
$$\cos\left(-4\pi - \frac{\pi}{3}\right) = \cos\left(-\frac{\pi}{3}\right)!$$

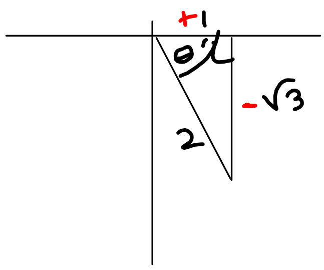
So $-\frac{13\pi}{3}$ is coterminal with $-\frac{\pi}{3}$!





$\theta' = 60° = \frac{\pi}{3}$ is reference angle, so this is a 60-30 right triangle





In QIV, ~~x &~~ y are < 0

Dummy!

$$\sin\theta = \frac{-\sqrt{3}}{2}$$

$$\cos\theta = \frac{1}{2}$$

$$\tan\theta = \frac{-\sqrt{3}}{1} = -\sqrt{3}$$