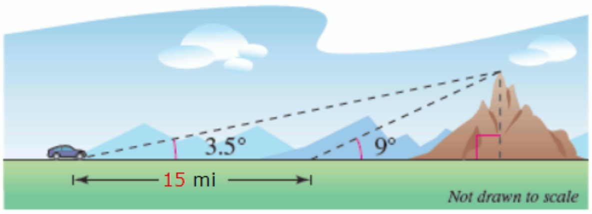
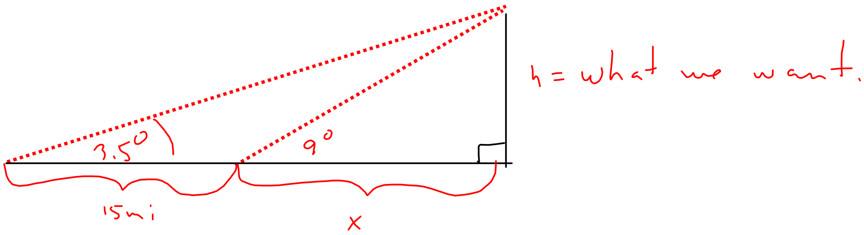
In traveling across flat land, you see a mountain directly in front of you. Its angle of elevation (to the peak) is 3.5°. After you drive 15 miles closer to the mountain, the angle of elevation is 9° (see figure). Approximate the height of the mountain. (Round your answer to one decimal place.)

1.5 mi





$$\frac{h}{x+15} = \tan(3.5^\circ) \qquad \frac{h}{x} = \tan(9^\circ)$$

$$h = (x+15)\tan(3.5^\circ) \qquad h = x\tan(9^\circ)$$

$h = h$!

Let $a = \tan(3.5^\circ)$ & $b = \tan(9^\circ)$

$$h = a(x+15) = bx \Longrightarrow$$

$$ax + 15a = bx \Longrightarrow$$

$$ax - bx = -15a$$

$$x(a-b) = -15a$$

$$x = \frac{-15a}{a-b} = \frac{-15\tan(3.5^\circ)}{\tan(3.5^\circ) - \tan(9^\circ)}$$

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-15tan(3.5)/(tan
(3.5)-tan(9))
        9.436557561
Ans*tan(9)
        1.494603888
```

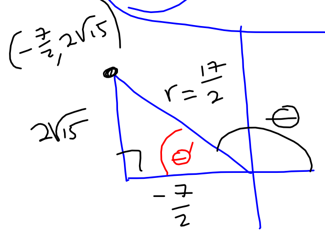
$$\Longrightarrow x \approx 9.436557561$$

$$\& \quad h = x\tan(9^\circ) \Longrightarrow$$

$$h \approx 1.494603888$$

$\left(-3\tfrac{1}{2}, 2\sqrt{15}\right) = (-3.5, 2\sqrt{15})$ → Hate mixed fractions in college.

$\left(-\tfrac{7}{2}, 2\sqrt{15}\right)$



θ = ANGLE

θ' = REFERENCE ANGLE

$\sin\theta = \dfrac{4\sqrt{15}}{17}$ $\qquad \csc\theta$

$\cos\theta$ $\qquad \sec\theta$

$\tan\theta$ $\qquad \cot\theta$

SCRATCH

$$\frac{2\sqrt{15}}{\frac{17}{2}} = \left(2\sqrt{15}\right)\left(\frac{2}{17}\right) = \frac{4\sqrt{15}}{17}$$

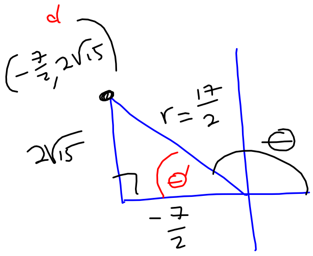
$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b}\cdot\frac{d}{c}$$

$$r^2 = \left(2\sqrt{15}\right)^2 + \left(-\tfrac{7}{2}\right)^2$$
$$= \left(2\sqrt{15}\right)^2 + \left(\tfrac{7}{2}\right)^2$$
$$= 2^2\sqrt{15}^2 + \frac{7^2}{2^2}$$
$$= 4\cdot 15 + \frac{49}{4} =$$
$$= 60 + \frac{49}{4} =$$
$$= \frac{240+49}{4} = \frac{289}{4} = r^2$$

$2\left(2+\sqrt{15}\right)$

$\left(2\sqrt{15}\right)^2$

$$\Rightarrow r = \pm\sqrt{\frac{289}{4}} = \pm\frac{17}{2}$$
$$= \pm\frac{\sqrt{289}}{\sqrt{4}}$$
$$\Rightarrow r = \frac{17}{2}$$



$\sin\theta = \dfrac{4\sqrt{15}}{17}$ $\qquad \csc\theta = \dfrac{17\sqrt{15}}{60}$

$\cos\theta = -\dfrac{7}{17}$ $\qquad \sec\theta = -\dfrac{17}{7}$

$\tan\theta = -\dfrac{4\sqrt{15}}{7}$ $\qquad \cot\theta = \dfrac{-7\sqrt{15}}{60}$

Scratch

$$\cos\theta = \frac{-\frac{7}{2}}{\frac{17}{2}} = \left(-\frac{7}{2}\right)\left(\frac{2}{17}\right) = -\frac{7}{17}$$

$$\tan\theta = \frac{2\sqrt{15}}{-\frac{7}{2}} = \left(2\sqrt{15}\right)\left(-\frac{2}{7}\right) = \frac{-4\sqrt{15}}{7}$$

$$\csc\theta = \frac{17}{4\sqrt{15}}\cdot\frac{\sqrt{15}}{\sqrt{15}} = \frac{17\sqrt{15}}{4\cdot 15} = \frac{17\sqrt{15}}{60}$$

$$\cot\theta = -\frac{7}{4\sqrt{15}}\cdot\frac{\sqrt{15}}{\sqrt{15}} = \frac{-7\sqrt{15}}{4\cdot 15} = \frac{-7\sqrt{15}}{60}$$

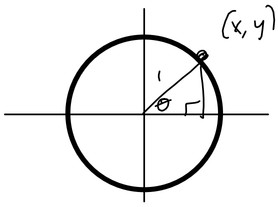
I'm ok with radicals in the denominator, usually, BUT WEBASSIGN IS <u>NOT</u>!

So Ryan, who's a real cool dude came in and taught me how to share sound in ZOOM, today. So embarrassing!

MOAR §1.3:

$$\frac{\tan\beta + \cot\beta}{\tan\beta} = \csc^2\beta$$

$\sin\theta = \frac{y}{r}$, $\cos\theta = \frac{x}{r}$, $\tan\theta = \frac{y}{x}$



$\sin\theta = \frac{y}{r} = \frac{y}{1} = y$

$\cos\theta = \frac{x}{r} = \frac{x}{1} = x$

Eq'n of circle of radius $r=1$, centered @ $(0,0)$

$x^2+y^2=1$

$\cos^2\theta + \sin^2\theta$ Pythagorean Identity.

$(\cos\theta)^2 + (\sin\theta)^2 = 1$

$\sec^2\theta = \tan^2\theta + 1$

$\csc^2\theta = \cot^2\theta + 1$

$$\tan\theta = \frac{y}{x} = \frac{y\cdot\frac{1}{r}}{x\cdot\frac{1}{r}} = \frac{\frac{y}{r}}{\frac{x}{r}} = \frac{\sin\theta}{\cos\theta}$$

$$\boxed{\tan^2\theta + 1 =} \left(\frac{\sin\theta}{\cos\theta}\right)^2 + 1 = \frac{\sin^2\theta}{\cos^2\theta} + 1$$

$$= \frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$= \frac{1}{(\cos\theta)^2} = \left(\frac{1}{\cos\theta}\right)^2 = \boxed{\sec^2\theta}$$

PROVE:

$$\frac{\tan\beta + \cot\beta}{\tan\beta} = \csc^2\beta$$

$$\frac{\tan\beta}{\tan\beta} + \frac{\cot\beta}{\tan\beta} = 1 + \frac{\frac{\cos\beta}{\sin\beta}}{\frac{\sin\beta}{\cos\beta}} = 1 + \frac{\cos\beta}{\sin\beta}\cdot\frac{\cos\beta}{\sin\beta} = 1 + \frac{\cos^2\beta}{\sin^2\beta}$$

$$\underbrace{= 1 + \left(\frac{\cos\beta}{\sin\beta}\right)^2 = 1 + (\cot\beta)^2}_{\text{optional}} = 1 + \cot^2\beta = \csc^2\beta \quad \blacksquare$$

Another way to approach it:

$$\frac{\tan\beta + \cot\beta}{\tan\beta} = \frac{\frac{\sin\beta}{\cos\beta} + \frac{\cos\beta}{\sin\beta}}{\frac{\sin\beta}{\cos\beta}} = \frac{\frac{\overbrace{\sin\beta\sin\beta + \cos\beta\cos\beta}^{\sin^2\beta + \cos^2\beta}}{\sin\beta\cos\beta}}{\frac{\sin\beta}{\cos\beta}}$$

$$= \frac{\frac{\sin^2\beta + \cos^2\beta}{\sin\beta\cos\beta}}{\frac{\sin\beta}{\cos\beta}} = \frac{\frac{1}{\sin\beta\cos\beta}}{\frac{\sin\beta}{\cos\beta}} = \frac{1}{\sin\beta\cancel{\cos\beta}}\cdot\frac{\cancel{\cos\beta}}{\sin\beta} = \frac{1}{\sin^2\beta} = \csc^2\beta \quad \blacksquare$$