Today: Secs 1.3, 1.4

1.3 - Trig in Quadrant I

1.4 - Trig in all 4 Quadrants.

$90^\circ = \frac{\pi}{2} \approx 1.7$

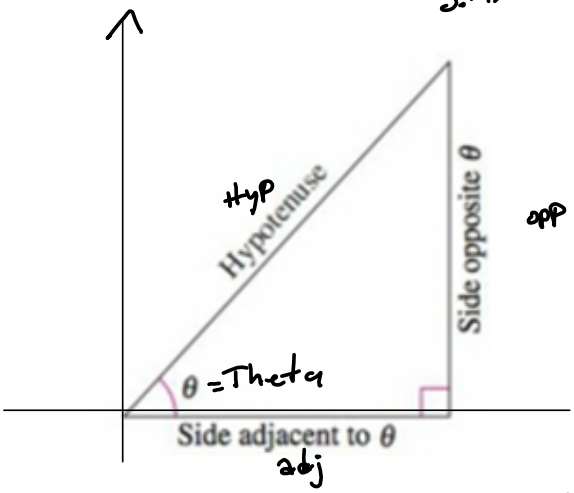
QII QI

$3.14 \approx 180^\circ = \pi$ $\quad 0^\circ \leftrightarrow 360^\circ$

$0 \leftrightarrow 2\pi \approx 6.28$

QIII QIV

$270^\circ = \frac{3\pi}{2} \approx 4.8$



$\sin\theta = \frac{opp}{hyp}$ $\quad \csc\theta = \frac{hyp}{opp}$

$\cos\theta = \frac{adj}{hyp}$ $\quad \sec\theta = \frac{hyp}{adj}$

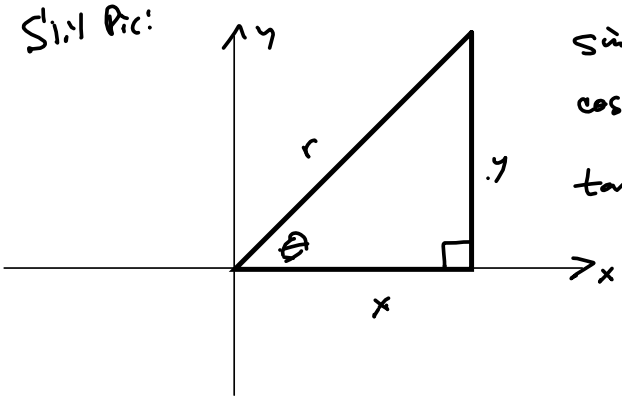
$\tan\theta = \frac{opp}{adj}$ $\quad \cot\theta = \frac{adj}{opp}$

Mnemonic: soh cah toa

Big 3: sine, cosine, tangent

Reciprocals: cosecant, secant, cotangent

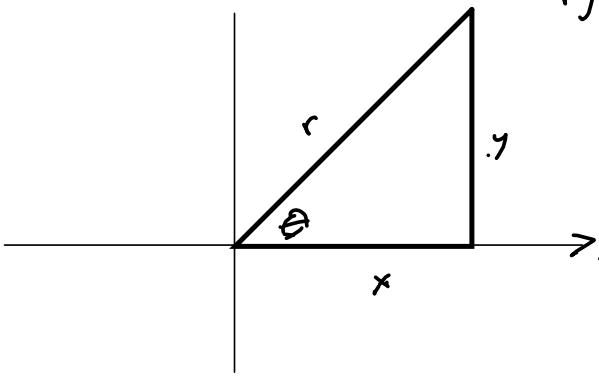
Unit Circle:



$\sin\theta = \frac{y}{r}$ $\quad \csc\theta = \frac{r}{y}$

$\cos\theta = \frac{x}{r}$ $\quad \sec\theta = \frac{r}{x}$

$\tan\theta = \frac{y}{x} = m$ $\quad \cot\theta = \frac{x}{y}$



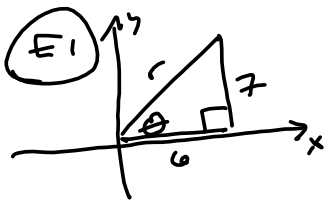
Pythagorus: $x^2+y^2=r^2 \Rightarrow r^2=x^2+y^2 \Rightarrow r=\pm\sqrt{x^2+y^2} = \sqrt{x^2+y^2}$ in QI,

$\sqrt{r^2}=|r|$

$\sqrt{r}^2 = r$ & $r\geq 0$

because $x, y, r \geq 0$

THIS CHANGES for x & y in §1.4! (All 4 quadrants)



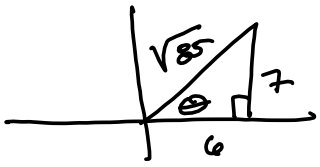
Find all 6 trig's.

$6^2+7^2=r^2=36+49=85$

$\Rightarrow r=\pm\sqrt{85}$

$\Rightarrow r=\sqrt{85}$, because $r\geq 0$

Find values of all 6 trig's?



until §6.

$\sin\theta=\frac{7}{\sqrt{85}}$ $\quad \csc\theta=\frac{\sqrt{85}}{7}$

$\cos\theta=\frac{6}{\sqrt{85}}$ $\quad \sec\theta=\frac{\sqrt{85}}{6}$

$\tan\theta=\frac{7}{6}$ $\quad \cot\theta=\frac{6}{7}$

I'm fine with that, but WebAssign probably wants you to rationalize denominators/

So, $\left(\frac{7}{\sqrt{85}}\right)\left(\frac{\sqrt{85}}{\sqrt{85}}\right)=\frac{7\sqrt{85}}{85}$

Departing from Textbook, but equivalent to this table

Sines, Cosines, and Tangents of Special Angles

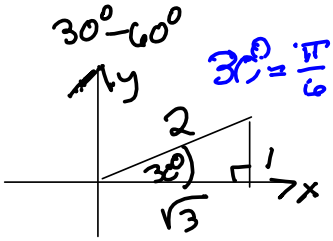
$\sin 30° = \sin\frac{\pi}{6} = \frac{1}{2}$ $\quad \cos 30° = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$ $\quad \tan 30° = \tan\frac{\pi}{6} = \frac{\sqrt{3}}{3}$

$\sin 45° = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$ $\quad \cos 45° = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$ $\quad \tan 45° = \tan\frac{\pi}{4} = 1$

$\sin 60° = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ $\quad \cos 60° = \cos\frac{\pi}{3} = \frac{1}{2}$ $\quad \tan 60° = \tan\frac{\pi}{3} = \sqrt{3}$

My advice: Don't memorize the table. Memorize three (actually 2) triangles:

Note $1 < \sqrt{3} < 2$



$30° = \frac{\pi}{6}$

$\sin\frac{\pi}{6} = \frac{1}{2}$

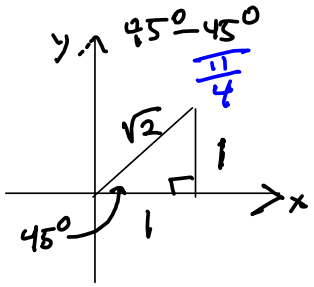
$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

$\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\csc\frac{\pi}{6} = 2$

$\sec\frac{\pi}{6} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\cot\frac{\pi}{6} = \sqrt{3}$

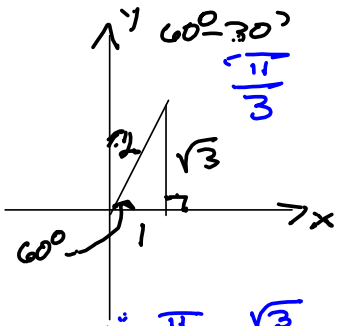


$\frac{\pi}{4}$

$\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$\tan\frac{\pi}{4} = 1$



$\frac{\pi}{3}$

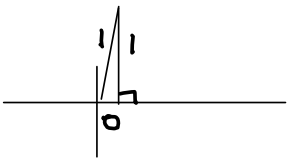
$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$\cos\frac{\pi}{3} = \frac{1}{2}$

$\tan\frac{\pi}{3} = \sqrt{3}$

Special: Quadrant Angles and drawing "degenerate triangles" for trig.

$90° = \frac{\pi}{2}$



$\sin\frac{\pi}{2} = 1$

$\cos\frac{\pi}{2} = 0$

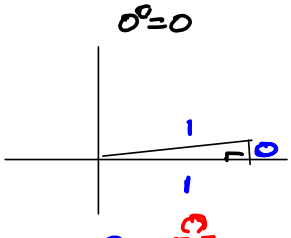
$\tan\frac{\pi}{2} = \frac{1}{0} \Rightarrow$ DNE

$\csc\frac{\pi}{2} = 1$

$\sec\frac{\pi}{2} = \frac{1}{1}$

$\cot\frac{\pi}{2} = \frac{0}{1} = 0$

$0° = 0$



$\sin 0 = \frac{0}{1}$

$\cos 0 = 1$

$\tan 0 = \frac{0}{1} = 0$

$\csc 0 =$

$\sec 0 = 1$

$\cot 0 \rightarrow \frac{1}{0}$

Degrees, Minutes, and Seconds.

First Trigonometric Identities

Fundamental Trigonometric Identities

Reciprocal Identities

$$\sin\theta = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{1}{\sec\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\frac{\sin\theta}{\cos\theta} = \frac{\frac{y}{r}}{\frac{x}{r}} = \frac{y}{r}\cdot\frac{r}{x} = \frac{y}{x}$$

Pythagorean Identities

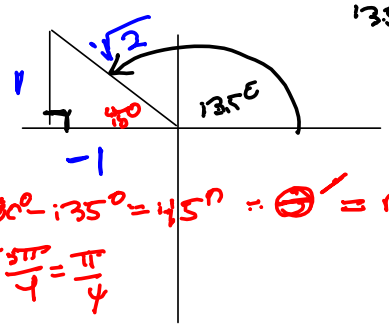
$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1+\tan^2\theta = 1+\left(\frac{\sin\theta}{\cos\theta}\right)^2 = 1+\frac{\sin^2\theta}{\cos^2\theta} = \frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}$$

$$= \frac{1}{\cos^2\theta} = \left(\frac{1}{\cos\theta}\right)^2 = \sec^2\theta$$

$135^\circ = \left(135^\circ\right)\left(\frac{\pi}{180^\circ}\right) = \frac{3\pi}{4}$



$180^\circ - 135^\circ = 45^\circ \therefore \theta' = \text{reference angle}$

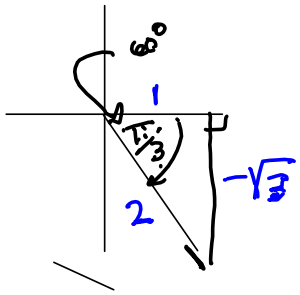
$\pi - \frac{3\pi}{4} = \frac{\pi}{4}$

$\sin 135^\circ = \frac{1}{\sqrt{2}}$

$\cos 135^\circ = \frac{-1}{\sqrt{2}}$

$\tan 135^\circ = \frac{1}{-1} = -1$

$\theta = \frac{5\pi}{3} \cdot \frac{180^\circ}{\pi} = 300^\circ$ $\quad \theta' = \frac{\pi}{3} = 300^\circ$



$\sin \frac{5\pi}{3} = \frac{-\sqrt{3}}{2}$

$\cos \frac{5\pi}{3} = \frac{1}{2}$

$\tan \frac{5\pi}{3} = \frac{-\sqrt{3}}{1}$

Is this better?
Yes!