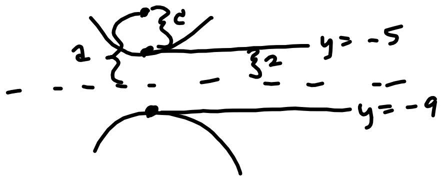
Last couple questions.

This one was covered, clumsily, last Thursday

Hyperbola, vertices @ $(5, \frac{3\pi}{2})$, $(9, \frac{3\pi}{2})$

A B



$e = \frac{c}{a}$. Center $= (x,y) = (0,-7)$ $\left((7, \frac{3\pi}{2}) \right)$

c = distance from focus (the pole) to the center $= 7$

$2a$ = distance between vertices $= 9 - 5 = 4 \Longrightarrow$ $\boxed{a = 2}$

Now, $e = \frac{c}{a} = \frac{7}{2}$

Directrix is horizontal: $\frac{ep}{1 \pm e\sin\theta}$

.. below the pole: $\frac{ep}{1 - e\sin\theta}$

$$\frac{\frac{7}{2}p}{1 - \frac{7}{2}\sin\left(\frac{3\pi}{2}\right)} = 5 \Longrightarrow$$

$$\frac{\frac{7}{2}p}{1 + \frac{7}{2}} = \frac{\frac{7}{2}p}{\frac{9}{2}} = \left(\frac{7}{2}p\right)\left(\frac{2}{9}\right) = \frac{7}{9}p = 5 \Longrightarrow$$

$$p = 5\left(\frac{9}{7}\right) = \frac{45}{7}$$

$$\Longrightarrow r = \frac{\frac{7}{2}\left(\frac{45}{7}\right)}{1 - \frac{7}{2}\sin\theta} = \frac{\frac{45}{2}}{1 - \frac{7}{2}\sin\theta} = \frac{45}{2 - 7\sin\theta}$$

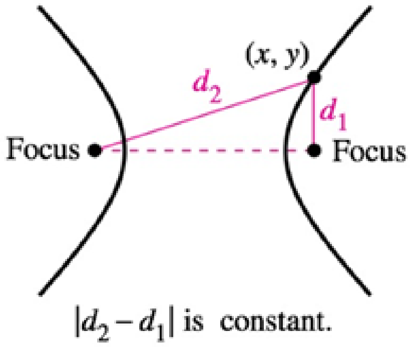


Figure 6.20

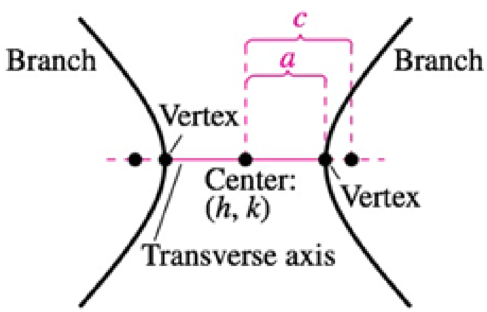


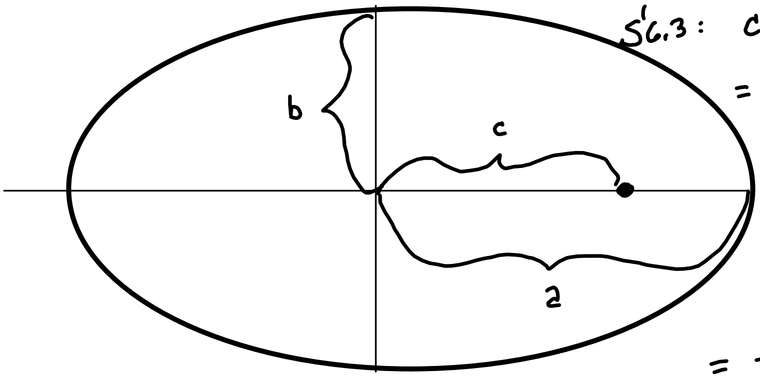
Figure 6.21

Polar 4 $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $r^2=\frac{b^2}{1-e^2\cos^2\theta}$

Find Polar Eq'n for

$\frac{x^2}{225}+\frac{y^2}{144}=1 \Longrightarrow a=15,\ b=12$

c = dist. from center to focus.



§6.3: $c^2=a^2-b^2=225-144$

$=81=9^2 \Longrightarrow c=9$

So, $e=\frac{c}{a}=\frac{9}{15}=\boxed{\frac{3}{5}=e}$

& $r^2=\frac{b^2}{1-e^2\cos^2\theta}$

$=\frac{12^2}{1-\left(\frac{3}{5}\right)^2\cos^2\theta}$

$=\frac{144}{1-\frac{9}{25}\cos^2\theta}$

$=\frac{144}{\frac{25-9\cos^2\theta}{25}}=\boxed{\frac{3600}{25-9\cos^2\theta}=r^2}$

Book accepted my original answer.

$=$ Book Answer $=\frac{9}{9}\left(\frac{3600}{25-9\cos^2\theta}\right)$

$=\frac{32400}{225-81\cos^2\theta}$

Find all 4^{th} roots of

$z = 16\left(\cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)\right)$

$r = 16 \Rightarrow \sqrt[4]{r} = \sqrt[4]{16} = 2$

inc: $\frac{2\pi}{n} = \frac{2\pi}{4} = \frac{\pi}{2} = \frac{6\pi}{12}$ $n=4$

$\sqrt[4]{z} = 2\left(\cos\left(\frac{\pi}{12}\right) + i\sin\left(\frac{\pi}{12}\right)\right)$

& others are $2\left(\cos\left(\frac{7\pi}{12}\right) + i\sin\left(\frac{7\pi}{12}\right)\right)$

$2\left(\cos\left(\frac{13\pi}{12}\right) + i\sin\left(\frac{13\pi}{12}\right)\right)$

$2\left(\cos\frac{19\pi}{12} + i\sin\left(\frac{19\pi}{12}\right)\right)$

$6+1=7$

$6+7=13$

$13+6=19$

$\frac{\frac{\pi}{3}}{\frac{4}{1}} = \frac{\pi}{3}\cdot\frac{1}{4} = \frac{\pi}{12}$

$\frac{5\pi}{63} = \frac{5\pi}{18}$ z_1

$\frac{2\pi}{3}\cdot\frac{6}{6} = \frac{12\pi}{18} =$ inc.

$5+12 \rightsquigarrow \sqrt[3]{3}\left(\cos\left(\frac{5\pi}{18}\right) + i\sin\left(\frac{5\pi}{18}\right)\right)$

$\sqrt[3]{3}\left(\cos\frac{17\pi}{18} + i\sin\left(\frac{17\pi}{18}\right)\right)$

$17+12$ $\sqrt[3]{3}\left(\cos\left(\frac{29\pi}{18}\right) + \ldots\right)$

$29+12$ $\sqrt[3]{3}\left(\cos\left(\frac{41\pi}{18}\right) + \ldots\right)$

$41+12 = 53$

$\frac{53\pi}{18} = \frac{(36+17)\pi}{18} = 2\pi + \frac{17\pi}{18}$

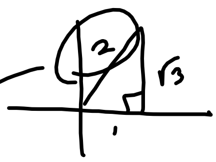
7^{th} roots:

$\frac{2\pi}{7}$ is increment

$$\sqrt[n]{r}\left(\cos\left(\frac{\theta+2k\pi}{n}\right) + i\sin\left(\frac{\theta+2k\pi}{n}\right)\right)$$

$k = 0, 1, \ldots, n-1$

$z = 1 + \sqrt{3}\, i$ Find its square roots $\to n = 2$



$\theta = \arcsin\left(\frac{\sqrt{3}}{2}\right) = 60^\circ = \frac{\pi}{3}$

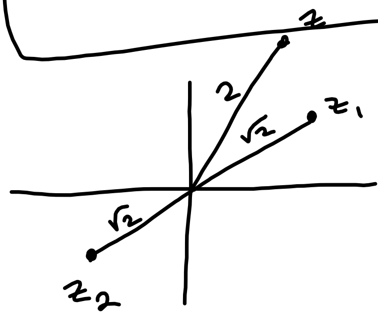
so $\frac{\theta}{2} = \frac{\pi}{6}$ & our increment is $\frac{2\pi}{2} = \pi = \frac{6\pi}{6}$

$2 = n$

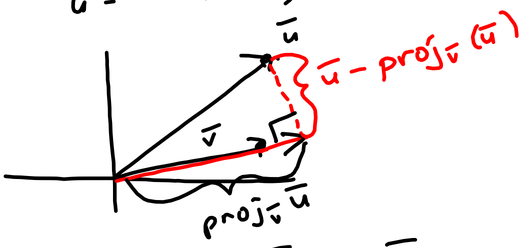
$r = 2$

$$\sqrt{z} = \boxed{\begin{array}{l} \sqrt{2}\left(\cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right)\right) = z_1 \\ \sqrt{2}\left(\cos\left(\frac{7\pi}{6}\right) + i\sin\left(\frac{7\pi}{6}\right)\right) = z_2 \end{array}}$$

$1 + 6 = 7 \rightsquigarrow$



$\bar{u} = \langle 7,7\rangle$, $\bar{v} = \langle 8,1\rangle$



$\sqrt{7^2+7^2} = \sqrt{2(49)} = 7\sqrt{2}$

$\sqrt{8^2+1^2} = \sqrt{65}$

$$\text{proj}_{\bar{v}} = \frac{\bar{u}\cdot\bar{v}}{\|\bar{u}\|\|\bar{v}\|}\bar{v} = \frac{56+7}{(7\sqrt{2})(\sqrt{65})}\langle 8,1\rangle$$

$$= \frac{63}{7\sqrt{130}}\langle 8,1\rangle = \text{proj}_{\bar{v}}(\bar{u})$$

2 | 130
5 | 65
13

No. You're remembering $\cos\theta = \dfrac{\bar{u}\cdot\bar{v}}{\|\bar{u}\|\|\bar{v}\|}$

But $\text{proj}_{\bar{v}}(\bar{u}) = \dfrac{\bar{u}\cdot\bar{v}}{\|\bar{v}\|^2}\bar{v}$

So $\text{proj}_{\bar{v}}\bar{u} = \dfrac{63}{(\sqrt{65})^2}\langle 8,1\rangle = \dfrac{63}{65}\langle 8,1\rangle$

$$= \boxed{\left\langle \frac{504}{65}, \frac{63}{65}\right\rangle = \text{proj}_{\bar{v}}(\bar{u})}$$

Now, $\bar{u} - \text{proj}_{\bar{v}}(\bar{u})$

$= \text{the other vector} = \langle 7,7\rangle - \left\langle \frac{504}{65}, \frac{63}{65}\right\rangle$

$$\left\langle \frac{455-504}{65}, \frac{455-63}{65}\right\rangle = \boxed{\left\langle \frac{-49}{65}, \frac{392}{65}\right\rangle}$$