Conic Sections is Last Topic

#20 §6.9 is a puzzler Don't know that I like my notes on it.

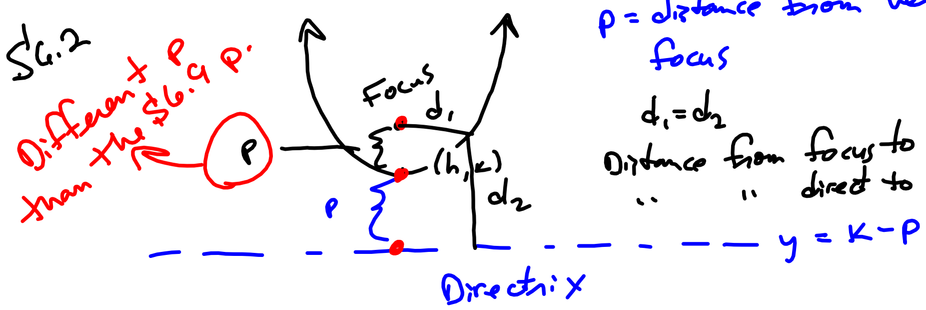
Rectangular Coordinates for Conic sections

$4p(x-h) = (y-k)^2$ on its side parabola

$4p(y-k) = (x-h)^2$ vertical Axis of symmetry

p = focal length

§6.2



p = distance from vertex to focus

$d_1 = d_2$

Distance from focus to point =
" " directrix to "

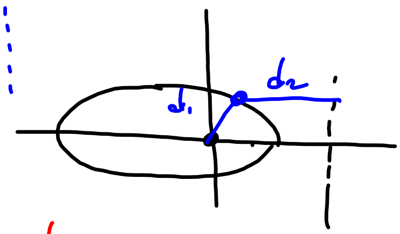
§6.9 is totally different:

Conics in Polar coords

$$r = \frac{ep}{1 \pm e\sin\theta}$$

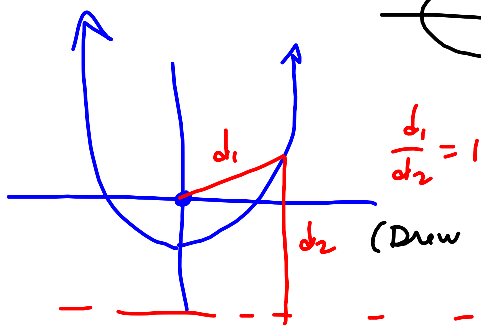
e = ratio between a point on the graph to the focus & the directrix

$$r = \frac{ep}{1 \pm e\cos\theta}$$



$\frac{d_1}{d_2}$ = constant

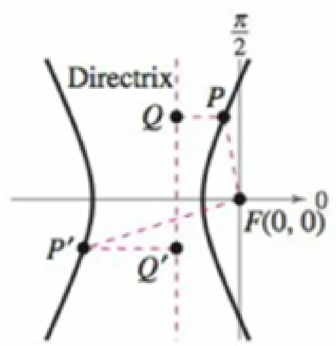
$\frac{d_1}{d_2} < 1 \rightarrow$ ellipse



$\frac{d_1}{d_2} = 1$ parabola

(Draw it longer than d_1 - DOH!)

$\frac{d_1}{d_2} > 1$ hyperbola



Hyperbola: $e > 1$

$$\frac{PF}{PQ} = \frac{P'F}{P'Q'} > 1$$

<u>Note</u> Focus is at the pole (origin) for <u>All</u> of those in §6.9.

Alternative Definition of a Conic

The locus of a point in the plane that moves such that its distance from a fixed point (focus) is in a constant ratio to its distance from a fixed line (directrix) is a **conic.** The constant ratio is the eccentricity of the conic and is denoted by e. Moreover, the conic is an **ellipse** when $0 < e < 1$, a **parabola** when $e = 1$, and a **hyperbola** when $e > 1$. (See the figures below.)

Classifying These Guys

p = distance from from focus to directrix
Different 'p' than in §6.2-6.5ish in rectangular coords

① $\frac{ep}{1+e\sin\theta} = r$ Horizontal Directrix above the focus.

+ : ABOVE / RIGHT

$r = \frac{3}{1+\sin\theta}$, $e=1$

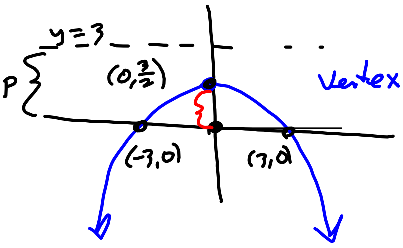
$ep = 3$

p = Dist. from focus to Directrix

$ep = 3$

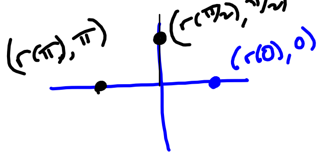
$1p = 3$

$\boxed{p=3}$



Vertex = ?

Build rectangular version of this parabola in standard form.



$r(0) = \frac{3}{1+\sin(0)} = \frac{3}{1} = 3$

$\rightsquigarrow (3,0) = (x,y)$

$r(\frac{\pi}{2}) = \frac{3}{1+\sin(\theta)} = \frac{3}{1+\sin(\frac{\pi}{2})}$

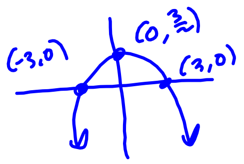
$= \frac{3}{2} \rightsquigarrow (x,y) = (0, \frac{3}{2})$

$r(\pi) = \frac{3}{1+\sin(\pi)} = \frac{3}{1} = 3$

$\rightsquigarrow (3,\pi) = (r,\theta)$

In rect:

$(-3,0)$



This says

$y = D(x-3)(x+3)$ for some # D

& $y(0) = \frac{3}{2} \Rightarrow$

$D(0-3)(0+3) = \frac{3}{2}$

$-9D = \frac{3}{2}$

$D = -\frac{1}{9}\cdot\frac{3}{2} = \frac{1}{3}\cdot\frac{1}{2} = \frac{1}{6} \Rightarrow$

$y = -\frac{1}{6}(x-3)(x+3) = -\frac{1}{6}(x^2-9) = -\frac{1}{6}x^2 + \frac{3}{2}$

$4p(y-k) = (x-h)^2$

$6y = -x^2 + 9$

$6y - 9 = -x^2$

$6(y - \frac{3}{2}) = -x^2$

$4p = 6 \Rightarrow$

$p = \frac{3}{2}$

$4(\frac{3}{2})(y - \frac{3}{2}) = -x^2 = -(x-h)^2$

says it opens down

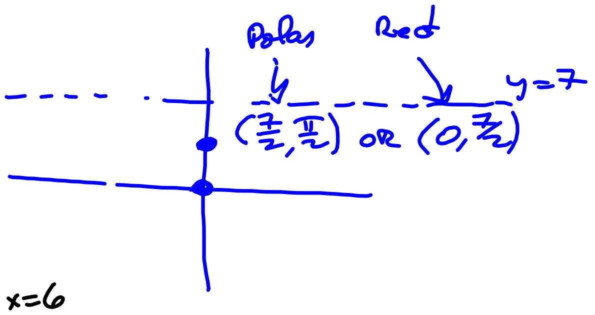
$(h,k) = \text{vertex} = (0, \frac{3}{2})$

(2) $\dfrac{ep}{1-e\sin\theta}$ Horizontal Directrix below focus

(3) $\dfrac{ep}{1+e\cos\theta}$ Vertical Directrix right of focus

(4) $\dfrac{ep}{1-e\cos\theta}$ Vertical Directrix left of focus.

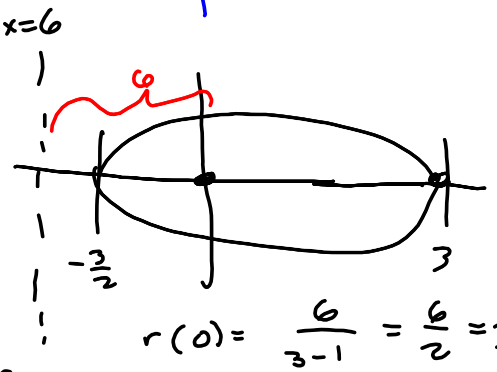
Like #7 $\dfrac{7}{1+\sin\theta}$



$ep = p = 7$

$r\left(\frac{\pi}{2}\right) = \dfrac{7}{1+1} = \dfrac{7}{2}$

Like #8 $\dfrac{6}{3-\cos\theta}$



$\dfrac{6}{3(1-\frac{1}{3}\cos\theta)} =$

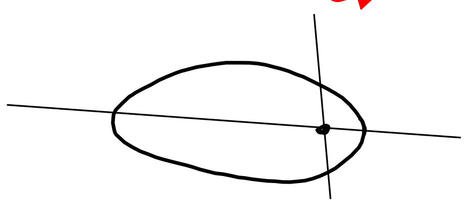
$= \dfrac{2}{1-\frac{1}{3}\cos\theta}$ $e=\frac{1}{3}$ Ellipse

$ep = 2 = \frac{1}{3}p = 2$

$p = 6$

$r(0) = \dfrac{6}{3-1} = \dfrac{6}{2} = 3$

$r(\pi) = \dfrac{6}{3-(-1)} = \dfrac{6}{4} = \dfrac{3}{2}$

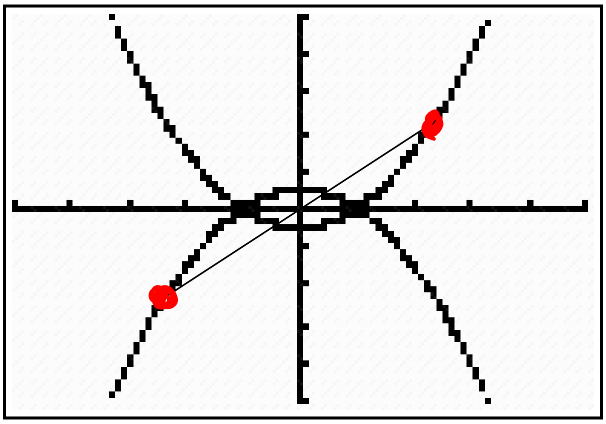


https://www.wolframalpha.com/ Can be a big help.

#11 $r = \frac{-1}{1-\sin\theta} = -\left(\frac{1}{1-\sin\theta}\right)$

which basically reflects $r = \frac{1}{1-\sin\theta}$ thru the pole.

Hor. Dir.
Below Focus



15. 0/1 points

Use a graphing utility to graph the polar equation. Identify the conic.

$$r = \frac{4}{-6 + 3\cos(\theta)}$$

Prep:

$$r = \frac{4}{-6(1-\frac{1}{2}\cos\theta)} = \frac{-\frac{2}{3}}{1-\frac{1}{2}\cos\theta} = -\boxed{\frac{\frac{2}{3}}{1-\frac{1}{2}\cos\theta}}$$

$-r$

Plot this & reflect thru the pole.

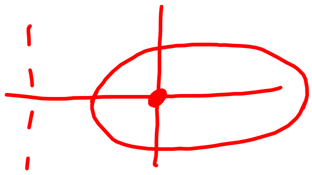
$$r = \frac{\frac{2}{3}}{1-\frac{1}{2}\cos\theta}$$

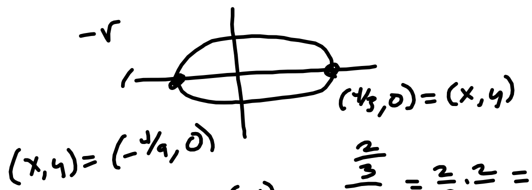
$e = \frac{1}{2} < 1$ Ellipse

$\boxed{\text{Vertical Directrix}}$ to left of focus

$$ep = \frac{2}{3} = \frac{1}{2}\cdot p = \frac{2}{3} \Rightarrow p = \frac{4}{3}$$

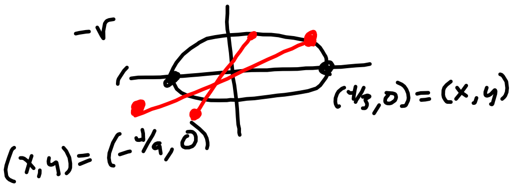
$$-r(0) = \frac{\frac{2}{3}}{1-\frac{1}{2}} = \frac{\frac{2}{3}}{\frac{1}{2}} = \frac{4}{3}$$

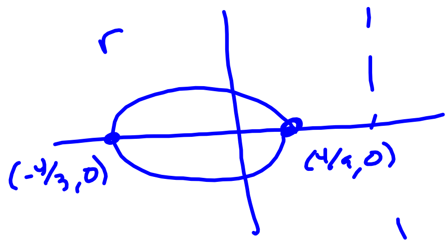


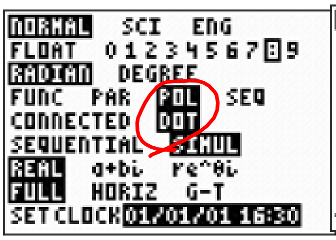


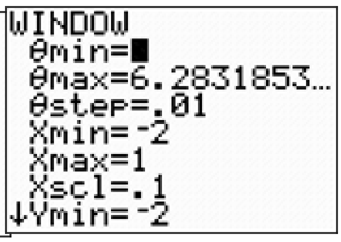
$$-r(\pi) = \frac{\frac{2}{3}}{\frac{3}{2}} = \frac{2}{3}\cdot\frac{2}{3} = \frac{4}{9}$$

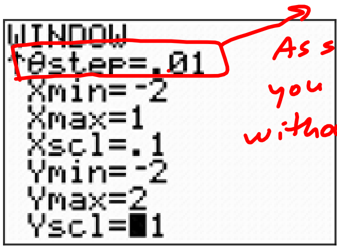
So if that's $-r$, then reflect thru the pole





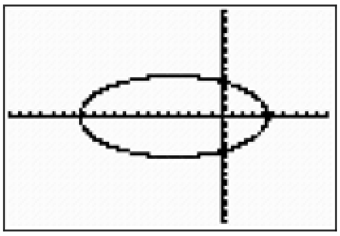






As small as you can make it, without choking

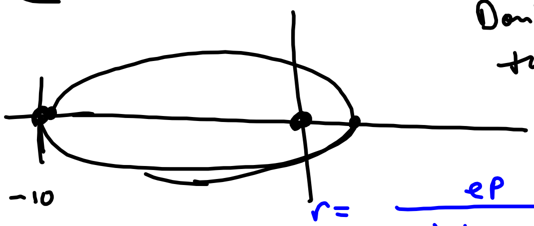
Took 10 secs with θ-step @ .001.



For best graphing-calculator results, do the pencil-and-paper work. That helps you set the window better. Hyperbolas, especially.

#20 $(2,0), (10,\pi) = (r,\theta)$

Don't know distance to the directrix p.



$e < 1$

$$r = \frac{ep}{1+e\cos\theta}$$

$$r(0) = \frac{ep}{1+e} = 2 \Rightarrow ep = 2(1+e)$$

$$r(\pi) = \frac{ep}{1-e} = 10 \Rightarrow ep = 10(1-e) \Rightarrow p = \frac{10(1-e)}{e} = \frac{\frac{10}{3}}{\frac{2}{3}} = \frac{10}{3}\cdot\frac{3}{2} = \boxed{5 = p}$$

$$2+2e = 10-10e$$

$$12e = 8$$

$$e = \frac{8}{12} = \frac{2}{3}$$

$$r = \frac{\frac{2}{3}p}{1+\frac{2}{3}\cos\theta}$$

$$r = \frac{\left(\frac{2}{3}\right)(5)}{1+\frac{2}{3}\cos\theta}\cdot\frac{3}{3}$$

$$= \frac{10}{3+2\cos\theta}$$