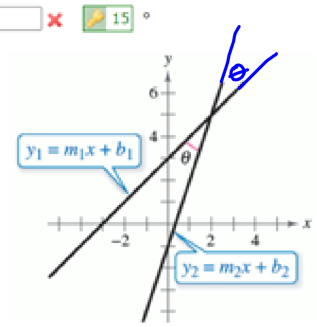
Homework Questions?

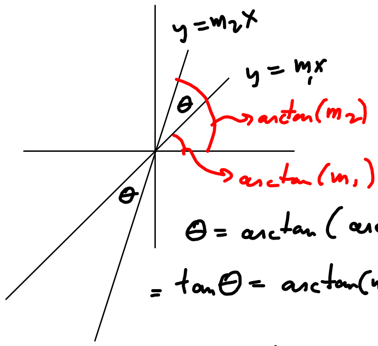
§2.4 #44

Use the figure, which shows two lines whose equations are $y_1 = m_1 x + b_1$ and $y_2 = m_2 x + b_2$. Assume that both lines have positive slopes. Derive a formula for the angle between the two lines. Then use your formula to find the angle between the given pair of lines.

$y = x$ and $y = \frac{1}{\sqrt{3}}x$

$\theta =$ [ ] ✗ 15 °





$\theta = \arctan(\arctan(m_2) - \arctan(m_1))$

$= \tan\theta = \arctan(m_2) - \arctan(m_1)$

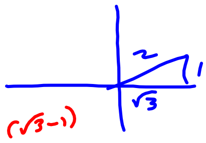
Given $m_1 = 1$, $m_2 = \frac{1}{\sqrt{3}}$

Then $\theta = \arctan(\arctan(1) - \arctan(\frac{1}{\sqrt{3}}))$

$= \arctan(1 - \frac{1}{\sqrt{3}}) = \arctan(\frac{\sqrt{3}-1}{\sqrt{3}})$

$\tan\theta = \tan(\arctan(1) + (-\arctan(\frac{1}{\sqrt{3}})))$

$= \dfrac{\tan(\arctan(1)) + \tan(-\arctan(\frac{1}{\sqrt{3}}))}{1 - \tan(\arctan(1))(\tan(-\arctan(\frac{1}{\sqrt{3}})))}$



If you can enter this, you could go straight to calculator.

$= \dfrac{1 - \frac{1}{\sqrt{3}}}{1 - (1)(-\frac{1}{\sqrt{3}})} = \dfrac{\sqrt{3}-1}{1+\sqrt{3}} = \tan\theta$

$\tan(u+v) = \dfrac{\tan(u) + \tan(v)}{1 - \tan(u)\tan(v)}$

$-(\sqrt{3}-1)$

$= \dfrac{(\sqrt{3}-1)(1-\sqrt{3})}{(1+\sqrt{3})(1-\sqrt{3})} = \dfrac{-(\sqrt{3}-1)^2}{1-3}$

$= \dfrac{-(3 - 2\sqrt{3} + 1)}{-2} = \dfrac{4 - 2\sqrt{3}}{2} = \dfrac{2-\sqrt{3}}{1}$

$= 2 - \sqrt{3}$

$(\sqrt{3}-1)^2 = (\sqrt{3})^2 - 2(\sqrt{3})(1) + 1^2$

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tan⁻¹(2-√(3))
      15.00000000
```

$= \tan\theta \Rightarrow$

$\theta = \arctan(2 - \sqrt{3})$

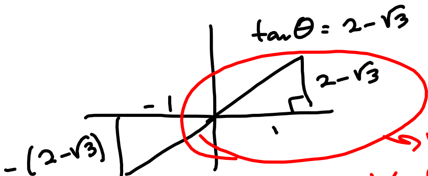
$= 15^0$

$\theta = \arctan(\tan\theta)$ because

$\theta \in QI$

when $\theta \notin QI$, you need to do more analysis.

$\tan\theta = 2 - \sqrt{3}$



We know this is the one b/c BOTH lines have positive slope, so $0 < \theta < 90^0$

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tan⁻¹(2-√(3))
      15.00000000
tan⁻¹((√(3)-1)/(1
+√(3))
      15.00000000
█
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If you can do $\arctan(\frac{\sqrt{3}-1}{\sqrt{3}+1})$ directly on your calculator

§2.3 #42

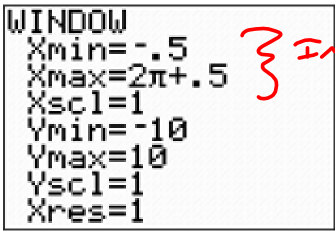
$f(x) = 4\cos^2(x) - \sin(x)$

EQN

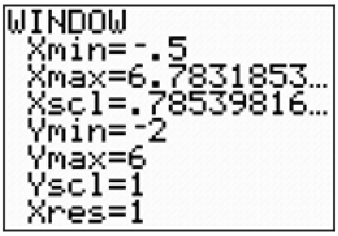
$-8\sin(x)\cos(x) - \cos(x) = 0$

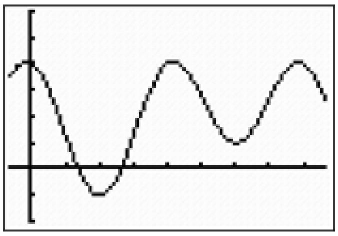
This is one we want

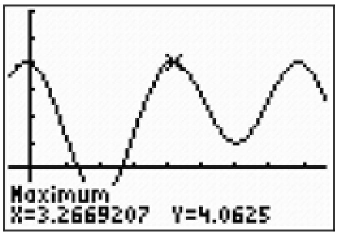
RADIANS MODE

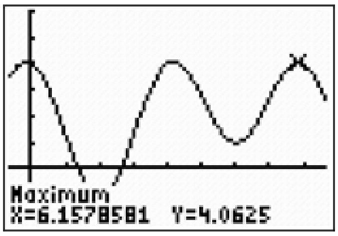


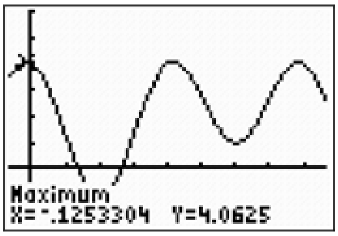
} Includes $[0, 2\pi]$











(3.2669, 4.0625), (6.1579, 4.0625)

To the left of $[0, 2\pi]$

Min ...

2^{nd}-TRACE gives "CALC" menu.

TI-83, TI-84

TRACE AND ZOOM Can also work, but it's SLOW.

Painful. But Do-Able.

DESMOS HAS DECENT FEATURES.

I demo the TI-84.

Section 2.4

Sum and Difference Formulas

$\sin(u+v) = \sin u \cos v + \cos u \sin v$

$\sin(u-v) = \sin u \cos v - \cos u \sin v$

$\cos(u+v) = \cos u \cos v - \sin u \sin v$

$\cos(u-v) = \cos u \cos v + \sin u \sin v$

$$\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v} \qquad \tan(u-v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

sine odd

cosine even

$\text{tangent} = \frac{\text{sine}}{\text{cosine}} = \frac{-}{+} = - = \text{odd}$

Section 2.5

Double-Angle Formulas

$$\sin 2u = 2 \sin u \cos u$$

$$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u}$$

$$\cos 2u = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$$

Power-Reducing Formulas

$$\sin^2 u = \frac{1 - \cos 2u}{2}$$

$$\cos^2 u = \frac{1 + \cos 2u}{2}$$

$$\tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}$$

Half-Angle Formulas

$$\sin \frac{u}{2} = \pm\sqrt{\frac{1 - \cos u}{2}} \qquad \cos \frac{u}{2} = \pm\sqrt{\frac{1 + \cos u}{2}}$$

$$\tan \frac{u}{2} = \frac{1 - \cos u}{\sin u} = \frac{\sin u}{1 + \cos u}$$

The signs of $\sin \frac{u}{2}$ and $\cos \frac{u}{2}$ depend on the quadrant in which $\frac{u}{2}$ lies.

Product-to-Sum Formulas

$$\sin u \sin v = \frac{1}{2}[\cos(u-v) - \cos(u+v)]$$

$$\cos u \cos v = \frac{1}{2}[\cos(u-v) + \cos(u+v)]$$

$$\sin u \cos v = \frac{1}{2}[\sin(u+v) + \sin(u-v)]$$

$$\cos u \sin v = \frac{1}{2}[\sin(u+v) - \sin(u-v)]$$

Double-Angle Formulas

$\sin 2u = 2 \sin u \cos u$

$\tan 2u = \dfrac{2 \tan u}{1 - \tan^2 u}$

$\cos 2u = \cos^2 u - \sin^2 u$
$= 2 \cos^2 u - 1$
$= 1 - 2 \sin^2 u$

$\sin(2\theta) = \sin(\theta + \theta) = \sin\theta\cos\theta + \sin\theta\cos\theta = \boxed{2\sin\theta\cos\theta = \sin(2\theta)}$

$\cos(2\theta) = \cos(\theta+\theta) = \cos\theta\cos\theta - \sin\theta\sin\theta =$

$\boxed{\cos(2\theta) = \cos^2\theta - \sin^2\theta}$

$= \cos^2\theta - (1-\cos^2\theta) = \cos^2\theta - 1 + \cos^2\theta$

$\boxed{\cos(2\theta) = 2\cos^2\theta - 1}$

$= 2(1-\sin^2\theta) - 1 = 2 - 2\sin^2\theta - 1$

$\boxed{\cos(2\theta) = 1 - 2\sin^2\theta}$

$\tan(2\theta) = \tan(\theta+\theta) = \dfrac{\tan\theta + \tan\theta}{1 - \tan\theta\tan\theta} = \dfrac{2\tan\theta}{1-\tan^2\theta}$

HALF-ANGLE VIA DOUBLE-ANGLE

$\cos(2\theta) = 2\cos^2\theta - 1$ solve for $\cos\theta$

$\Rightarrow 2\cos^2\theta - 1 = \cos(2\theta)$

$\Rightarrow 2\cos^2\theta = \cos(2\theta) + 1$

$\Rightarrow \cos^2\theta = \frac{\cos(2\theta)+1}{2}$ Power Reduction Formula

$\Rightarrow \sqrt{\cos^2\theta} = \sqrt{\frac{\cos(2\theta)+1}{2}}$

$|\cos\theta| = \sqrt{\frac{\cos(2\theta)+1}{2}}$

$\boxed{\cos\theta = \pm\sqrt{\frac{\cos(2\theta)+1}{2}}}$ & $\pm$ Depends on what quadrant θ's in, i.e., what quadrant 2θ's in.

$\boxed{\cos\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1+\cos(u)}{2}}}$

$\cos(2\theta) = 1 - 2\sin^2\theta$ Solve for $\sin\theta$:

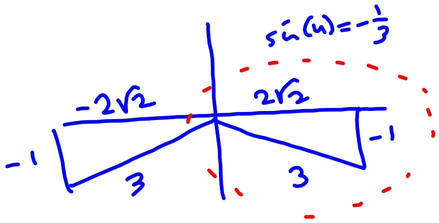
$1 - 2\sin^2\theta = \cos(2\theta)$

$-2\sin^2\theta = \cos(2\theta) - 1$

$\sin^2(\theta) = \frac{1-\cos(2\theta)}{2}$ Power Reduction Formula

$\sin\theta = \pm\sqrt{\frac{1-\cos(2\theta)}{2}} \Rightarrow \sin\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1-\cos(u)}{2}}$

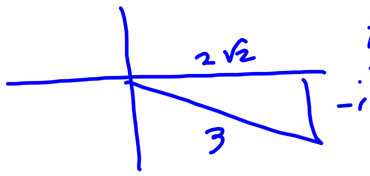
Determine $\sin\left(\frac{u}{2}\right)$ if $\sin(u) = -\frac{1}{3}$ & $\tan(u) < 0$



$3^2 - 1^2 = 8$

$\sqrt{8} = 2\sqrt{2}$

AND $\tan(u) < 0$



is the one

So, is $\sin\left(\frac{u}{2}\right)$ + OR − ?

Key Analysis for written work.

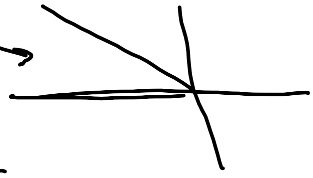
Analysis: $u \in \overline{QIV}$

$\frac{3\pi}{2} < u < 2\pi$

$\frac{3\pi}{4} < \frac{u}{2} < \pi$

$135^\circ < \frac{u}{2} < 180^\circ$

$\frac{u}{2}$ in here →



sine is +

cosine is −

$$\sin\left(\frac{u}{2}\right) = +\sqrt{\frac{1-\cos(u)}{2}}$$

↗ $\overline{QII}$

$$= \sqrt{\frac{1 - \frac{2\sqrt{2}}{3}}{2}}$$

$$= \sqrt{\frac{\frac{3-2\sqrt{3}}{3}}{\frac{2}{1}}} = \sqrt{\frac{3-2\sqrt{3}}{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}}$$

$$= \frac{\sqrt{(3-2\sqrt{3})\sqrt{6}}}{\sqrt{6\cdot 6}} = \frac{\sqrt{3\sqrt{6} - 2\sqrt{18}}}{6}$$

$$= \frac{\sqrt{3\sqrt{6} - 2\cdot 3\sqrt{2}}}{6} = \frac{\sqrt{3\sqrt{6} - 6\sqrt{2}}}{6}$$

$$\cos\left(\frac{u}{2}\right) = -\frac{\sqrt{3\sqrt{6} + 6\sqrt{2}}}{6}$$

↗ From $\frac{u}{2} \in \overline{QII}$

Want to do a more calculator-dependent version.