

S 4.1

$$z\bar{z} = a^2 + b^2 \text{ if } z = a+bi$$

55.  $\frac{2}{1+i} - \frac{3}{1-i} = \left(\frac{2}{1+i}\right)\left(\frac{1-i}{1-i}\right) - \left(\frac{3}{1-i}\right)\left(\frac{1+i}{1+i}\right) = \frac{2-2i}{2+2} - \frac{3+3i}{2}$

$$= \frac{2-2i-3-3i}{2} = \frac{-1-5i}{2} = -\frac{1}{2} - \left(\frac{5}{2}\right)i$$

solve w/ Quadratic Formula

71.  $4x^2 + 16x + 21 = 0$

$$\begin{aligned} a &= 4, b = 16, c = 21 \Rightarrow \\ b^2 - 4ac &= 16^2 - 4(4)(21) = \\ &= 256 - 336 = -80 \\ x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-16 \pm 4\sqrt{5}}{2(4)} \\ &= \frac{16 \pm 4\sqrt{5}}{8} = \frac{4(4 \pm \sqrt{5})}{8} \\ &= \boxed{\frac{4 \pm i\sqrt{5}}{2} = x} \end{aligned}$$

$$4x^2 + 16x + 21 =$$

$$= 4(x^2 + 4x + 2^2) + 21 - 16$$

$$= 4(x+2)^2 + 5 = 0 \Rightarrow$$

$$4(x+2)^2 = -5$$

$$(x+2)^2 = -\frac{5}{4}$$

$$x+2 = \pm \sqrt{-\frac{5}{4}} = \pm i \frac{\sqrt{5}}{2}$$

$$\Rightarrow x = -2 \pm \frac{\sqrt{5}}{2}i = \frac{-4 \pm i\sqrt{5}}{2}$$

86.  $(-i)^6 = ((-1)(i))^6 = (-1)^6(i)^6 = i^6 = (i^2)^3 = (-1)^3 = -1$

$$\begin{array}{r} 16 \\ 21 \\ \hline 16 \\ 320 \\ \hline 336 \end{array}$$

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31,

$$\begin{array}{r} 2 \\ | \\ 1747240 \\ 2 \\ \hline 873620 \\ 2 \\ \hline 436810 \\ 5 \\ \hline 218405 \\ 11 \\ \hline 43681 \\ 11 \\ \hline 3971 \\ 19 \\ \hline 361 \\ 19 \end{array}$$

$$\begin{array}{r} 19 \\ 22 \\ \hline 38 \\ 38 \\ \hline 418 \end{array}$$

$$\sqrt{1747240} = 2 \cdot 11 \cdot 19 \sqrt{2 \cdot 5}$$

$$= \boxed{418\sqrt{10}}$$

How I built question:

$$2^3 \cdot 5 \cdot 11^2 \cdot 19^2 = 1747240$$

## Section 4.2

Factor Theorem: If  $x = c$  is a zero of a polynomial  $f(x)$ , then  $x - c$  is a factor.

$$\begin{aligned} f(x) &= ax^n + bx^{n-1} + cx^{n-2} + \dots + yx + z \\ &= (x-c) \underbrace{(ax^{n-1} + bx^{n-2} + cx^{n-3} + \dots + yx + z)}_{\substack{\text{Solv off factor of } x-c \\ \text{Depressed polynomial of degree } n-1, \text{ with same leading coefficient}}} \\ &= a(x-c)(x^{n-1} + \hat{b}x^{n-2} + \hat{c}x^{n-3} + \dots + \hat{y}x + \hat{z}) \end{aligned}$$

**The Fundamental Theorem of Algebra**

If  $f(x)$  is a polynomial of degree  $n$ , where  $n > 0$ , then  $f$  has at least one zero in the complex number system.

$$a(x-c_1)(x-c_2)(x^{n-2} + \tilde{b}x^{n-3} + \tilde{c}x^{n-4} + \dots + \tilde{y}x + \tilde{z})$$

**Linear Factorization Theorem**

If  $f(x)$  is a polynomial of degree  $n$ , where  $n > 0$ , then  $f(x)$  has precisely  $n$  linear factors

$$f(x) = a_n(x - c_1)(x - c_2) \cdots (x - c_n)$$

$f(x)$  splits into linear factors

where  $c_1, c_2, \dots, c_n$  are complex numbers.

Polynomial theory from College Algebra Writing Project:

<https://harryzaims.com/121-online/121-online-spring-21/writing-projects/Writing-Project-3-spring-21.pdf>



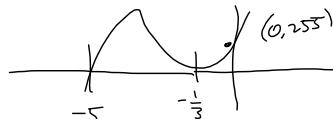
Its Solutions:

<https://harryzaims.com/121-online/121-online-spring-21/writing-projects/Writing-Project-3-spring-21-solns.pdf>



$$f(x) = 9x^5 - 75x^4 - 224x^3 + 2172x^2 + 1511x + 255$$

Graph shows



$x+5 \leftrightarrow x = -5$  is zero

$x + \frac{1}{3} \leftrightarrow x = -\frac{1}{3}$  .. "

$$\begin{array}{r} 9 \quad -75 \quad -224 \quad +2172 \quad 1511 \quad 255 \\ -5 \quad \quad \quad 600 \quad -1880 \quad -1460 \quad -255 \\ \hline -45 \quad 600 \quad -1880 \quad -1460 \quad -255 \end{array}$$

$$\begin{array}{r} 9 \quad -120 \quad 1376 \quad 292 \quad 51 \quad 0 \\ -\frac{1}{3} \quad \quad \quad -3 \quad 41 \quad -139 \quad -51 \\ \hline 9 \quad -123 \quad 417 \quad 153 \quad 0 \end{array}$$

$$\begin{array}{r} 9 \quad -123 \quad 417 \quad 153 \quad 0 \\ -\frac{1}{3} \quad \quad \quad -3 \quad 42 \quad -153 \\ \hline 9 \quad -126 \quad 459 \quad 0 \end{array}$$

This says  $f(x) = (x+5)(x + \frac{1}{3})^2 (9x^2 - 126x + 459)$

i

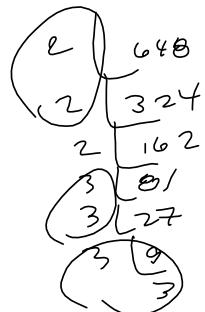
$$9x^2 - 126x + 459 = 0$$

$$a = 9, b = -126, c = 459$$

$$(-126)^2 - 4(9)(459) = -648$$

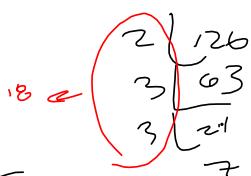
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{126 \pm 18i\sqrt{2}}{2(9)}$$

$$= \frac{18(7 \pm i\sqrt{2})}{18} = 7 \pm i\sqrt{2}$$



$$\therefore \sqrt{648} = 2 \cdot 3 \cdot 3 \sqrt{2}$$

$$= 18\sqrt{2}i$$



THIS SAYS:

$$f(x) = 9(x+5)\left(x+\frac{1}{3}\right)^2\left(x-(7+i\sqrt{2})\right)\left(x-(7-i\sqrt{2})\right)$$

This is  $f(x)$ , split into linear factors.

To get into:

$$\begin{aligned}
 &= (x+5)\left(3\right)^2\left(x+\frac{1}{3}\right)^2(\underline{\quad}) \\
 &= (x+5)\left(3\left(x+\frac{1}{3}\right)\right)^2(\underline{\quad}) \\
 &= (x+5)\left(3x+1\right)^2\left(x-(7+i\sqrt{2})\right)\left(x-(7-i\sqrt{2})\right)
 \end{aligned}$$

Conjugate Pairs Theorem: If all the coefficients of a polynomial are real, and  $x = a + bi$  is a root (zero) of  $f$ , then so is  $a - bi$

16. + 0/1 points

Use the given zero to find all the zeros of the function.  
in your answer.)

Function	Zero	Divide by
$g(x) = 5x^3 + 29x^2 + 44x - 10$	$-3 + i$	$x - (-3 + i)$
$\begin{array}{r} -3+i \\ \hline 5 \end{array}$	$\begin{array}{r} 29 & 44 & -10 \\ -15 + 5i & -47 - i & 0 \end{array}$	$\begin{array}{r} 361 \\ Ans/17 \\ 21,23529412 \\ Ans*17/19 \\ 19 \\ (-3+i)*(14+5i) \\ -47-i \end{array}$
$\begin{array}{r} -3-i \\ \hline 5 \end{array}$	$\begin{array}{r} 14+5i & -3-i & 0 \\ -15 - 5i & 3+i & \end{array}$	$(-3+i)(14+5i)$
$\begin{array}{r} 5 \\ \times \\ \hline -1 \\ C \\ \hline 0 \end{array}$	$\begin{array}{r} 0 \\ R \\ \hline 0 \end{array}$	$= -42 - 15i + 14i + 5i^2$
$\Rightarrow f(x) = (x - (-3 + i))(x - (-3 - i))(5x - 1)$		
$\boxed{-3 \pm i, \frac{1}{5}}$		
$= -42 - i - 5 = -47 - i$		
$(-3+i)(-3-i) = 3^2 + i^2$		

Rotations  $\longleftrightarrow$  Products.

$$\text{Recall } \|z_1 z_2\| = \|z_1\| \|z_2\|$$

If  $\theta_1 = \arg(z_1)$  &  $\theta_2 = \arg(z_2) = \text{Direction angle of } z_2$

$$\text{Then } \arg(z_1 z_2) = \theta_1 + \theta_2 = \arg(z_1) + \arg(z_2)$$

DeMoivre Says:  $z_1 = r_1 (\cos \theta_1 + i \sin \theta_1)$  &  
 $z_2 = r_2 (\cos \theta_2 + i \sin \theta_2) \implies$

$$z_1 z_2 = r_1 r_2 (\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2))$$

$$z_1 = 3 \left( \cos\left(\frac{\pi}{6}\right) + i \sin\left(\frac{\pi}{6}\right) \right) \text{ &}$$

$$z_2 = 7 \left( \cos\left(\frac{3\pi}{4}\right) + i \sin\left(\frac{3\pi}{4}\right) \right) \implies$$

$$z_1 z_2 = 21 \left( \cos\left(\frac{11\pi}{12}\right) + i \sin\left(\frac{11\pi}{12}\right) \right)$$

$$\frac{\pi}{6} + \frac{3\pi}{4} = \frac{2\pi + 9\pi}{12} = \frac{11\pi}{12}$$

Powers:  $z_1^7 = 3^7 \left( \cos(7\theta_1) + i \sin(7\theta_1) \right)$   
 $= 3^7 \left( \cos\left(\frac{7\pi}{6}\right) + i \sin\left(\frac{7\pi}{6}\right) \right)$

If  $\|z_2\| = 1$ , then

$z_1 z_2 = z_2 z_1$  is a Pure Rotation of  $z_1$ .

Next time: Everything about roots.