S 6.8 Graphs in Polar Coordinates

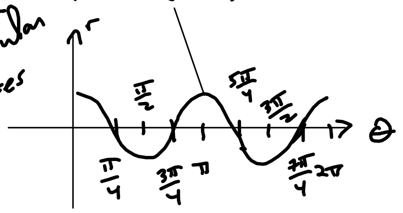
Tests for Symmetry in Polar Coordinates

The graph of a polar equation is symmetric with respect to the following when the given substitution yields an equivalent equation.

1. The line $\theta = \pi/2$: Replace (r, θ) with $(r, \pi - \theta)$ or $(-r, -\theta)$.
2. The polar axis: Replace (r, θ) with $(r, -\theta)$ or $(-r, \pi - \theta)$.
3. The pole: Replace (r, θ) with $(r, \pi + \theta)$ or $(-r, \theta)$.

EXAMPLE $r = \cos(2\theta)$

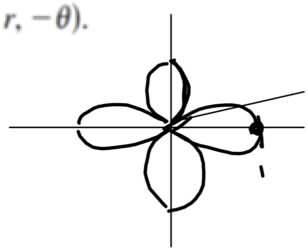
plot rectangular in rectangular coordinates



using Rectangular Graph to obtain polar graph.

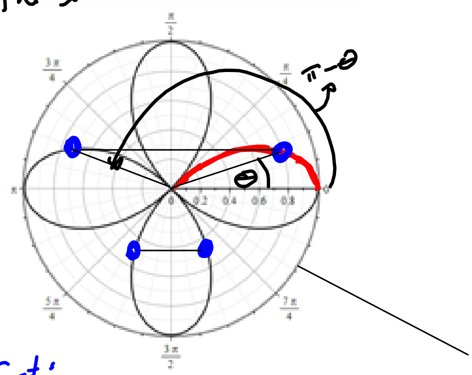
$r = \cos(2\theta)$ 4-petal Polar Plot

1. The line $\theta = \pi/2$: Replace (r, θ) with $(r, \pi - \theta)$ or $(-r, -\theta)$.



SYMMETRY ROSE

The line $\theta = \frac{\pi}{2}$



Test:

Replace θ by $\pi - \theta$ & see if it's equivalent eq'n

$r = \cos(2\theta)$

$r = \cos(2(\pi - \theta))$

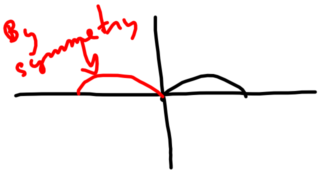
$= \cos(2\pi - 2\theta)$

$= \cos(2\pi)\cos(-2\theta) - \sin(2\pi)\sin(2\theta)$

$= (1)\cos(2\theta) - (0)\sin(2\theta)$

$= \cos(2\theta)$! SAME!

By symmetry



OR

Replace (r, θ) by $(-r, -\theta)$

$r = \cos(2\theta)$

$-r = \cos(2(-\theta))$

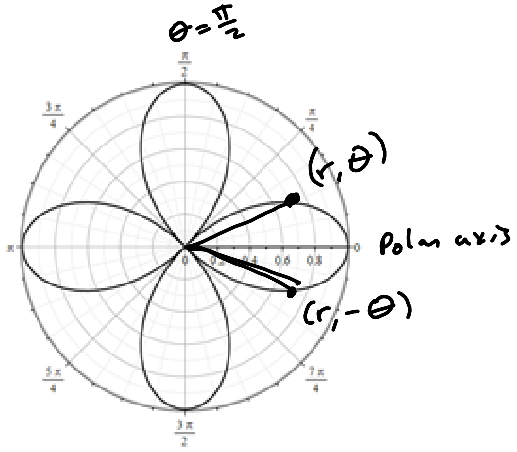
$-r = \cos(2\theta)$

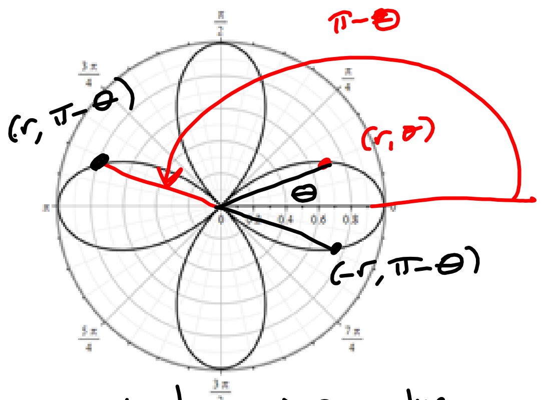
This test fails!

Only need one to work

In "real life," I'd've stopped with the work on the left to ascertain symmetry w.r.t. the line $\theta = \frac{\pi}{2}$.

2. The polar axis: Replace (r, θ) with $(r, -\theta)$ or $(-r, \pi - \theta)$.



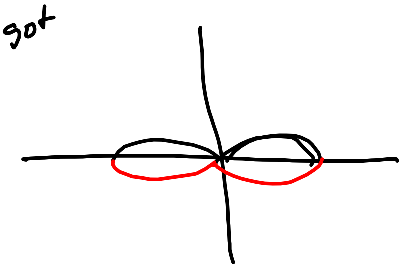


$r = \cos(2\theta)$

$r = \cos(2(-\theta))$

$r = \cos(2\theta)$ sweet!

By this work & previous, we've got



$-r = \cos(2(\pi - \theta))$

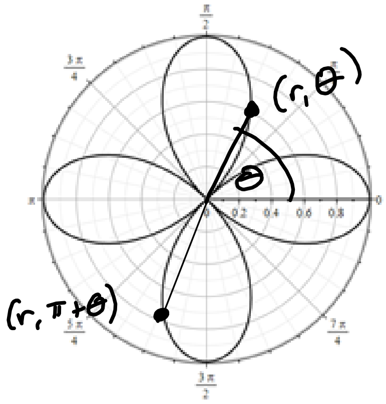
$-r = \cos(2\pi - 2\theta)$

$-r = \cos(2\pi)\cos(-2\theta) - \cancel{\sin(2\pi)\sin(-2\theta)}$

$-r = \cos(2\theta)$ FAILED TEST

(We just need one to work.)

3. The pole: Replace (r, θ) with $(r, \pi + \theta)$ or $(-r, \theta)$.



$r = \cos(2(\pi+\theta))$
$= \cos(2\pi + 2\theta)$
$= \cos(2\theta)$
Same!

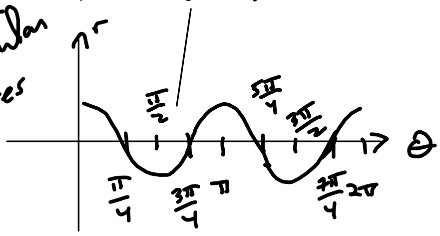
Cosine has period 2π, idiot!

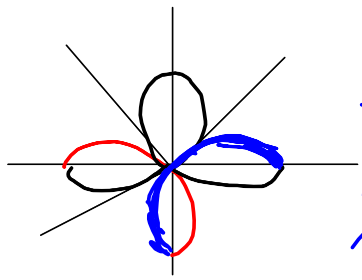
$-r = \cos(2\theta)$ Nowp!

So $\cos(2\theta) = r$ has all 3 types of symmetry. You still need to do some point plotting, old-school.

EXAMPLE $r = \cos(2\theta)$

plot rectangular in coordinates





Plotting from 0 to $\frac{\pi}{2}$ is all we really needed!

The rest is symmetry!

$r = \cos(2\theta)$
$r = 0 + \cos(2\theta)$

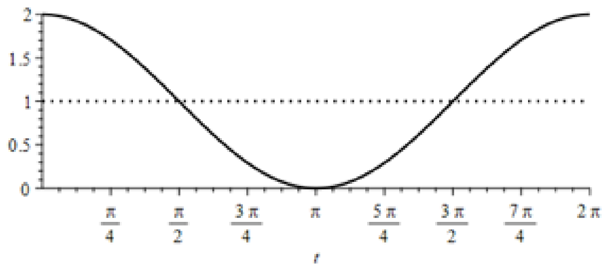
Loops: The sine/cosine piece is bigger than the constant
$a + b\cos(c\theta)$
$b > a$
Same for $a + b\sin(c\theta)$

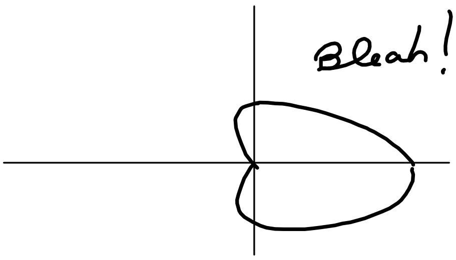
Quick Tests for Symmetry in Polar Coordinates

1. The graph of $r = f(\sin\theta)$ is symmetric with respect to the line $\theta = \frac{\pi}{2}$.
2. The graph of $r = g(\cos\theta)$ is symmetric with respect to the polar axis.

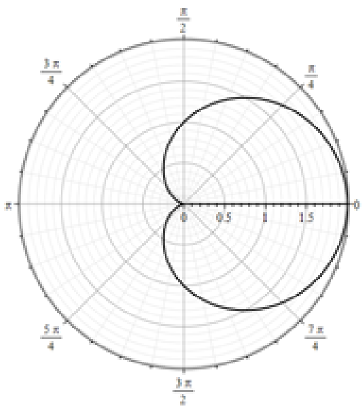
Cardioid:

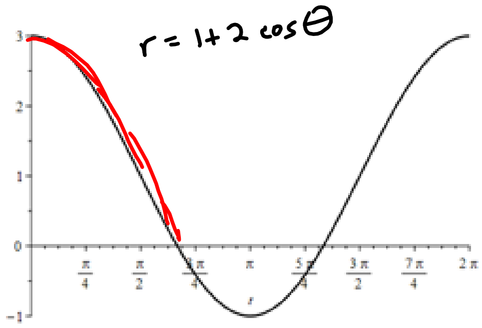
$r = 1 + \cos\theta$

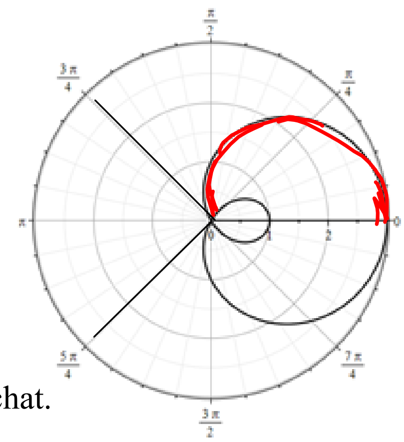




Bleah!







Are ya with me? Say something in the chat.