

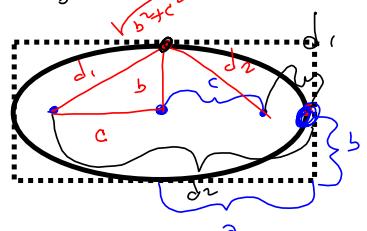
$$\text{S 6.2} \quad (x-h)^2 = 4p(y-k)$$

$p = \text{focal length}$

Distance to focus from pt  
 Distance to directrix from P+

S 6.3

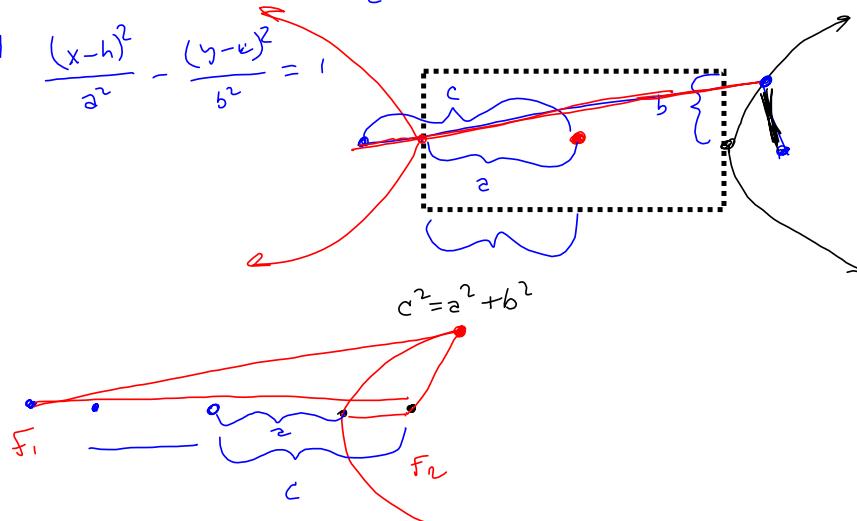
$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$



$$\begin{aligned} d_1^2 &= b^2 + c^2 \\ d_1 + d_2 &= 2\sqrt{b^2 + c^2} = a - c + a + c = 2a \\ 2\sqrt{b^2 + c^2} &= 2a \\ a^2 &= b^2 + c^2 \end{aligned}$$

S 6.4

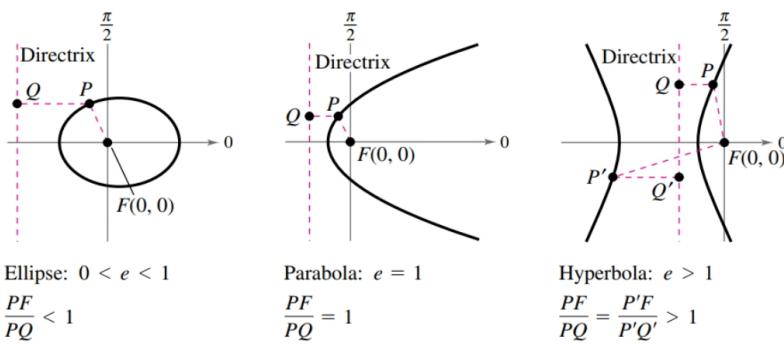
$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$



S<sub>6,9</sub>

## Alternative Definition of a Conic

The locus of a point in the plane that moves such that its distance from a fixed point (focus) is in a constant ratio to its distance from a fixed line (directrix) is a **conic**. The constant ratio is the *eccentricity* of the conic and is denoted by  $e$ . Moreover, the conic is an **ellipse** when  $0 < e < 1$ , a **parabola** when  $e = 1$ , and a **hyperbola** when  $e > 1$ . (See the figures below.)



## Polar Equations of Conics

The graph of a polar equation of the form

$$1. r = \frac{ep}{1 \pm e \cos \theta} \quad \text{or} \quad 2. r = \frac{ep}{1 \pm e \sin \theta}$$

vert. Dir.                      hor. directrix

is a conic, where  $e > 0$  is the eccentricity and  $|p|$  is the distance between the focus (pole) and the directrix.

1. Horizontal directrix above the pole:  $r = \frac{ep}{1 + e \sin \theta}$
  2. Horizontal directrix below the pole:  $r = \frac{ep}{1 - e \sin \theta}$
  3. Vertical directrix to the right of the pole:  $r = \frac{ep}{1 + e \cos \theta}$
  4. Vertical directrix to the left of the pole:  $r = \frac{ep}{1 - e \cos \theta}$

18.

0/1 points

Find a polar equation of the indicated conic in terms of  $r$  with

Conic

Eccentricity

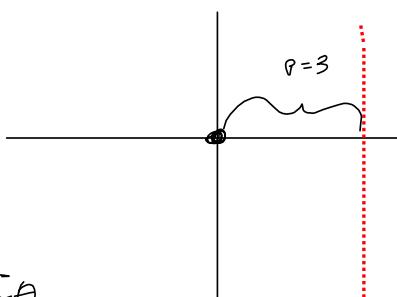
Directrix

Ellipse

$$e = \frac{1}{4}$$

 $x = 3$ 


$$\times \quad r = \frac{3}{\cos(\theta) + 4}$$



$p = \text{distance from focus to directrix}$

$$\begin{aligned} \frac{ep}{1+e\cos\theta} &= \frac{3e}{1+e\cos\theta} \\ &= \frac{3(\frac{1}{4})}{1+\frac{1}{4}\cos\theta} = \frac{\frac{3}{4}}{1+\frac{1}{4}\cos\theta} \text{ Me} \\ &= \frac{3}{4+4\cos\theta} \end{aligned}$$

19.

0/1 points

Find a polar equation of the indicated conic in terms of  $r$  w|Conic  
Hyperbola

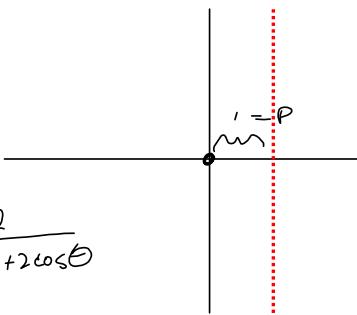
Eccentricity

Directrix

$e = 2$

 $x = 1$ 


$$\times \quad r = \frac{2}{2\cos(\theta) + 1}$$

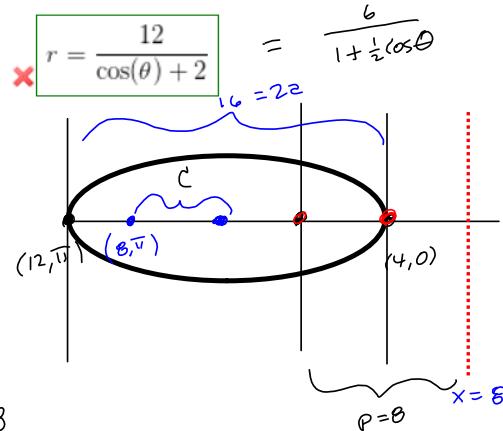


$$\frac{ep}{1+e\cos\theta} = \frac{2p}{1+2\cos\theta} = \frac{2}{1+2\cos\theta}$$

20. + 0/1 pointsFind a polar equation of the conic in terms of  $r$  with its focus at the pole.

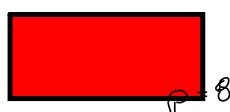
Conic  
Ellipse

Vertices  
 $(4, 0), (12, \pi)$

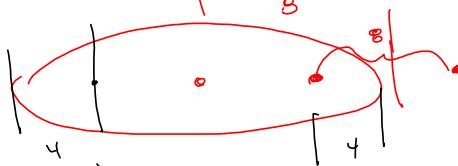


$$\frac{ep}{1+e\cos\theta}$$

$$\begin{aligned} p &= 4 \\ 2z &= 16 \\ z &= 8 \\ e &= \frac{c}{a} = \frac{1}{2} \\ a^2 &= b^2 + c^2 \end{aligned}$$



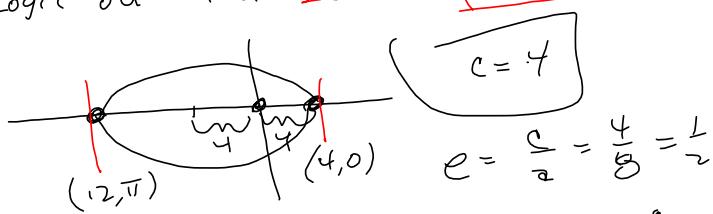
$$r = \frac{8e}{1+e\cos\theta} = \frac{4}{1+\frac{1}{2}\cos\theta} = \frac{8}{1+\cos\theta}$$



#20  
cont'd

Let's do this Jocelyn's way:

Logic out that  $2a = 16 \Rightarrow a = 8$



Here's Jocelyn's method for determining  $\rho$ :

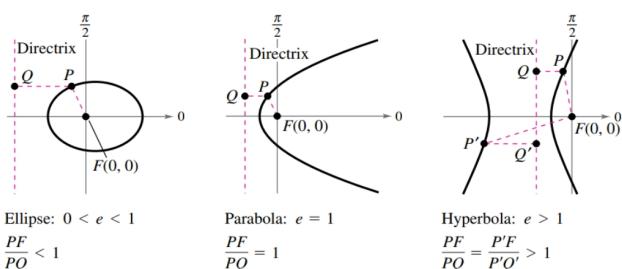
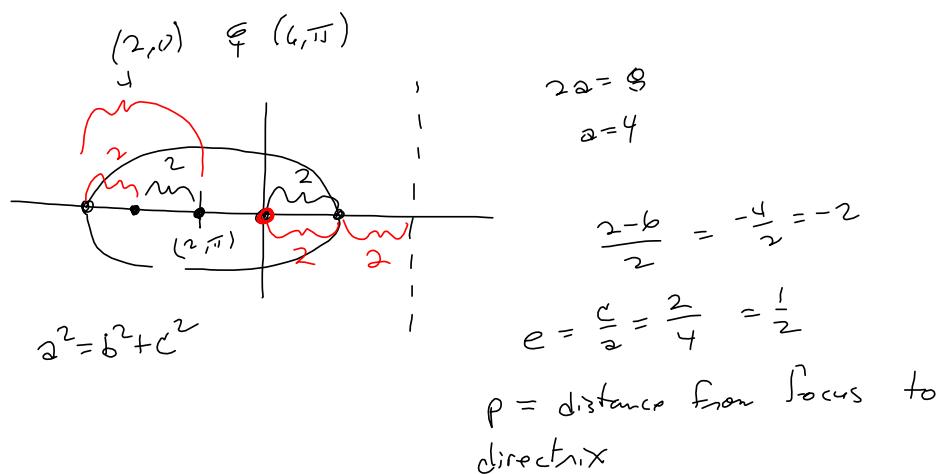
Use what you know:  $(4,0) = (r, \theta)$  is on the graph!

$$r = 4 = \frac{e\rho}{1+e\cos(\theta)} = \frac{\frac{1}{2}\rho}{1+\frac{1}{2}\cos\theta} = \frac{\frac{1}{2}\rho}{1+\frac{1}{2}} = \frac{\frac{1}{2}\rho}{\frac{3}{2}} = \frac{\rho}{3} = 4$$

$$\Rightarrow \rho = 12 \Rightarrow r = \frac{\frac{1}{2} \cdot 12}{1 + \frac{1}{2} \cos\theta} = \frac{6}{1 + \frac{1}{2} \cos\theta} = \frac{12}{2 + \cos\theta}$$

Finally!

My issue: Blurring  $\rho$  in  $S^{6.2}$   
with  $\rho$  in  $S^{6.9}$ !



$$\frac{6}{2+\cos\theta} = \frac{3}{1+\frac{1}{2}\cos\theta}$$

You got  $ep = 3$   
 $= 3(\frac{1}{2})(\frac{2}{1}) = 6 \cdot \frac{1}{2}$

$\boxed{P=6}$

$$r = 2 = \frac{ep}{1+e\cos\theta} = \frac{\frac{1}{2}P}{1+\frac{1}{2}\cos\theta} = \frac{P}{2+\cos\theta}$$

$$= \frac{P}{2+\cos\theta} = \frac{P}{2+1} = \frac{P}{3} \Rightarrow \boxed{P=6}$$

22. + 0/1 points

Find a polar equation of the conic in terms of  $r$  with its focus at the pole.

Conic  
Hyperbola

Vertices

 $(1, 3\pi/2), (9, 3\pi/2)$ 

$$\boxed{r = \frac{9}{4 - 5 \sin(\theta)}}$$

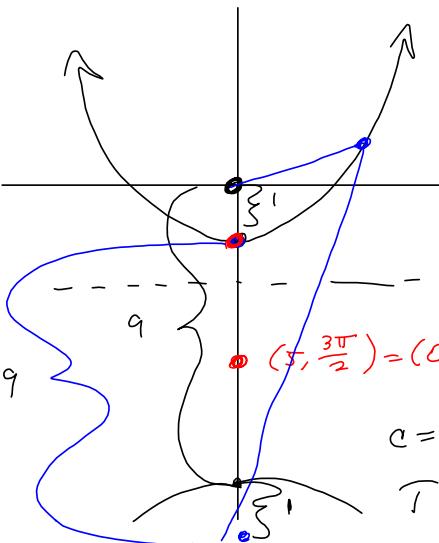
$$e = \frac{PF}{PQ} = \frac{9}{1} = 9$$

$$r = \frac{ep}{1 - e \sin \theta} = \frac{9p}{1 - 9 \sin \theta}$$

$$r = \frac{9p}{1 + 9 \sin \left(\frac{3\pi}{2}\right)} = \frac{9p}{1 + 9} = \frac{9p}{10}$$

$$= \frac{9p}{10} = 1 \Rightarrow p = \frac{10}{9}, \text{ so}$$

$$r = \frac{10}{1 - 9 \sin \theta} \quad \text{New P!}$$



$$2a = 8 \\ a = 4$$

$$c = 5, a = 4$$

Joselyn Says: Do  $\frac{c}{a}$ , idiot!

$$e = \frac{c}{a} = \frac{5}{4}, \text{ so}$$

$$r = 1 = \frac{ep}{1 - e \sin \theta} = \frac{\frac{5}{4}p}{1 - \frac{5}{4} \sin \left(\frac{3\pi}{2}\right)} = 1 = \frac{\frac{5}{4}p}{1 - \left(-\frac{5}{4}\right)} = \frac{\frac{5}{4}p}{\frac{9}{4}} = \frac{5}{9}p = 1$$

$$r = \frac{\left(\frac{5}{4}\right)\left(\frac{9}{5}\right)}{1 - \frac{5}{4} \sin \theta} = \frac{9}{4 - 5 \sin \theta}$$

$$\boxed{P = \frac{9}{5}}$$

24. + 0/1 points

The polar equation of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ is } r^2 = \frac{b^2}{1 - e^2 \cos^2 \theta}.$$

Use the above information to write the polar form of the equation of the conic.

$$\frac{x^2}{169} + \frac{y^2}{144} = 1$$

X 
$$r^2 = \frac{24336}{169 - 25 \cos^2(\theta)}$$

$$\begin{aligned} a &= 13, & b &= 12 \\ c^2 &= 13^2 - 12^2 & &= 169 - 144 = 25 \\ c &= 5 \end{aligned}$$

$$\begin{aligned} r^2 &= \frac{144}{1 - \left(\frac{5}{13}\right)^2 \cos^2 \theta} & e &= \frac{c}{a} = \frac{5}{13} \\ &= \frac{144}{169 - 25 \cos^2 \theta} & &= \frac{(144)(169)}{169 - 25 \cos^2 \theta} \end{aligned}$$

