Questions about Chapter 3?

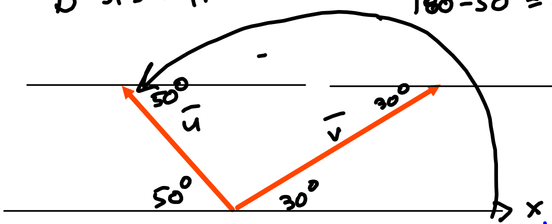
Stuff I can think of that you might want/need to see live:

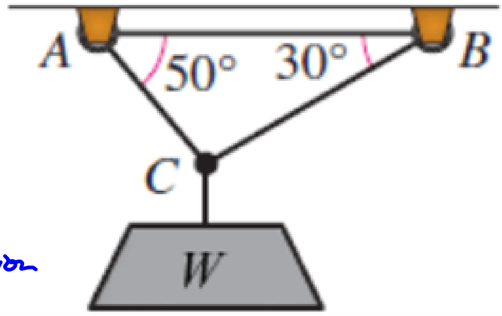
$\text{proj}_{\bar{v}}\bar{u}$

$\text{comp}_{\bar{v}}\bar{u}$

Finding Resultants & hanging weight questions.

§ 3.3 App's #2 $180^\circ - 50^\circ = 130^\circ$ $w = 2295$ lbs



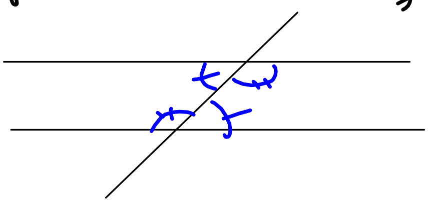


A vector may be expressed as the scalar product of its length and a unit vector in the direction of the vector.

Common unit vectors:

$\langle \cos\theta, \sin\theta \rangle$, since $\sqrt{\cos^2\theta + \sin^2\theta} = \sqrt{1} = 1$

Geometry: opposite interior angles of a transversal of 2 parallel lines are congruent



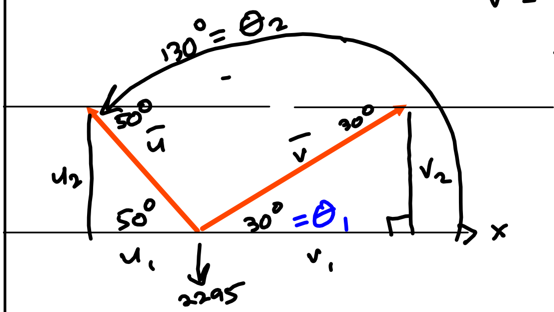
$\bar{v} = \langle v_1, v_2 \rangle = \langle \|\bar{v}\|\cos\theta_1, \|\bar{v}\|\sin\theta_1 \rangle$

$= \|\bar{v}\| \langle \cos\theta_1, \sin\theta_1 \rangle$

$\bar{u} = \langle u_1, u_2 \rangle = \|\bar{u}\| \langle \cos\theta_2, \sin\theta_2 \rangle$

$\frac{v_1}{\|\bar{v}\|} = \cos\theta_1 \Rightarrow v_1 = \|\bar{v}\|\cos\theta_1$

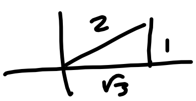
$\frac{v_2}{\|\bar{v}\|} = \sin\theta_1 \Rightarrow v_2 = \|\bar{v}\|\sin\theta_1$



$\bar{u} + \bar{v} = \langle 0, 2295 \rangle$

Notation: $a = \|\bar{u}\|$, $b = \|\bar{v}\|$ are what we want to find

$\bar{u} = a\langle \cos 30^\circ, \sin 30^\circ \rangle = \cancel{a\langle \cos 30^\circ, \sin 30^\circ \rangle}$



$\bar{v} = b\langle \cos 130^\circ, \sin 130^\circ \rangle$

$\bar{u} + \bar{v} = \langle 0, 2295 \rangle \Rightarrow$

$\langle a\cos 30^\circ, a\sin 30^\circ \rangle + \langle b\cos 130^\circ, b\sin 130^\circ \rangle = \langle 0, 2295 \rangle$

$a\cos 30^\circ + b\cos 130^\circ = 0$ solve for a & b.

$a\sin 30^\circ + b\sin 130^\circ = 2295$

$a\cos 30^\circ = -b\cos 130^\circ \Rightarrow$

$a = \dfrac{-b\cos 130^\circ}{\cos 30^\circ}$

$\Rightarrow \dfrac{-b\cos 130^\circ}{\cos 30^\circ}\sin 30^\circ + b\sin 130^\circ = 2295$

$\Rightarrow b\left(-\dfrac{\cos 130^\circ \sin 30^\circ}{\cos 30^\circ} + \sin 130^\circ\right) = 2295$

$\Rightarrow b = \dfrac{2295}{-\dfrac{\cos 130^\circ \sin 30^\circ}{\cos 30^\circ} + \sin 130^\circ}$

$\approx \dfrac{2295}{1.13715804} \approx 2018.189129$

$\approx 2018.2 \approx \|\bar{v}\|$

```
-cos(130)sin(30)
/cos(30)+sin(130
)
        1.13715804
2295/Ans
       2018.189129
```

$\&\quad a = \dfrac{-b\cos 130^\circ}{\cos 30^\circ} = \|\bar{u}\| = \text{tension}$

$\approx 2018.189129 \left(\dfrac{\cos 130^\circ}{\cos 30^\circ}\right) \approx - 1497.954865$? No

use the value still sitting on your calculator

```
       2018.189129
Ans*cos(130)/cos
(30)
      -1497.954865
```

You dropped the negative here

Functions f as you know them:

$f : \mathbb{R} \to \mathbb{R}$

$f(x) = x^2$

Feed it reals. It spits out reals.

$\mathbb{R} \times \mathbb{R}$ = the xy-plane. $= \{ (x,y) \mid x, y \in \mathbb{R} \}$

= Cartesian Product of $\mathbb{R}$ with itself.

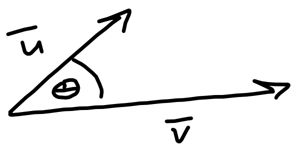
Now: $\bullet : (\mathbb{R} \times \mathbb{R}) \times (\mathbb{R} \times \mathbb{R}) \longrightarrow \mathbb{R}$

$\langle u_1, u_2 \rangle \bullet \langle v_1, v_2 \rangle = u_1 v_1 + u_2 v_2$

$\bullet : \underbrace{\mathbb{R}^2 \times \mathbb{R}^2}_{\text{Domain}} \longrightarrow \underbrace{\mathbb{R}}_{\text{Range.}}$

Recall

$\cos \theta = \dfrac{\bar{u} \cdot \bar{v}}{\|\bar{u}\| \|\bar{v}\|}$



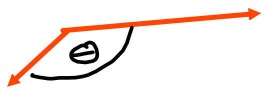
$0 \le \theta < 180^\circ$

Note if

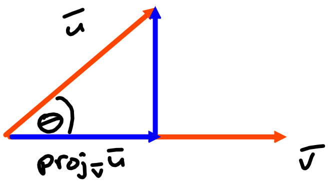
$90^\circ < \theta < 180^\circ$

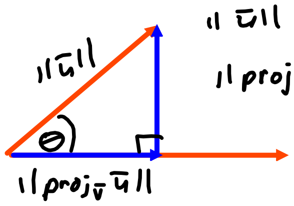
$\cos \theta < 0$.

i.e., $\theta \in Q\,\mathrm{II}$



Projections





$$\frac{\|proj_{\bar{v}}\bar{u}\|}{\|\bar{u}\|} = \cos\theta$$

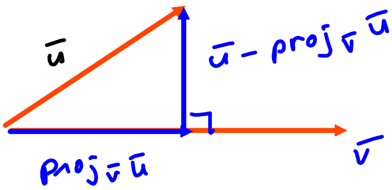
$$\|proj_{\bar{v}}\bar{u}\| = \|\bar{u}\|\cos\theta$$

$$proj_{\bar{v}}\bar{u} = \|\bar{u}\|(\cos\theta)(\text{unit vector in direction of } \bar{v})$$

$$= \|\bar{u}\| \cdot \frac{\bar{u} \cdot \bar{v}}{\|\bar{u}\|\|\bar{v}\|} \cdot \frac{\bar{v}}{\|\bar{v}\|} = \frac{\bar{u} \cdot \bar{v}}{\|\bar{v}\|^2}\bar{v}$$

$\frac{\bar{v}}{\|\bar{v}\|}$ is "ugly", but $\frac{1}{\|\bar{v}\|}\bar{v}$ is scalar times vector.

We can decompose $\bar{u}$ into the sum of a vector parallel to $\bar{v}$ & a vector perpendicular to $\bar{v}$.



Find **u** · **v**.

$9\bar{i}-3\bar{j}$ is a linear combo on $\bar{i}, \bar{j}$

$\mathbf{u} = 9\mathbf{i} - 3\mathbf{j} = \langle 9, -3 \rangle$

$\mathbf{v} = \mathbf{i} - \mathbf{j} = \langle 1, -1 \rangle$

$(9)(1) + (-3)(-1) = \boxed{12 = \bar{u} \cdot \bar{v}}$

$\|\bar{u}\| = \sqrt{u_1^2 + u_2^2} = \sqrt{\bar{u} \cdot \bar{u}}$

$\bar{u} \cdot \bar{u} = \|\bar{u}\|^2$

$\bar{u} = \langle 4, 3 \rangle, \quad \bar{v} = \langle 6, 0 \rangle$ $\qquad \|\bar{u}\| = \sqrt{4^2+3^2} = \sqrt{25} = 5$

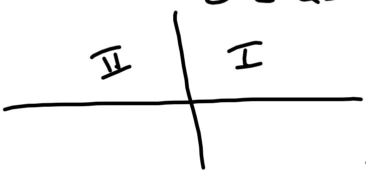
$\|\bar{v}\| = \sqrt{0^2+6^2} = 6$

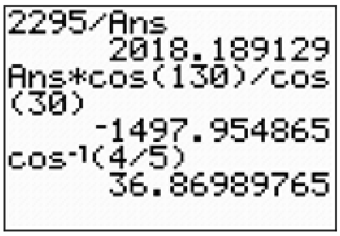
$\cos\theta = \dfrac{\bar{u}\cdot\bar{v}}{\|\bar{u}\|\|\bar{v}\|} = \dfrac{24+0}{5\cdot 6} = \dfrac{24}{30} = \dfrac{4}{5}$

$\rightarrow \theta = \cos^{-1}\left(\frac{4}{5}\right) \approx 36.86989765^\circ$

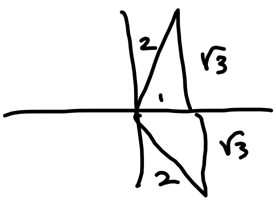
NOTE We're ALWAYS between 0° & 180°, so $\cos^{-1}(x)$ gives us θ.

$\theta \in$ QI or QII always




2295/Ans
2018.189129
Ans*cos(130)/cos(30)
-1497.954865
cos⁻¹(4/5)
36.86989765

Recall: $\cos\theta = \frac{1}{2}$



$\theta = 60^\circ$ OR 300° OR -60°

but $\cos^{-1}(x)$ only sees 60° & it'll be right.