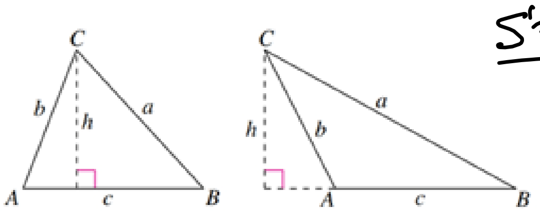
Law of Sines *(p. 262)*

If ABC is a triangle with sides a, b, and c, then

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$



A is acute. *A* is obtuse.

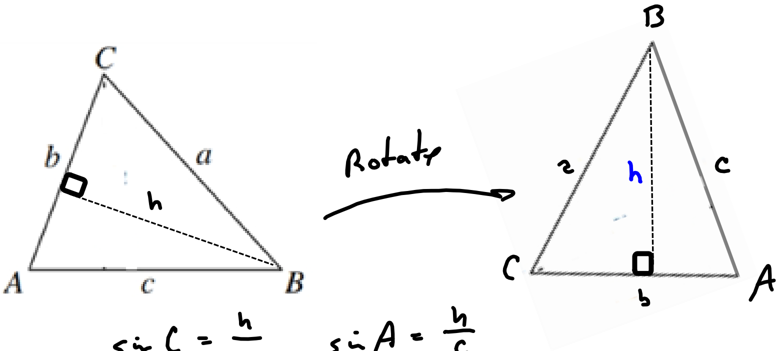
§3.1 Preview

Proof

$\sin A = \frac{h}{b}$, $\sin B = \frac{h}{a} \implies$

$h = b\sin A = a\sin B \implies$

$\frac{\sin A}{a} = \frac{\sin B}{b}$. OR $\frac{a}{\sin A} = \frac{b}{\sin B}$ (Good for finding lengths)



$\sin C = \frac{h}{a}$, $\sin A = \frac{h}{c}$

$\implies \dots \implies \frac{\sin A}{a} = \frac{\sin C}{c}$ ($= \frac{\sin B}{b}$, by previous work & transitivity of the relational operator "=".

ArithMETic Operators

$+, -, *, \div$

$<, >$

7. 0/3 points LarTrig10 3.1.026. [388274

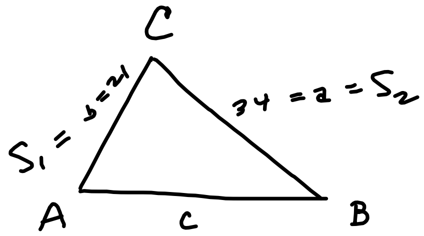
Use the Law of Sines to solve (if possible) the triangle. If two solutions exist, find both. Round your answers to two decimal places. (If not possible, enter IMPOSSIBLE.)

$A = 78°$, $a = 34$, $b = 21$

$B =$ 37.17 °

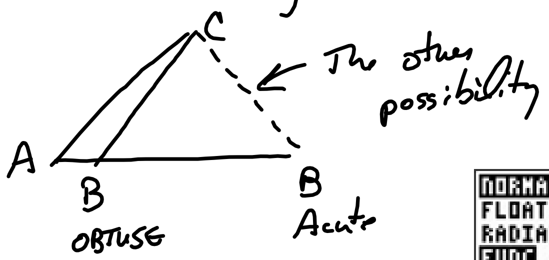
$C =$ 64.83 °

$c =$ 31.46



This has only ONE sol'n because $S_2 > S_1$

$34 > 21$, so S_2 ($a =$) is long enough to reach C & Too long to fit here;



Need: side c, C, B

$\frac{\sin B}{b} = \frac{\sin A}{a} \rightarrow$

$\sin B = \frac{b \sin A}{a}$

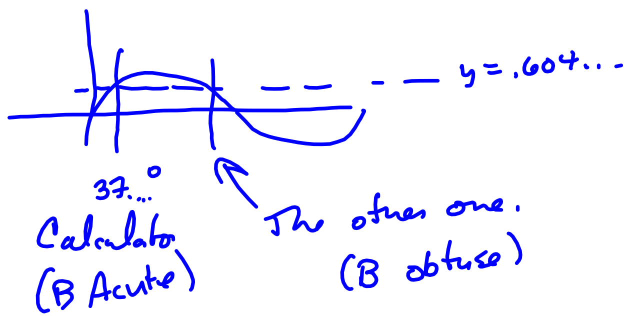
$= \frac{21 \sin(78°)}{34} \approx \sin(B)$

NORMAL SCI ENG
FLOAT 0 1 2 3 4 5 6 7 8 9
RADIAN
FUNC PAR POL SEQ
CONNECTED DOT
SEQUENTIAL SIMUL
REAL a+bi re^θi
FULL HORIZ G-T
SET CLOCK 01/01/01 13:30

```
21sin(78)/34
        .60414999
sin⁻¹(Ans)
        37.16770048
```

$\approx \sin(B)$

$\approx B$, if B is acute



Note: If $B = 37.\ldots^\circ$, then $C = 180^\circ - 78^\circ - 37.\ldots^\circ$

≈ 64.83229952

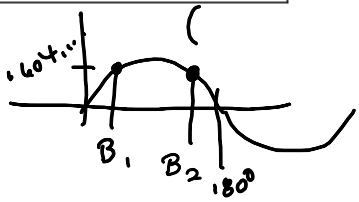
```
21sin(78)/34
          .60414999
sin⁻¹(Ans)
        37.16770048
180-78-Ans
        64.83229952
```

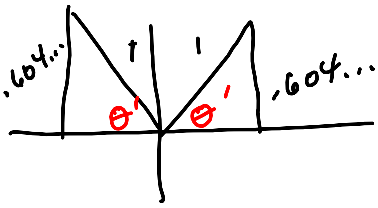
I didn't round to do my calculations

Here's why there's only one sol'n

If there WERE a 2^{nd} B, then

$B =$





$\theta' = 37.\ldots^\circ$, then

$B_1 = 37.\ldots^\circ$

$B_2 = 180^\circ - 37.\ldots^\circ \approx 142.8322995^\circ$

& so $C = 180^\circ - 78^\circ - 142.\ldots^\circ$

$\approx$

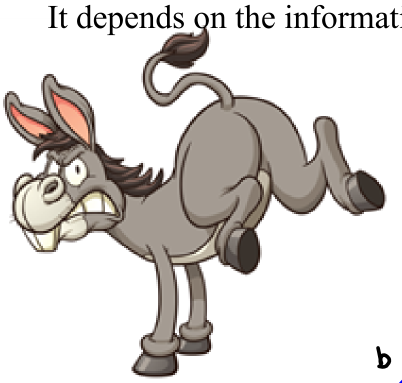
```
        37.16770048
180-78-Ans
        64.83229952
180-37.16770048
         142.832295
180-78-Ans
       -40.83229952
```

$C < 0$?! Un-Possible!

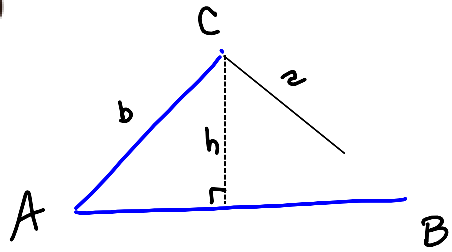
So if you mistakenly think $\exists$ 2 sol'ns, you'll get a negative angle for C.

Law of Sines is great for AAS, ASA, which uniquely determine the triangle.

In the case of ASS, it *might* work, but it might give you *two* solutions or have *no solutions*. It depends on the information given.



AS_1S_2 case is the only one that requires any hard-core analysis. Basically it comes down to whether S_2 is long enough (one or possibly 2 solutions), super-long (a unique solution), or not long enough (no solution).



$\frac{h}{b} = \sin A$

$h = b \sin A$

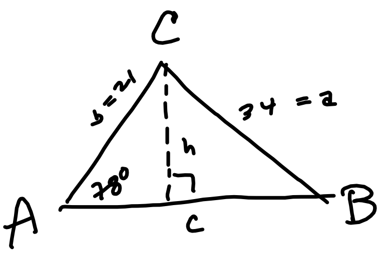
a has to reach:

$\underline{\text{Need } a \geq h = b \sin A}$. Then $\exists$ 2 possibilities

(1) $a < b \Rightarrow 2$ Sol'ns.

(2) $a > b \Rightarrow 1$ Sol'n.

The one <u>we</u> did was case (1)




```
21sin(78)
        20.54109962
```

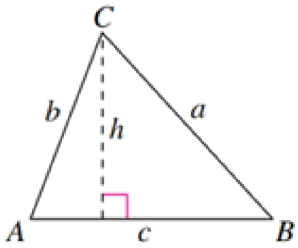
$\frac{h}{21} = \sin(78°)$

$h = 21 \sin(78°) \approx 20.54109962$

Now $a = 34 > 20....$, so long enough to reach, but too long to give us a 2nd sol'n, because $34 > 21$

Area of an oblique triangle

$$\text{Area} = \frac{1}{2}bc\sin A$$



$= \frac{1}{2}\text{base}\cdot\text{height}$

$= \frac{1}{2}ch$

$= \frac{1}{2}cb\sin A$

$$\boxed{\text{Area} = \frac{1}{2}bc\sin A}$$

$\frac{h}{b} = \sin A \rightarrow$

$h = b\sin A$

Writing Project #2:

"Bonus" questions are questions that count.

There are 105 points available. I'm grading on 100 points, so you can get 105%!!!

0/3 points

Use the Law of Sines to solve the triangle. Round your answers to two decimal places.

$A = 85° 50'$, $C = 51.5°$, $c = 19.3$

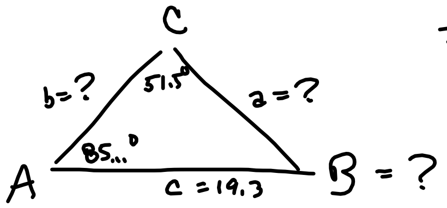
$B =$ 42.67 °

$a =$ 24.60

$b =$ 16.71

$$85° 50' = 85° + \left(\frac{50}{60}\right)° = 85.8\bar{3}°$$

$$(50 \text{ min})\left(\frac{1 \text{ degree}}{60 \text{ min}}\right) = \frac{50}{60}°$$



This is AAS

we have C & c

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$a = \frac{c \sin A}{\sin C} = \frac{19.3 \sin(85.8\bar{3}°)}{\sin(51.5°)}$$

```
85+50/60
              85.83333333
19.3*sin(Ans)/si
n(51.5)
              24.59594667
```

≈ 24.59594667

$\approx \boxed{24.60 \approx a}$

$$B = 180° - A - C = 180° - 85.8\bar{3}° - 51.5°$$
$$= 42.\bar{6}°$$

```
19.3*sin(Ans)/si
n(51.5)
              24.59594667
85+50/60
              85.83333333
Ans+51.5-180
             -42.66666667
```

$\approx 42.6667°$

$\approx \boxed{42.67° \approx B}$

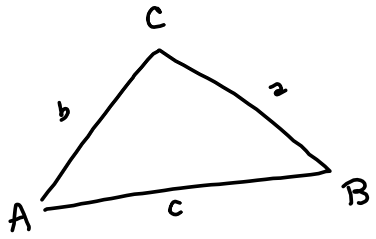
$$\frac{b}{\sin B} = \frac{c}{\sin C} \Rightarrow$$

$$b = \frac{c \sin B}{\sin C} = \frac{19.3 \sin(42.\bar{6})}{\sin(51.5°)}$$

```
ns+51.5-180
            -42.66666667
Ans
             42.66666667
9.3sin(Ans)/sin
51.5)
             16.71363564
```

≈ 16.71363564

$\approx \boxed{16.71 \approx b}$



Law of Cosines

SSS

$a^2 = b^2 + c^2 - 2bc\cos A$

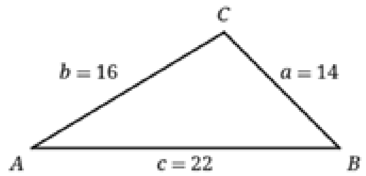
4. 0/3 points LarTrig10 3.2.005. [3

Use the Law of Cosines to solve the triangle. Round your answers to two decimal places.

$A =$ 39.40 °

$B =$ 46.50 °

$C =$ 94.10 °



Book wants you to have a version of this that has $\cos A$ "solved-for."

$a^2 = b^2 + c^2 - 2bc\cos A$

$2bc\cos A = b^2 + c^2 - a^2$

$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

Meh. Easy to manipulate to this form

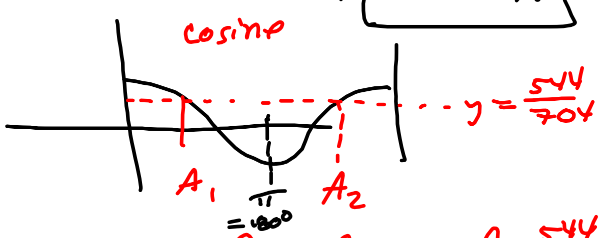
$\cos A = \dfrac{16^2 + 22^2 - 14^2}{2(16)(22)}$

$= \dfrac{256 + 484 - 196}{704} = \dfrac{544}{704} \longrightarrow$

$\cos^{-1}(\cos A) = \cos^{-1}\left(\frac{544}{704}\right) \approx 39.40056875^\circ \approx A$

$\approx \boxed{39.40^\circ \approx A}$

```
      704.0000000
256+484-196
      544.0000000
14²
      196.0000000
cos⁻¹(544/704
      39.40056875
```



2 sol'ns for $\cos A = \frac{544}{704}$

but one of them will be greater than 180°

Impossible!

Back to §3.1 for an IMPOSSIBLE situation.

5. 0/3 points LarTrig10 3.1.024.MI. [3882638]

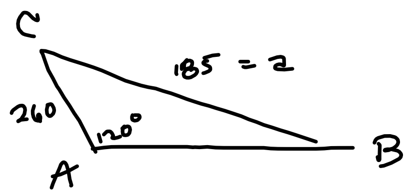
Use the Law of Sines to solve (if possible) the triangle. If two solutions exist, find both. Round your answers to two decimal places. (If a triangle is not possible, enter IMPOSSIBLE in each corresponding answer blank.)

$A = 120°$, $a = 185$, $b = 260$

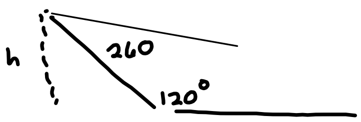
$B =$ ✗ IMPOSSIBLE °

$C =$ ✗ IMPOSSIBLE °

$c =$ ✗ IMPOSSIBLE



Side a has to be long enough to reach



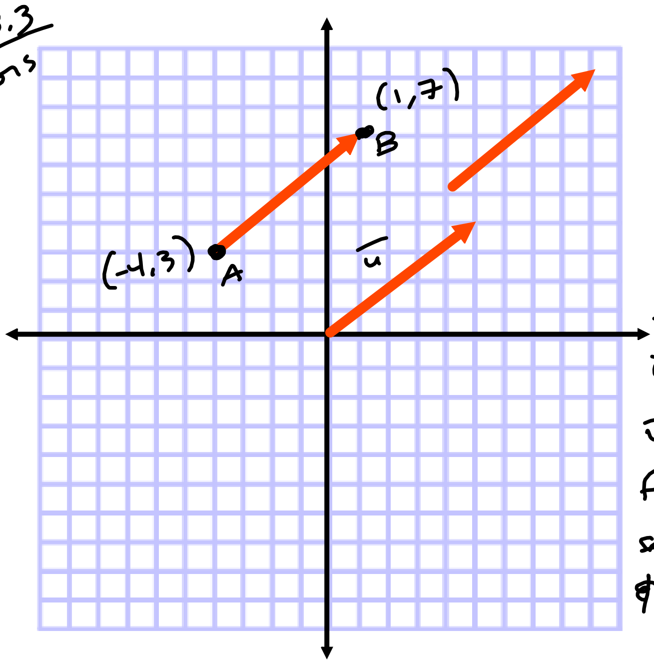
$$\frac{h}{260} = \sin 120°$$

$$h = 260 \sin 120° \approx 225.17$$

$> 185 = a \Rightarrow$ side 'a' can't "reach"

Impossible! side 'a' isn't long enough.

§3.3
Vectors



$A = (-4,3)$
$B = (1,7)$

We want to talk about the directed line segment $\overrightarrow{AB} = \overline{u}$

$\overline{u}$ is called a <u>vector</u>.

$\overline{u}$ is the representative for all directed line segments with same length & direction as $\overrightarrow{AB}$

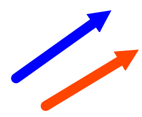
To find $\overline{u}$'s standard form:
Endpoint − starting point:

$$\langle 1-(-4),\ 7-3 \rangle = \boxed{\langle 5, 4 \rangle = \overline{u}}$$

Vector Operations

$\bar{u} + \bar{v}$:

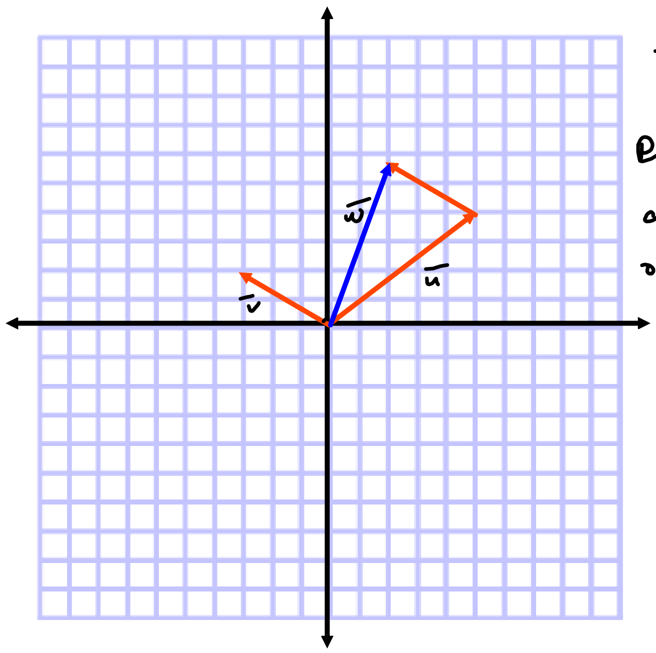
$$\langle u_1, u_2 \rangle + \langle v_1, v_2 \rangle = \langle u_1 + v_1, u_2 + v_2 \rangle$$



$\bar{u} = \langle 5, 4 \rangle$, $\bar{v} = \langle -3, 2 \rangle$, then

$$\bar{u} + \bar{v} = \langle 5-3, 4+2 \rangle = \langle 2, 6 \rangle = \bar{u} + \bar{v} = \bar{w}$$

= Resultant

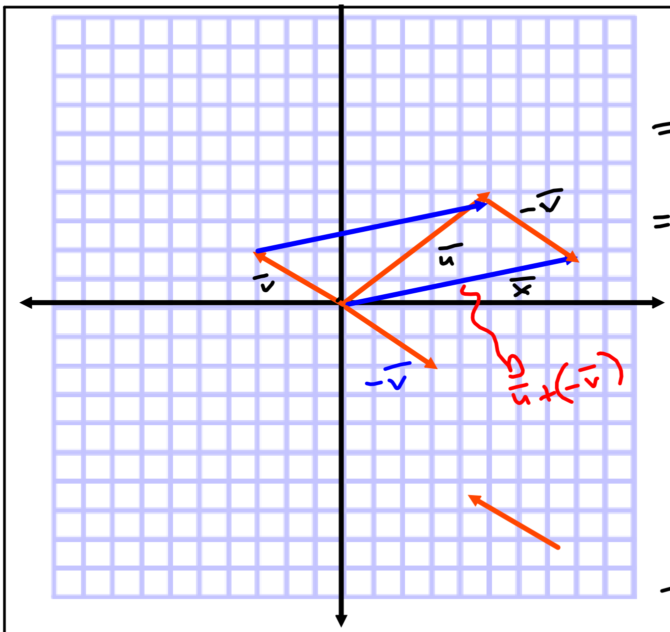


$\bar{w}$ is obtained by placing the butt of $\bar{v}$ against the terminal point of $\bar{u}$.

Subtraction:

$$\bar{u} - \bar{v} = \bar{u} + (-\bar{v})$$

$-\bar{v}$ is same length, but opposite direction



$\bar{u}-\bar{v} = \bar{x}$

$= \langle 5,4\rangle - \langle -3,2\rangle$

$= \langle 5-(-3), 4-2\rangle = \langle 8,2\rangle = \bar{x}$

Remember this:

$\bar{u}-\bar{v}$ is obtained by the vector from the tip of $\bar{v}$ to the tip of $\bar{u}$.

$\bar{u}-\bar{v}$ always points @ the tip of $\bar{u}$.

we'll need that when we do Law of Cosines in vector form.

Ops on vectors:

$\pm$ $\quad \bar{u} \pm \bar{v}$

Scalar multiplication

$5\bar{u} = 5\langle 5,4\rangle = \langle 25,20\rangle.$

scalar "size" "number"

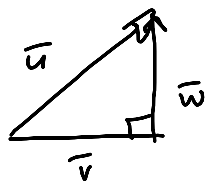
vector "size & direction"

Next time:

Dot Product,

Projections

(Gram-Schmidt)



$\bar{u} = \bar{v} + \bar{w}$

Write $\bar{u}$ as the sum of 2 vectors that are perpendicular.

$\bar{i} = \langle 1,0 \rangle$
$\bar{j} = \langle 0,1 \rangle$ } Standard unit vectors.

Any vector can be written as a linear combo of $\bar{i}$ & $\bar{j}$:

$\langle 5,4 \rangle = 5\langle 1,0 \rangle + 4\langle 0,1 \rangle$
$= 5\bar{i} + 4\bar{j}$

$\|\bar{u}\|$ = $\bar{u}$'s magnitude.

$\|\bar{u}\| = \sqrt{u_1^2 + u_2^2} = \sqrt{5^2 + 4^2} = \sqrt{25+16} = \sqrt{41}$

= length of $\bar{u}$.

Next time: Dot Product

$\bar{u} = \langle 5,4 \rangle$
$\bar{v} = \langle 2,-3 \rangle$

$\bar{u} \cdot \bar{v} = \langle 5,4 \rangle \bullet \langle 2,-3 \rangle = 5 \cdot 2 + 4 \cdot (-3)$
$= 10 - 12 = \boxed{-2 = \bar{u} \cdot \bar{v}}$

Notice $\bar{u} \cdot \bar{u} = \|\bar{u}\|^2$

$\langle 5,4 \rangle \circ \langle 5,4 \rangle = 5^2 + 4^2 = 41$