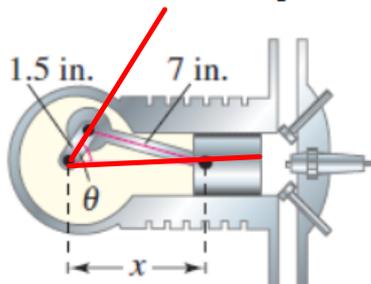


12. + 0/4 points

An engine has a seven-inch connecting rod fastened to a crank (see figure).



$$x^2 = 1.5^2 + 7^2 - 2(1.5)(7) \cos(\theta)$$

$$7^2 = 1.5^2 + x^2 - 2(1.5)(x) \cos \theta$$

$$\theta = 0 \Rightarrow x = 8.5'' \quad \theta = \pi \Rightarrow x = 5.5''$$

$$\theta = \frac{\pi}{2} \Rightarrow x = 7''$$

(a) Use the Law of Cosines to write an equation giving the relationship between x and θ .

$$x^2 - 3x \cos(\theta) - 46.75 = 0$$

(b) Write x as a function of θ . (Select the sign that yields positive values of x .)

$$\frac{1}{2} \left(3 \cos(\theta) + \sqrt{9 \cos^2(\theta) + 187} \right)$$

(c) Use a graphing utility to graph the function in part (b).

$$7^2 = 1.5^2 + x^2 -$$

$$2(1.5)(x) \cos \theta = 2.25 + x^2 + (3 \cos(\theta))x = 49$$

Make x a function of θ

Let $b = 3 \cos(\theta)$ Then

$$x^2 + bx - 46.75 = 0$$

$$a=1, b=b, c=-46.75$$

$$x = \frac{-b \pm \sqrt{b^2 + 187}}{2}$$

$$b^2 = 4ac = b^2 - 4(1)(-46.75)$$

$$= b^2 + 187$$

$$x = \frac{-3 \cos \theta \pm \sqrt{9 \cos^2 \theta + 187}}{2} = x(\theta), \text{ except the } \pm \text{ thingie.}$$

$$x = \frac{-3 \cos \theta + \sqrt{9 \cos^2 \theta + 187}}{2}$$

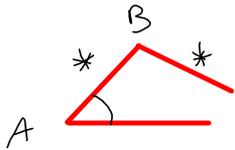
$$\sin^4(4x) = (\sin^2(4x))^2 = \left(\frac{1 - \cos(8x)}{2}\right)^2$$

$$= \frac{1}{4} (1 - 2\cos(8x) + \cos^2(8x)) \quad \text{Power Reduction Formula}$$

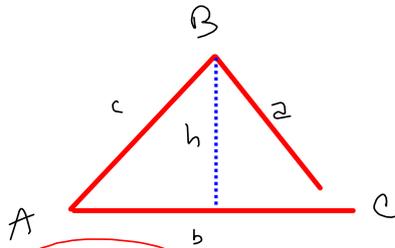
$$= \frac{1}{4} \left(1 - 2\cos(8x) + \frac{1 + \cos(16x)}{2}\right)$$

§3.1 Law of Sines

SAS, ASA: uniquely determines triangle



SSA:
3 possibilities!

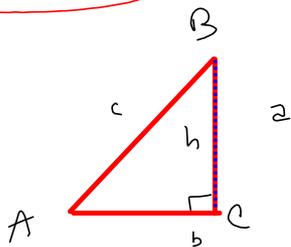


① $a < h$: Can't make a triangle NO SOLN

$$\frac{h}{c} = \sin A$$

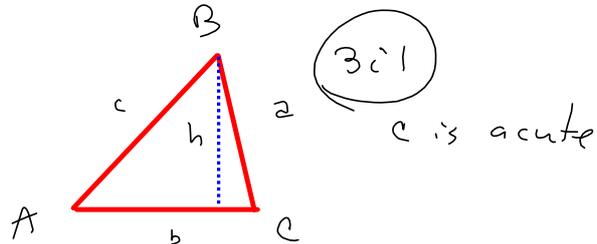
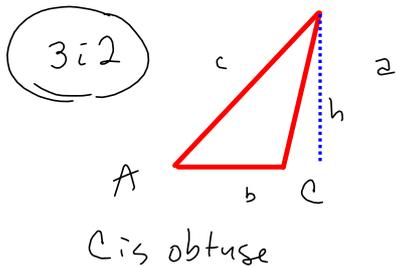
$$h = c \sin A$$

② $a = h$: One sol'n \Rightarrow Really Rare. Right Triangle!



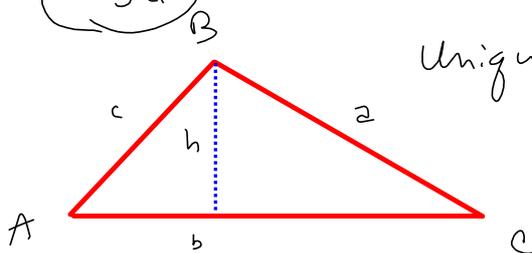
③ $a > h$
 $h < a$

① $h < a < c$ 2 solutions!
1 - C is acute
2 - C is obtuse



③ii $h < a$ & $a > c$

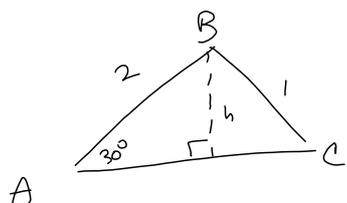
Unique solution & C is acute.



$$A = 30^\circ$$

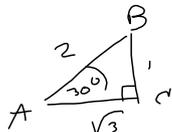
$$a = 1$$

$$c = 2$$



$$\frac{h}{2} = \sin 30^\circ = \frac{1}{2} \Rightarrow h = 1 = a!$$

Unique solution.



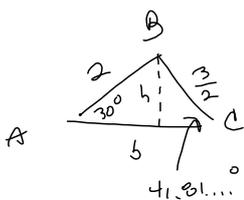
Law of Sines:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin A}{a} = \frac{\sin 30^\circ}{1} = \frac{1}{2}$$

$$\frac{\sin C}{c} = \frac{\sin 90^\circ}{2} = \frac{1}{2}$$

$$\frac{\sin B}{b} = \frac{\sin 60^\circ}{\sqrt{3}} = \frac{\frac{\sqrt{3}}{2}}{\sqrt{3}} = \frac{1}{2}$$



How many solns?

$$h = 2 \sin 30^\circ = 1$$

$$\frac{\sin 30^\circ}{\frac{3}{2}} = \frac{\sin C}{2} \Rightarrow$$

$$2 \cdot \frac{2}{3} \sin 30^\circ = \sin C$$

$$\frac{4}{3} \left(\frac{1}{2}\right) = \frac{2}{3} = \sin C$$

$$\arcsin\left(\frac{2}{3}\right) \approx 41.81031490^\circ$$

$$C \text{ is acute? } C \approx \underline{41.81031490^\circ}$$

NEVER round this

before calculating the 3rd angle!

$$\angle B = 180^\circ - 30^\circ - C \approx 108.1896851^\circ$$

Round to 3 digits:

$A = 30^\circ$ $B \approx 108.190^\circ$ $C \approx 41.810^\circ$	$a = \frac{3}{2}$ $c = 2$ $b \approx 2.850$
---	---

$$\frac{b}{\sin B} = \frac{a}{\sin A} \Rightarrow$$

$$b = \frac{a \sin B}{\sin A} = \frac{\frac{3}{2} \sin(108.190^\circ)}{\sin(30^\circ)}$$

$$\approx 2.850084796 \approx b$$

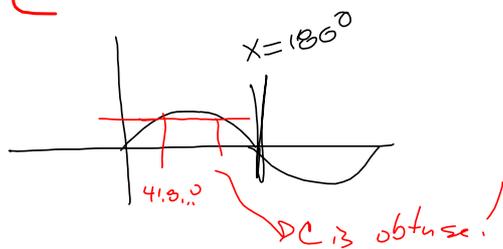
How do we find the "C is obtuse solution?"



The idea:

$$\frac{\sin A}{a} = \frac{\sin C}{c} \text{ is still going}$$

to be true.

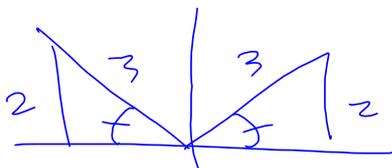
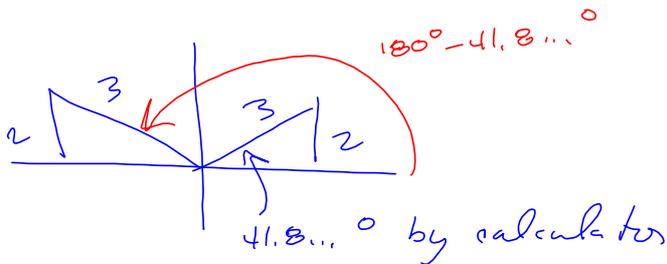


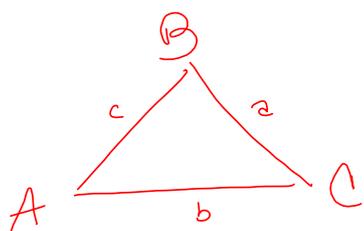
Acute $C_1 = 41.8...^\circ \rightarrow$

Obtuse $C_2 = 180^\circ - 41.8...^\circ$

Another way to see it:

$$\frac{4}{3} \left(\frac{1}{2} \right) = \frac{2}{3} = \sin C$$





LAW of COSINES

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$