Today: 2.3, 2.4

§2.3 $2\sin^2(2x) = 1$

Find all solns in $[0, 2\pi)$

$0 \le x \le 2\pi \implies$

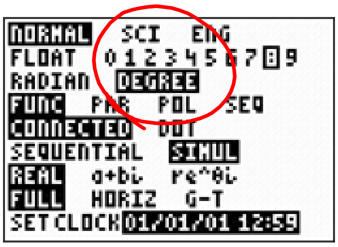
$0 \le 2x \le 4\pi$ is where we look for $2x$'s.

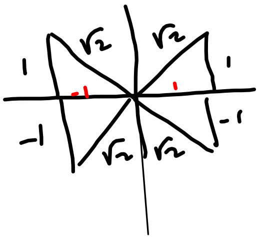
$\sin^2(2x) = \frac{1}{2}$

$\sqrt{\sin^2(2x)} = \sqrt{\frac{1}{2}} = \frac{\sqrt{1}}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

$|\sin(2x)| = \frac{1}{\sqrt{2}}$

$\sin(2x) = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$





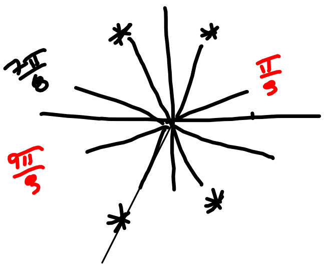
$2x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{9\pi}{4},$

$\frac{11\pi}{4}, \frac{13\pi}{4}, \frac{15\pi}{4}$

$0 \le 2x \le 4\pi$

$\implies x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}$

Finds all solns for $x \in [0, 2\pi)$



Each of these plus $2\pi n$

$\frac{\pi}{8} + 2\pi n$, $\frac{3\pi}{8} + 2\pi n$, $\frac{5\pi}{8} + 2\pi n$, $\frac{7\pi}{8} + 2\pi n$,

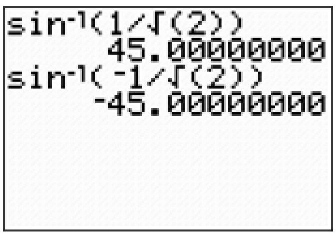
$\frac{9\pi}{8} + 2\pi n$, $\frac{11\pi}{8} + 2\pi n$, $\frac{13\pi}{8} + 2\pi n$, $\frac{15\pi}{8} + 2\pi n$ is fine.

But WebAssign's looking for $\frac{\pi}{8} + n\pi$, $\frac{3\pi}{8} + n\pi$, $\frac{5\pi}{8} + n\pi$, $\frac{7\pi}{8} + n\pi$

B/c $\frac{\pi}{8}$ & $\frac{\pi}{8} + \pi$ are solns in $[0, 2\pi)$

$\frac{3\pi}{8}$ & $\frac{3\pi}{8} + \pi$ " " " "

calculator-dependent work:



Now convert to Pi Radians, like they (and I) want:

$\left(\cancel{45}^{1}\right)\left(\frac{\pi}{\cancel{180}_{4}}\right) = \frac{\pi}{4}$ & the rest follows

§ 2.4 Sum Formulas

$$\boxed{\sin(u+v) = \sin(u)\cos(v) + \sin(v)\cos(u)}$$

They give you a separate difference:

$$\boxed{\sin(u-v)} = \sin(u+(-v)) = \sin(u)\cos(-v) + \sin(-v)\cos(u)$$

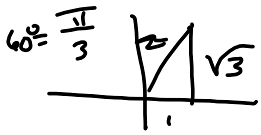
All you have to remember is sine is odd & cosine is even.

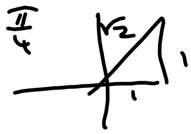
$$= \boxed{\sin(u)\cos(v) - \sin(v)\cos(u)}$$

Find the <u>EXACT</u> value of $\sin\left(\frac{7\pi}{12}\right)$

$$\frac{7}{12} = \frac{1}{12} + \frac{6}{12} = \frac{2}{12} + \frac{5}{12} = \frac{3}{12} + \frac{4}{12} = \frac{1}{4} + \frac{1}{3}$$

$$= \sin\left(\frac{3\pi}{12} + \frac{4\pi}{12}\right) = \sin\left(\frac{\pi}{4} + \frac{\pi}{3}\right) = \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{3}\right) + \sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right)$$





$$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$$

$$= \frac{1+\sqrt{3}}{2\sqrt{2}} = \left(\frac{1+\sqrt{3}}{2\sqrt{2}}\right)\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{\sqrt{2}+\sqrt{3}\sqrt{2}}{2\cdot 2} = \frac{\sqrt{2}+\sqrt{6}}{4}$$

```
sin⁻¹(-1/√(2))
          -45.00000000
(√(2)+√(6))/4
            .96592583
sin(7π/12*180/π)
            .96592583
```

Check.

Converting to degrees, since I'm in degrees MODE.

$\cos(u+v) = \cos(u)\cos(v) - \sin(u)\sin(v)$

$\tan(u+v) = \dfrac{\sin(u+v)}{\cos(u+v)}$, so TECHNICALLY, all you have to know are sine & cosine versions

Sum and Difference Formulas

$\sin(u + v) = \sin u \cos v + \cos u \sin v$

$\sin(u - v) = \sin u \cos v - \cos u \sin v$ Meh

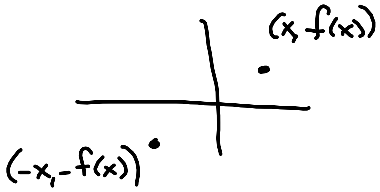
$\cos(u + v) = \cos u \cos v - \sin u \sin v$

$\cos(u - v) = \cos u \cos v + \sin u \sin v$ Meh

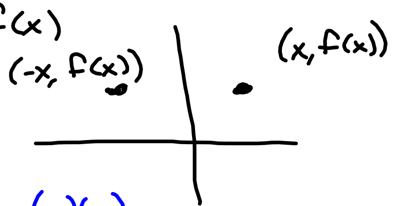
$\tan(u + v) = \dfrac{\tan u + \tan v}{1 - \tan u \tan v}$ $\quad$ $\tan(u - v) = \dfrac{\tan u - \tan v}{1 + \tan u \tan v}$ Meh

A little bit about odd & even

ODD : $f(-x) = -f(x)$



EVEN: $f(-x) = f(x)$



$$\frac{(x^2+1)(x^3-x)}{\cos(x)\sin(x)} = \frac{(+)(-)}{(+)(-)} = + \quad \text{EVEN}$$

$$\tan(x) = \frac{\sin(x)}{\cos(x)} = \frac{-}{+} = - \quad \text{ODD}$$

$\sin(\frac{\pi}{2}-x) = \cos(x)$ was proved by transforming the graph of $\sin(x)$ into the graph of $\sin(\frac{\pi}{2}-x) = \cos(x)$

$$\sin(\tfrac{\pi}{2}-x) = \sin(\tfrac{\pi}{2})\cos(-x) + \sin(-x)\cos(\tfrac{\pi}{2})$$
$$= (1)\cos(x) - (\sin(x))(0) = \cos(x).$$

S 2.3 #15 on WebAssign:

cos 5x(2 cos x + 1) = 0 AMBIGUOUS NOTATION

sucks. Should be written properly:

$$(\cos(5x))(2\cos(x)+1) = 0$$

Double-Angle Formula

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cos(2x) = \cos^2(x) - \sin^2(x) = \cos^2(x) - (1 - \cos^2(x))$$

DOUBLE-ANGLE

$$= \cos^2(x) - 1 + \cos^2(x) = 2\cos^2(x) - 1$$

From this

$$\cos(2x) = 2\cos^2(x) - 1 = \cos(2x)$$

$$\Rightarrow 2\cos^2(x) = \cos(2x) + 1$$

$$\Rightarrow \cos^2(x) = \frac{\cos(2x) + 1}{2}$$

$$\Rightarrow \cos(x) = \pm\sqrt{\frac{1 + \cos(2x)}{2}}$$

is the HALF-ANGLE FORMULA