Questions over Chapter 2?

Buehler?

why is $\boxed{\sin(-u) = -\sin(u)}$? $\rightarrow f(-x) = -f(x)$

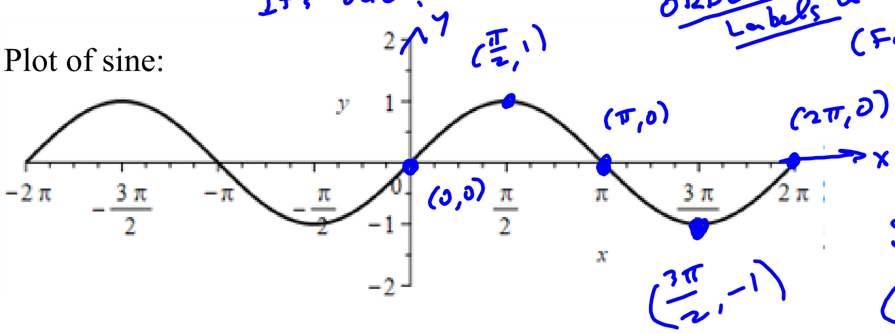
What property of sine is this?

It's odd!

"OPLs"

ORDERED-PAIR Labels in blue. (For hand sketch)

Plot of sine:

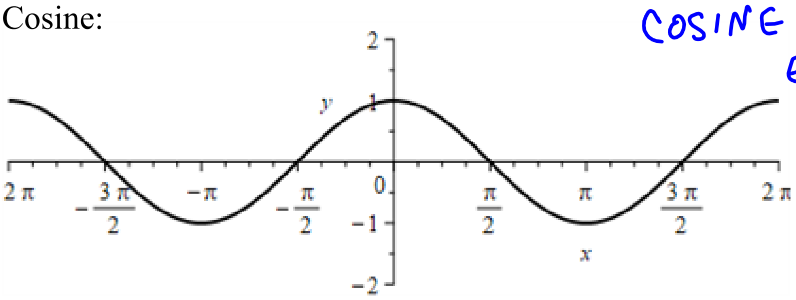


SINE IS ODD

Writing Project #2, long postponed, covers Chapter 3 and is due April Fool's Day.

Plot of Cosine:

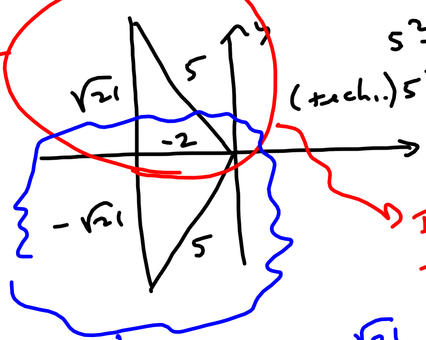
COSINE IS EVEN



Use the given conditions to find the values of all six trigonometric functions. (If an answer is undefined, enter UNDEFINED.)

$\sec(x) = -\frac{5}{2}$, $\tan(x) < 0$

$\cos(x) = -\frac{2}{5}$



$5^2 - 2^2 = 25 - 4 = 21 = b^2$

(tech.) $5^2 - (-2)^2 \Rightarrow b = \pm\sqrt{21}$

$3 \,\lfloor 21$

7

This is the one

$\tan(x) = \frac{-\sqrt{21}}{-2} = \frac{\sqrt{21}}{2} > 0$

$\sin(x) = \frac{\sqrt{21}}{5}$ $\quad$ $\csc(x) = \frac{5}{\sqrt{21}} \cdot \frac{\sqrt{21}}{\sqrt{21}} = \frac{5\sqrt{21}}{21}$

$\cos(x) = -\frac{2}{5}$ $\quad$ $\sec(x) = -\frac{5}{2}$

$\tan(x) = \frac{\sqrt{21}}{-2}$ $\quad$ $\cot(x) = -\frac{2}{\sqrt{21}} \cdot \frac{\sqrt{21}}{\sqrt{21}} = \frac{-2\sqrt{21}}{21}$

$\sqrt{21}\,\sqrt{21} = \sqrt{21 \cdot 21} = \sqrt{21^2} = 21$

$$\frac{\cos(x)}{1+\sin(x)} - \frac{\cos(x)}{1-\sin(x)} =$$

$$\boxed{LCD = (1+\sin(x))(1-\sin(x))}$$

$$\frac{\cos(x)}{1+\sin(x)} \cdot \frac{1-\sin(x)}{1-\sin(x)} - \frac{\cos(x)}{1-\sin(x)} \cdot \frac{1+\sin(x)}{1+\sin(x)}$$

$$= \frac{\cos(x)(1-\sin(x)) - \cos(x)(1+\sin(x))}{1-\sin^2(x)}$$

$$(a-b)(a+b) = a^2 - b^2$$

$$= \frac{\cos(x) - \cos(x)\sin(x) - [\cos(x) + \cos(x)\sin(x)]}{\cos^2(x)}$$

$$= \frac{\cos(x) - \cos(x)\sin(x) - \cos(x) - \cos(x)\sin(x)}{\cos^2(x)}$$

$$= \frac{-2\cos(x)\sin(x)}{\cos^2(x)} = \frac{-2\sin(x)}{\cos(x)} = -2\tan(x)$$

Verify the identity.

$$6\cos(\sin^{-1} x) = \sqrt{36 - 36x^2}$$

No, Steve. Inside of arcsine, x is not an angle! and $-\frac{\pi}{2} \le \arcsin(x) \le \frac{\pi}{2}$

x is $\frac{\text{opp}}{\text{hyp}} = \frac{h}{r}$ is trig ratio.

Rather than "prove" this identity, I'm more likely to ask and in Calculus II you're more likely to have to provide "an algebraic expression that is equivalent to $6\cos(\sin^{-1}(x))$

(I would write $\cos(\arcsin(x))$ to avoid any confusion with $\frac{1}{\sin(x)} \ne \arcsin(x) = \sin^{-1}(x)$ even though $\sin^{-2}(x) = \frac{1}{\sin^2(x)}$)

Something left out of this question is the fact that they're assuming $\cancel{0 \le x < \frac{\pi}{2}}$,

No, $-1 \le x \le 1$

i.e., we're in QI, where $\sin(x)$ & $\arcsin(x)$ are true inverse functions.

$6\cos(\arcsin(x))$

Let $\theta = \arcsin(x)$. Then

$\sin\theta = \sin(\arcsin(x)) = x$ $\boxed{\text{(Not True if } \theta \in \text{QII or III)}}$

You're confusing angles with trig ratios, Steve!

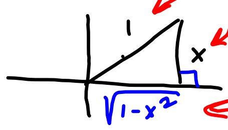
Yes, it is!

$\arcsin(\theta) = \theta$ only if $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$. TRUE

$6\cos(\theta) =$

This is arcsin(x) pic.

Triangle for $\cos(\arcsin(x))$



$\sqrt{1^2 - x^2}$

This is 3rd side from Pythagorus

$\cos(\theta) = \frac{\sqrt{1-x^2}}{1} \Rightarrow$

Product of radicals is...

$6\cos\theta = 6\sqrt{1-x^2} = \sqrt{6^2}\sqrt{1-x^2}$

$= \sqrt{36(1-x^2)} = \sqrt{36 - 36x^2}$

radical of the product.

$$6\cos(\sin^{-1} x) = \sqrt{36 - 36x^2}$$

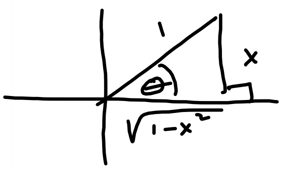
$\sin^{-1}(x) = \arcsin(x) =$ angle whose sine is x.

Let $\theta = \sin^{-1} x \Rightarrow \sin\theta = x = \frac{x}{1}$. Thus,

$6\cos(\sin^{-1} x) = 6\cos(\boxed{\theta}$ ✗ $\boxed{\theta})$

$= 6\sqrt{1 - \boxed{x^2}}$ ✗ $\boxed{x^2}$

$= \sqrt{36 - 36x^2}$



Solve the equation. (Enter your answers as a comma-separated list. Use *n* as an integer constant. Enter your response in radians.)

$$12\sin^2(x) + 18\sin(x) + 6 = 0$$

I have no problem treating $\sin(x)$ as "the variable." Books & teachers will make some fuss about making a substitution, more or less formally.

Here's how "they" would do it:

Let $u = \sin(x)$. Then

$12u^2 + 18u + 6 = 0$

$\Rightarrow 2u^2 + 3u + 1 = 0$

$\Rightarrow (2u+1)(u+1) = 0$

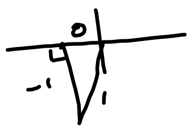
You can really "see" it's quadratic in 'u.' I want you to SEE the original eq'n is quadratic in 'sin(x).'

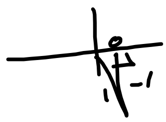
$\Rightarrow 2u + 1 = 0$ OR $u + 1 = 0$

$2u = -1$ $\qquad$ $u = -1$

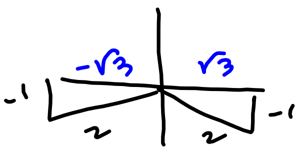
$u = -\frac{1}{2}$ $\qquad$ $\sin(x) = -1$

$\sin(x) = -\frac{1}{2}$





Either way you draw this degenerate triangle, the angle is $270° = \frac{3\pi}{2}$

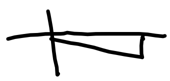


Calculator in degree mode

$\sin^{-1}(-\frac{1}{2}) = -30° = -\frac{\pi}{6} \Rightarrow \frac{\pi}{6}$ is reference angle & we're in QIII & QIV

Make it between 0 & 2π

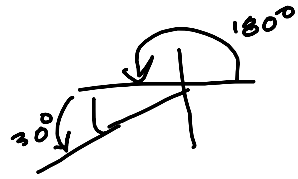
$\frac{2\pi}{1}\cdot\frac{6}{6} - \frac{\pi}{6} = \frac{11\pi}{6}$



The other one is

$180^\circ + 30^\circ = 210^\circ$

$= \frac{\pi}{1} \cdot \frac{6}{6} + \frac{\pi}{6} = \frac{7\pi}{6}$

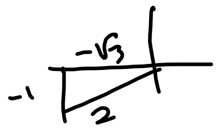


$\boxed{x = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{3\pi}{2}}$ $\Rightarrow$ Gives all solutions $x \in [0, 2\pi)$

$12\sin^2(x) + 18\sin(x) + 6 = 0$

$12\sin^2(\frac{7\pi}{6}) + 18\sin(\frac{7\pi}{6}) + 6$

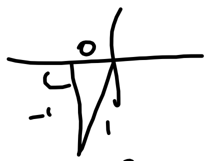
$= 12(-\frac{1}{2})^2 + 18(-\frac{1}{2}) + 6 = \frac{12}{4} - 9 + 6 = 3 - 9 + 6 = 0$ ✓



$\frac{11\pi}{6}$ works, too.

$x = \frac{3\pi}{2}$

$12\sin^2(\frac{3\pi}{2}) + 18\sin(\frac{3\pi}{2}) + 6$



$= 12(-1)^2 + 18(-1) + 6$

$= 12 - 18 + 6 = 0$ ✓

So, no extraneous solutions!

So to find all sol'ns, we need the $2\pi n$ thing to capture ALL sol'ns that are coterminal with the ones we found.

WebAssign wants:

$\frac{7\pi}{6} + 2\pi n, \frac{11\pi}{6} + 2\pi n, \frac{3\pi}{2} + 2\pi n.$

Mills likes:

$x \in \{u + \frac{2\pi}{n} \mid u = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{3\pi}{2}, n \in \mathbb{Z}\}$

presents the set of all solutions, not just conditions for membership.

Use the Quadratic Formula to find all solutions of the equation in the interval $[0, 2\pi)$. (Enter your answers as a comma-separated list. Round each answer to four decimal places.)

$$12 \sin^2(x) - 17 \sin(x) + 6 = 0$$

$x =$ ✗ 0.7297, 0.8481, 2.2935, 2.4119

Let $u = \sin(x)$ ⟶

$12u^2 - 17u + 6 = 0$ ⟶

$a = 12,\ b = -17,\ c = 6$

$b^2 - 4ac = (-17)^2 - 4(12)(6)$

$= 289 - 288 = 1$

$$\begin{array}{r} 48 \\ \times\ 6 \\ \hline 288 \end{array}$$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{17 \pm \sqrt{1}}{2(12)}$

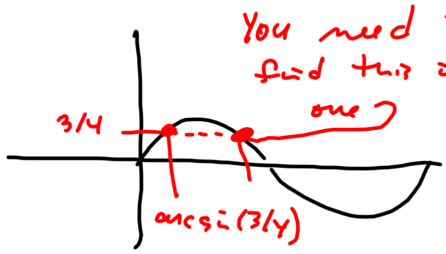
$\dfrac{17+1}{24} = \dfrac{18}{24} = \dfrac{3}{4} = .75$

$\dfrac{17-1}{24} = \dfrac{16}{24} = \dfrac{2}{3}$

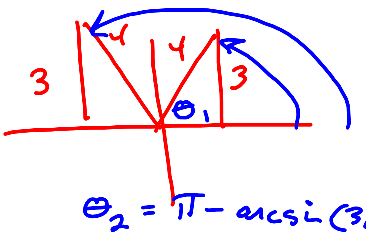
We need answers in <u>Decimal Radians</u>, so go <u>radians MODE</u> on your calculator.

$\sin(x) = .75$

$x = \arcsin(.75) \approx$



You need to find this other one



$\theta_1 \approx$ 0.8480620790

$\theta_2 = \pi - \arcsin(3/4) \approx$ 2.293530575