Questions?

Today, 6.6, 6.7, 6.8

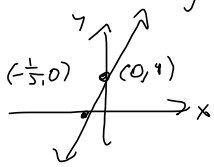
6.6 Parametric Equations:

Classic:

$x = t$

$y = 5t + 1$

a line $y = 5x + 1$



S 6.6 #10

$x = 1 + \cos\theta$

$y = 1 + 6\sin\theta$

Eliminate the parameter & sketch.

Big trick: Pythagorus: $\sin^2\theta + \cos^2\theta = 1$

$x - 1 = \cos\theta$

$y - 1 = 6\sin\theta$

$\frac{y-1}{6} = \sin\theta$

$(x-1)^2 + \left(\frac{y-1}{6}\right)^2 = 1$

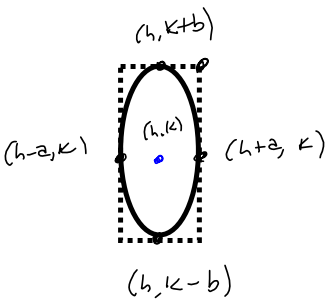
$(x-1)^2 + \frac{(y-1)^2}{6^2} = 1$

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

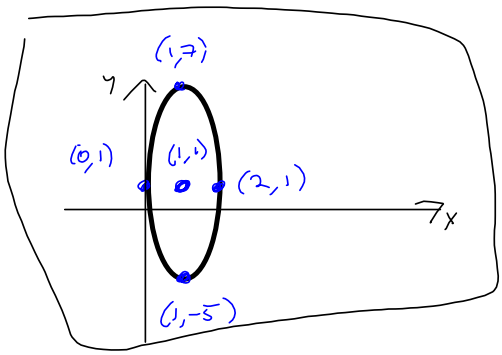
(h, k) = center

a = left/right

b = up/down.



$$\boxed{\frac{(x-1)^2}{1^2} + \frac{(y-1)^2}{6^2} = 1}$$



§6.7 Polar Coordinates

(x,y) becomes (r,θ) via

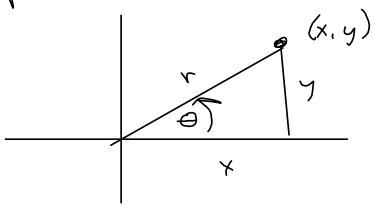
$x^2+y^2=r^2$

$\cos\theta = \frac{x}{r}$

$x = r\cos\theta$

$\sin\theta = \frac{y}{r}$

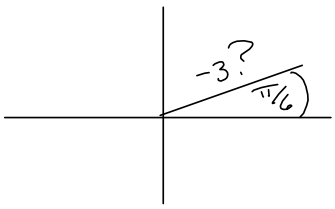
$y = r\sin\theta$

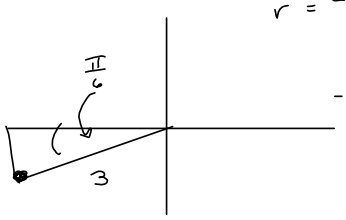


$\frac{y}{x} = \tan\theta$

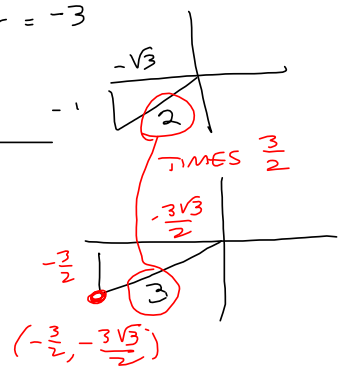
Weird Thing: $-r$ values.

$(r,\theta) = (-3, \frac{\pi}{6})$ Plot it.





$r = -3$



OR $x = r\cos\theta$

$= -3\cos(\frac{\pi}{6}) = -3(\frac{\sqrt{3}}{2})$

$= \frac{-3\sqrt{3}}{2}$

& $y = r\sin\theta = -3\sin(\frac{\pi}{6}) =$

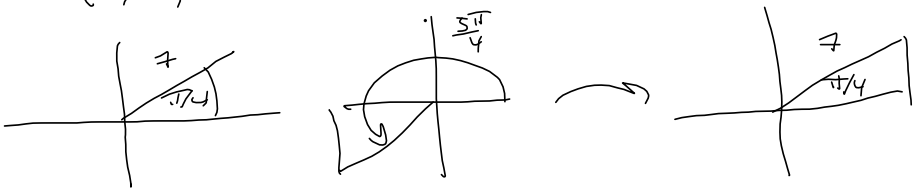
$= -3(\frac{1}{2}) = -\frac{3}{2}$, so

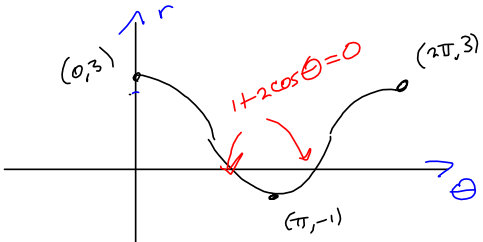
$(x,y) = (-\frac{3\sqrt{3}}{2}, -\frac{3}{2})$ is rectangular coords

for $(r,\theta) = (-3, \frac{\pi}{6})$

Polar Coordinate Representations are not unique.

$(7, \frac{\pi}{4}) = (-7, \frac{5\pi}{4}) = (7, \frac{9\pi}{4}) = (7, -\frac{7\pi}{4})$ = same point!

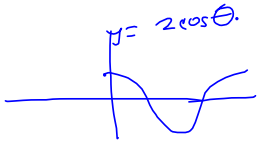


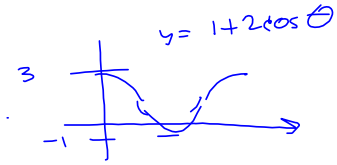


"Rectangular" picture

$r = 1 + 2\cos\theta$

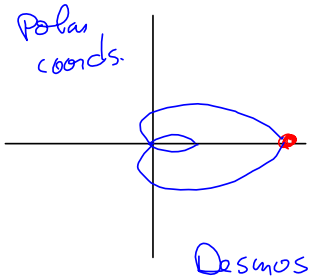
Plot it in rectangular coords. first





Polar coordinates Picture:

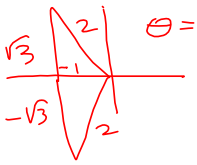
$r = 1 + 2\cos\theta$ in Polar coords.



$1 + 2\cos\theta = 0$

$2\cos\theta = -1$

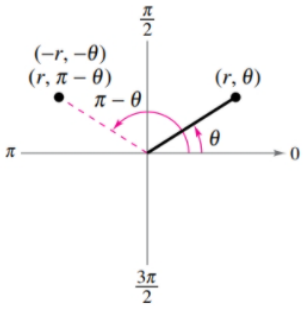
$\cos\theta = -\frac{1}{2}$



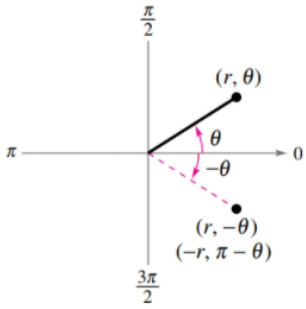
$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

$= 120°, 240°$

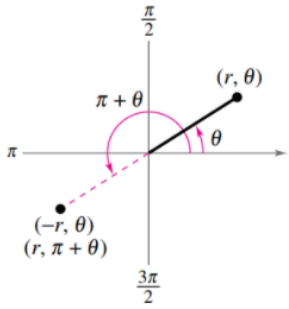
Desmos Does Polar Graphs (Graphs in Polar Coords)



Symmetry with Respect to the Line $\theta = \frac{\pi}{2}$



Symmetry with Respect to the Polar Axis



Symmetry with Respect to the Pole

Tests for Symmetry in Polar Coordinates

The graph of a polar equation is symmetric with respect to the following when the given substitution yields an equivalent equation.

1. The line $\theta = \pi/2$: Replace (r, θ) with $(r, \pi - \theta)$ or $(-r, -\theta)$.
2. The polar axis: Replace (r, θ) with $(r, -\theta)$ or $(-r, \pi - \theta)$.
3. The pole: Replace (r, θ) with $(r, \pi + \theta)$ or $(-r, \theta)$.

$r = 1 + 2\sin\theta$

$r = 1 + 2\sin(\pi - \theta)$

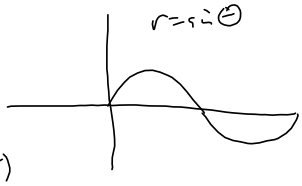
$= 1 + 2\sin\pi\sin\cos(-\theta)$

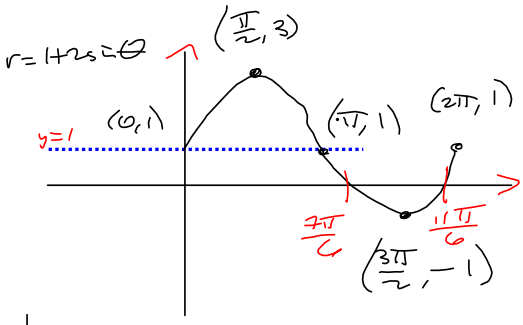
$+ 2\sin(-\theta)\cos(\pi)$ $\to -1$

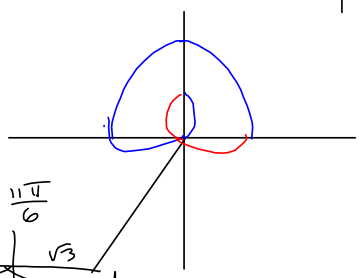
$= 1 - 2\sin\theta \cdot (-1)$

$= 1 + 2\sin\theta = r$ SAME!

Symmetric about $\theta = \frac{\pi}{2}$!

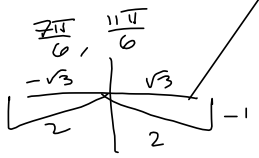






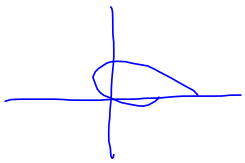
$1 + 2\sin\theta = 0$

$\sin\theta = -\frac{1}{2}$

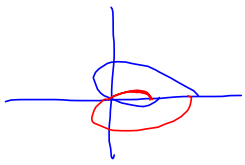


(r,θ) $\quad r = 1+2\cos\theta$

$r = 1+2\cos(-\theta) = 1+2\cos\theta$ is equivalent to the original. So, when I got to here:



I know I had this:



Graphing from 0 to π actually gives the whole picture, by symmetry.

$-r = 1+2\cos(\pi-\theta)$

$\cos(\pi-\theta) = \cos\pi\cos(-\theta) - \sin(\pi)\sin(-\theta)$

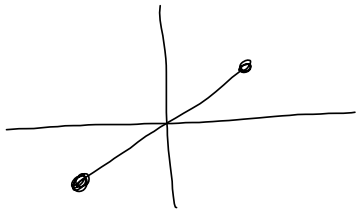
$-r = 1+2[-1\cos\theta + 0]$

$-r = 1-2\cos\theta$

$r = -1+2\cos\theta$ is NOT Equivalent to original

Sometimes one test fails & the other one doesn't.

Didn't Do Symmetry thru the origin.
in §6.8



Please have questions next time!