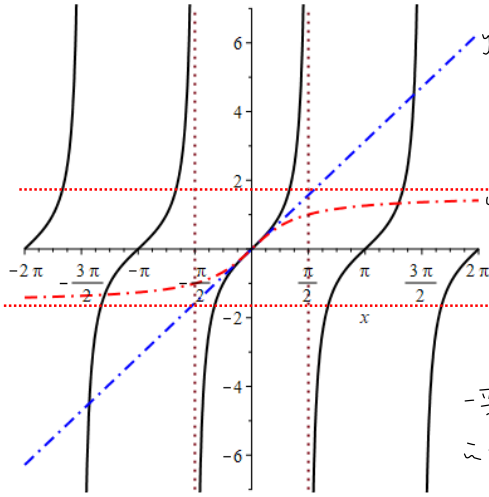


$y=x$ is a good "guide" function

$y=\arctan(x)$ $y=\frac{\pi}{2}$

or "$\tan^{-1}(x)$"

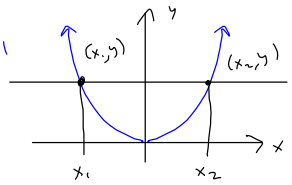
$y=-\frac{\pi}{2}$

$\tan(x)$ is most interesting between $-\frac{\pi}{2}$ & $\frac{\pi}{2}$. And $(-\frac{\pi}{2}, \frac{\pi}{2})$ is the interval of outputs for arctangent.

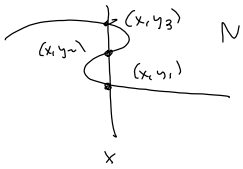
If $f(x)$ is 1-to-1, then $f^{-1}(x)$ is a function.

$f(x)=x^2$ is NOT 1-to-1.

Not 1-to-1



A relation is a function if for every x in its domain, there's exactly on y in its range.



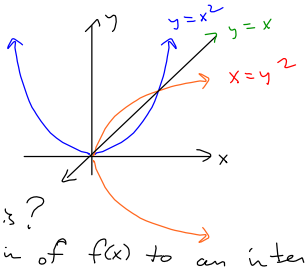
Not function vertical line test!

A function is 1-to-1 if for each y in its range, there's exactly one x in its domain.

The inverse relation for $y=x^2$ is $x=y^2$

$y=x^2 \Rightarrow$

$x=\pm\sqrt{y}$

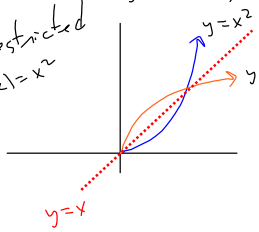


How do we handle this?

We restrict the domain of $f(x)$ to an interval on which it is 1-to-1.

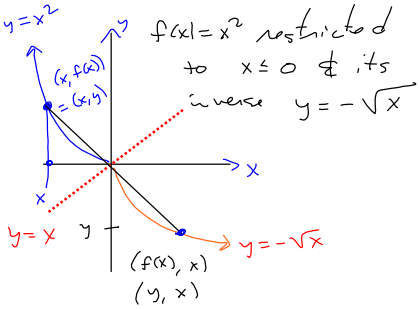
$f(x)=x^2$, restricted to $x\in[0,\infty)$, i.e. $x\geq 0$.

Restricted $f(x)=x^2$



is a function!

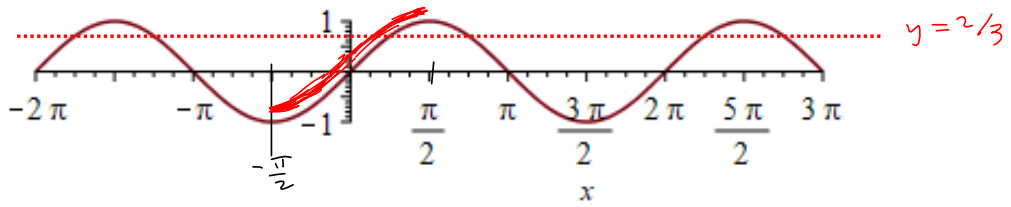
The graph of $f^{-1}(x)$ is the reflection of the graph of $y=f(x)$ about the line $y=x$.



$f(x)=x^2$ restricted to $x\leq 0$ & its inverse $y=-\sqrt{x}$

Why? Because we want to solve $\sin(x) = \frac{2}{3}$

But $\sin(x)$ isn't 1-to-1!



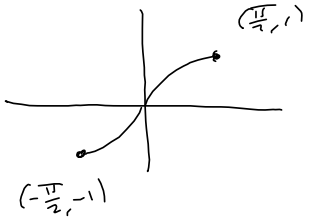
$\sin(x) = \frac{2}{3}$ has an ∞ # of solutions!

We restrict $\sin(x)$ to $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$, and so

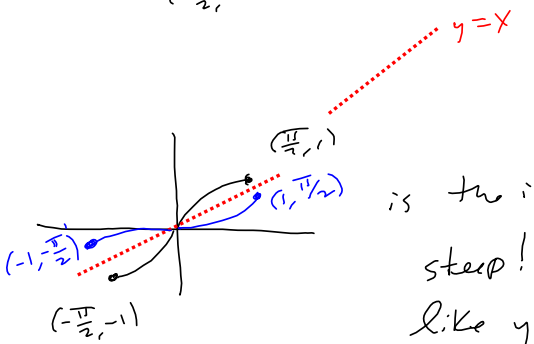
$\arcsin(x) = \sin^{-1}(x)$ spits out ANGLES between (and including) $-\frac{\pi}{2}$ & $\frac{\pi}{2}$.

(↳ calculator)

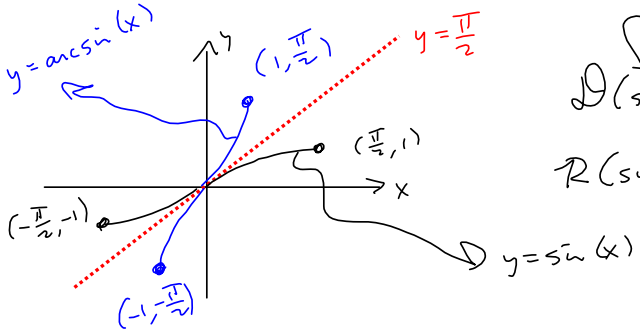
Graph of restricted sine function



is the idea



is the idea But I made sine too steep! Its slope @ $x=0$ is $m=1$, like $y=x$.

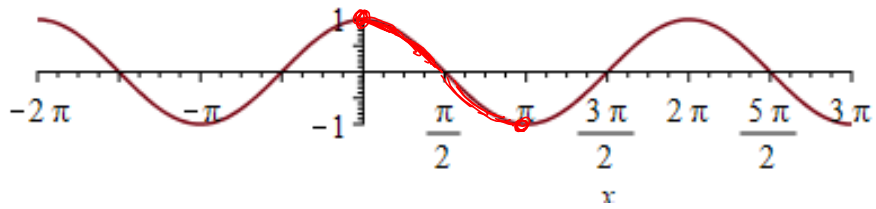


Restricted to be 1-to-1

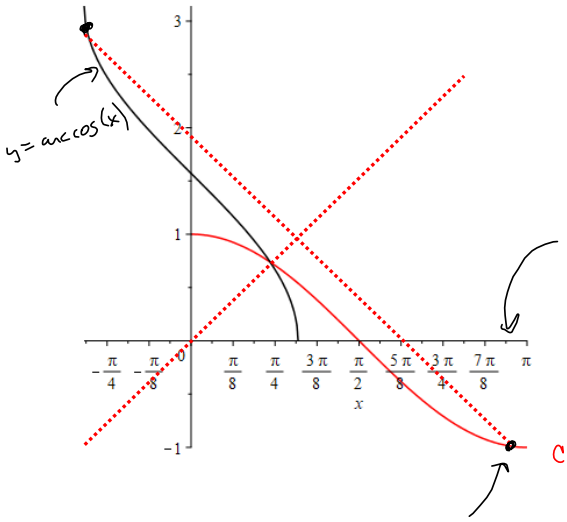
$\mathcal{D}(\sin(x)) = [-\frac{\pi}{2}, \frac{\pi}{2}] = \mathcal{R}(\arcsin(x))$

$\mathcal{R}(\sin(x)) = [-1, 1] = \mathcal{D}(\arcsin(x))$

Same game for cosine:



Restrict to $x \in [0, \pi]$



$\cos(x) = -.9$

$\Rightarrow x = \arccos(-.9)$ *

≈ 2.690565842

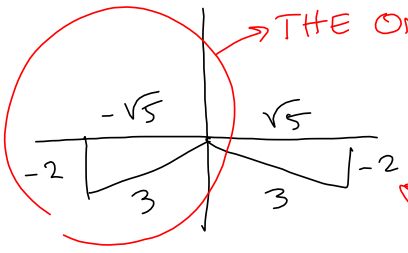
$\approx 154.1580672^{\circ}$

check

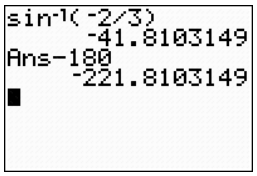
$\cos(2.690565842) \approx -.9000000001$

Now, $\arccos(\cos(x))$ gives you AN x-value satisfying the equation $\cos(x) = .9$, but not all, and maybe not even the one you want.

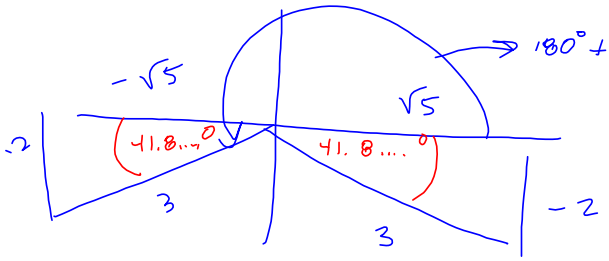
Tell me, with 4-decimal-place precision, what θ is, if $\sin(\theta) = -\frac{2}{3}$ and I know $\cos\theta < 0$, and $\theta \in [0°, 360°]$. $= [0, 2\pi]$



$9 - 4 = 5 \rightsquigarrow \sqrt{5}$

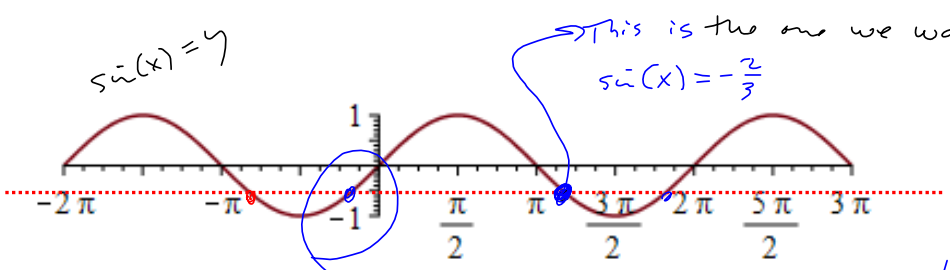


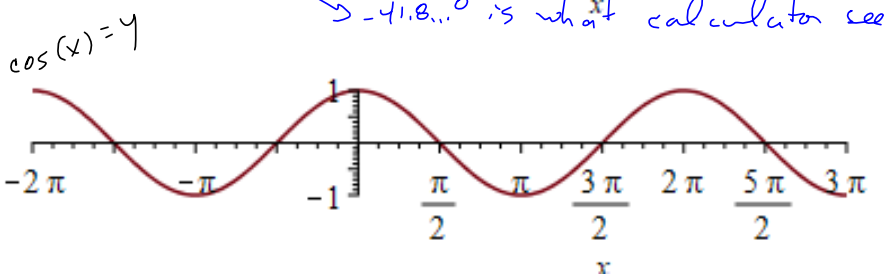
Calculator sees the wrong.



$\rightarrow 180° + 41.8...° \approx 221.8103149°$

$\approx 221.8103° \approx \theta$





$\arcsin(\sin(x)) = x$ ONLY if $x \in [-\frac{\pi}{2}, \frac{\pi}{2}] = [-90^\circ, 90^\circ]$