C2 Questions?

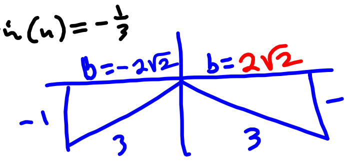
Writing Project #2 will cover Chapter 3

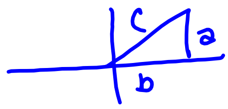
I may insert some C2 on it.

Finish last example, using more calculator.

Determine $\sin\left(\frac{u}{2}\right)$ if $\sin(u) = -\frac{1}{3}$ & $\tan(u) < 0$

$\sin(u) = -\frac{1}{3}$





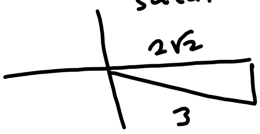
$a^2+b^2=c^2$

$b^2 = c^2 - a^2 = 3^2 - 1^2 = 8 = b^2$

$8 = 2 \cdot 4 = 2 \cdot 2 \cdot 2 \to 2\sqrt{2}$

$\Rightarrow b = \pm\sqrt{8} = \pm 2\sqrt{2}$

$\sin(u) = -\frac{1}{3}$ AND $\tan(u) = \frac{-1}{2\sqrt{2}} < 0$



Assume $0 \le u < 2\pi$, i.e., $0 \le u < 360^\circ$

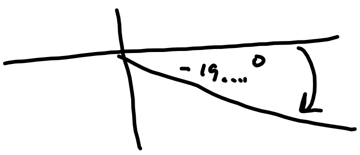
```
      SCI  ENG
FLOAT 0123456789
RADIAN DEGREE
FUNC PAR POL SEQ
CONNECTED DOT
SEQUENTIAL SIMUL
REAL a+bi re^θi
FULL HORIZ G-T
SET CLOCK 01/01/01 13:22
```

```
sin⁻¹(-1/3)
         -.33983691
Ans*180/π
        -19.47122063
```

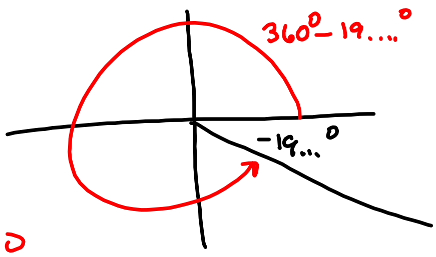
Do $\sin^{-1}\left(-\frac{1}{3}\right) \approx -.33983691 \approx -19.47122063^\circ$

Calculator's $\sin^{-1}$ only gives $u \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] = [-90^\circ, 90^\circ]$, so we need to interpret

$-19.\ldots^\circ \notin [0^\circ, 360^\circ)$



So we want angle coterminal with $-19.\ldots^\circ$ that's in $[0^\circ, 360^\circ)$




```
sin⁻¹(-1/3)
         -.33983691
Ans*180/π
        -19.47122063
360+Ans
         340.5287794
```

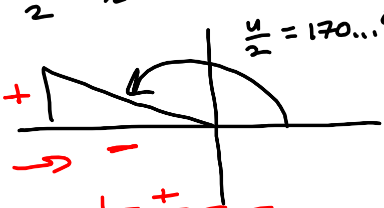
$\rightsquigarrow u = 340\ldots^\circ$

```
         -.33983691
Ans*180/π
        -19.47122063
360+Ans
         340.5287794
Ans/2
         170.2643897
```

So, immediately,

$\frac{u}{2} = \frac{340\ldots^\circ}{2} \approx 170.2643897^\circ$, i.e., $\frac{u}{2} \in$ QII

$\frac{u}{2} = 170\ldots^\circ$



sine $\to +$

cosine $\to -$

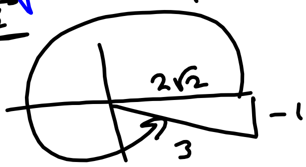
& so tangent $= \frac{+}{-} = -$

uGE

$$\sin\left(\frac{u}{2}\right) = +\sqrt{\frac{1-\cos(u)}{2}} = \sqrt{\frac{1-\frac{2\sqrt{2}}{3}}{\frac{2}{1}}} = \sqrt{\frac{3-2\sqrt{2}}{6}} = \frac{\sqrt{18-12\sqrt{2}}}{6}$$

$$\sqrt{\frac{6}{6}}$$

$$\cos\left(\frac{u}{2}\right) = -\sqrt{\frac{1+\frac{2\sqrt{2}}{3}}{2}} = \dots = -\frac{\sqrt{18+12\sqrt{2}}}{6}$$



After we locate $u \in Q\overline{IV}$

& $\frac{u}{2} = 170\dots^\circ \in Q\overline{II}$

18 -

Alternatively, use the formula for $\tan\left(\frac{u}{2}\right)$, which WebAssign will prefer:

$$\tan\frac{u}{2} = \frac{1-\cos u}{\sin u} = \frac{\sin u}{1+\cos u} = \frac{1-\frac{2\sqrt{2}}{3}}{-\frac{1}{3}} = \frac{\frac{3-2\sqrt{2}}{3}}{-\frac{1}{3}}$$

$$= \frac{3-2\sqrt{2}}{3}\cdot\left(-\frac{3}{1}\right) = -(3-2\sqrt{2})$$ is So much cleaner! $\boxed{2\sqrt{2}-3}$

OR $$= \frac{-\frac{1}{3}}{1+\frac{2\sqrt{2}}{3}} = \frac{-\frac{1}{3}}{\frac{3+2\sqrt{2}}{3}} = -\frac{1}{3}\left(\frac{3}{3+2\sqrt{2}}\right) = -\frac{1}{3+2\sqrt{2}}$$

$$= -\left(\frac{1}{3+2\sqrt{2}}\right)\cdot\left(\frac{3-2\sqrt{2}}{3-2\sqrt{2}}\right) = -\frac{3-2\sqrt{2}}{3^2-(2\sqrt{2})^2} = -\left(\frac{3-2\sqrt{3}}{9-8}\right) = -\left(\frac{3-2\sqrt{3}}{+1}\right)$$

$$2^2\sqrt{2}^2 = 4\cdot 2 = 8$$

$$= -(3-2\sqrt{2}) = \boxed{2\sqrt{2}-3}$$ Neg. ✓

Other way! $-\sqrt{\frac{18-12\sqrt{2}}{18+12\sqrt{2}}}$ still not simplified radical form

My "easy" way kinda sucks.

```
8+2√(2)))
       -.85346765
2√(2)-3
       -.17157288
-√((18-12√(2))/(
18+12√(2)))
       -.17157288
```

→ Same Both ways

Book way is better!

$$\sqrt{\frac{18-12\sqrt{2}}{18+12\sqrt{2}}} = \sqrt{\frac{3-2\sqrt{2}}{3+2\sqrt{2}}} \cdot \sqrt{\frac{3-2\sqrt{2}}{3-2\sqrt{2}}} = \sqrt{\frac{(3-2\sqrt{2})^2}{9-8}}$$

$$= \frac{|3-2\sqrt{2}|}{\sqrt{1}} = 3-2\sqrt{2}\ !$$

stick the "$-$" out front: ... $2\sqrt{2}-3$ ✓

Half-Angle Formulas

$$\sin\frac{u}{2} = \pm\sqrt{\frac{1-\cos u}{2}} \qquad \cos\frac{u}{2} = \pm\sqrt{\frac{1+\cos u}{2}}$$

$$\tan\frac{u}{2} = \frac{1-\cos u}{\sin u} = \frac{\sin u}{1+\cos u}$$

The signs of $\sin\frac{u}{2}$ and $\cos\frac{u}{2}$ depend on the quadrant in which $\frac{u}{2}$ lies.

Product-to-Sum Formulas

$$\sin u \sin v = \frac{1}{2}[\cos(u-v) - \cos(u+v)]$$

$$\cos u \cos v = \frac{1}{2}[\cos(u-v) + \cos(u+v)]$$

$$\sin u \cos v = \frac{1}{2}[\sin(u+v) + \sin(u-v)]$$

$$\cos u \sin v = \frac{1}{2}[\sin(u+v) - \sin(u-v)]$$

Sum-to-Product Formulas

$$\sin u + \sin v = 2\sin\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$

$$\sin u - \sin v = 2\cos\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$

$$\cos u + \cos v = 2\cos\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$

$$\cos u - \cos v = -2\sin\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$

$\cos(u-v) - \cos(u+v) =$

$\cos(u)\cos(-v) - \sin(u)\sin(-v)$

$- (\cos(u)\cos(v) - \sin(u)\sin(v))$

$= \cos(u)\cos(v) + \sin(u)\sin(v)$

$- \cos(u)\cos(v) + \sin(u)\sin(v)$

$= 2\sin(u)\sin(v)$

Let $x = u+v$, $y = u-v$. Then

$\sin(u)\cos(v) = \frac{1}{2}[\sin(u+v) + \sin(u-v)]$

$= \sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) = \frac{1}{2}[\sin(x) + \sin(y)]$ ■

from $x+y = u+v+u-v = 2u \Rightarrow u = \frac{x+y}{2}$

& $x-y = u+v-(u-v) = 2v$

$\Rightarrow v = \frac{x-y}{2}$

Calculus

$\int \sin(5x)\sin(7x)\,dx$ is HARD! $u = 5x$, $v = 7x$

But $\frac{1}{2}\int(\cos(-2x) - \cos(12x))\,dx = \frac{1}{2}\int\cos(2x)\,dx - \frac{1}{2}\int\cos(12x)\,dx$

$= \frac{1}{2}\cdot\frac{1}{2}\sin(2x) - \frac{1}{2}\cdot\frac{1}{12}\sin(12x) + C$

This is "easy" for Calc I & II.

Solving a Trigonometric Equation In Exercises 33–42, find all solutions of the equation in the interval $[0, 2\pi)$.

33. $2\cos^2 x - \cos x = 1$
34. $2\cos^2 x + 3\cos x = 0$
35. $\cos^2 x + \sin x = 1$
36. $\sin^2 x + 2\cos x = 2$
37. $2\sin 2x - \sqrt{2} = 0$
38. $2\cos\frac{x}{2} + 1 = 0$
39. $3\tan^2\left(\frac{x}{3}\right) - 1 = 0$
40. ~~$\sqrt{3}\tan 3x = 0$~~
41. $\cos 4x(\cos x - 1) = 0$
42. $3\csc^2 5x = -4$

Looks Hard!

Work some of these next Tuesday.

34) $2\cos^2(x) + 3\cos(x) = 0$

Let $u = \cos(x) \implies$

$2u^2 + 3u = 0 \implies$

$u(2u+3) = 0 \implies$

Looking only for $x \in [0, 2\pi)$

$u = 0$ or $2u + 3 = 0$

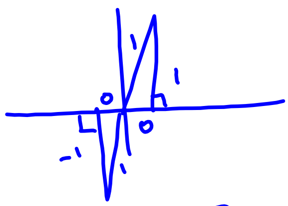
$\cos(x) = 0$

$2\cos(x) + 3 = 0$

$2\cos(x) = -3$

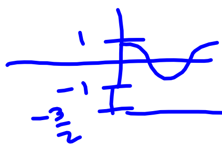
$\cos(x) = -\frac{3}{2} < -1$?!

Un possible!



$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$90^\circ, 270^\circ$



$y = -\frac{3}{2}$, Never touches!

$3\csc^2(5x) = -4$ Want all $x \in [0, 2\pi)$, i.e., $0 \le x < 2\pi$ $\Longrightarrow$ we want all $5x \in [0, 10\pi)$, i.e. $0 \le 5x \le 10\pi$

$\csc^2(5x) = -\frac{4}{3}$ has no real solutions!

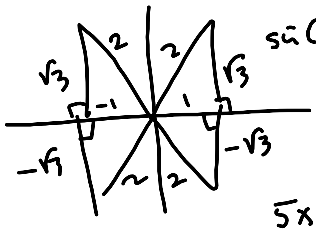
$(\text{😐})^2$ = Neg?! Nevah!

One w/ real sol'ns:

$$3\csc^2(5x) = +4$$

$$\csc^2(5x) = \frac{4}{3}$$

$$\csc(5x) = \pm\sqrt{\frac{4}{3}} = \pm\frac{2}{\sqrt{3}} = \pm\frac{2\sqrt{3}}{3}$$



$\sin(5x) = \pm\frac{\sqrt{3}}{2}$

$\csc(5x) = \pm\frac{2}{\sqrt{3}}$

$\Longrightarrow \sin(5x) = \pm\frac{\sqrt{3}}{2}$

$2\pi = \frac{6\pi}{3}$

$5x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3},$

$\frac{7\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}, \frac{11\pi}{3},$

$\frac{13\pi}{3}, \frac{14\pi}{3}, \frac{16\pi}{3}, \frac{17\pi}{3},$

$\frac{19\pi}{3}, \frac{20\pi}{3}, \frac{22\pi}{3}, \frac{23\pi}{3},$

$\frac{25\pi}{3}, \frac{26\pi}{3}, \frac{28\pi}{3}, \frac{29\pi}{3}$

$\frac{31\pi}{3}$ is past it!

So $x = \frac{\pi}{15}, \frac{2\pi}{15}, \frac{4\pi}{15}, \frac{5\pi}{15} = \frac{\pi}{3},$ ... Keep going we're outta time!