

Questions?

S 2.5 Power-Reducing Formulas

$$\cos(u+v) = \cos u \cos v - \sin u \sin v$$

$$\cos(2u) = \cos^2 u - \sin^2 u = \cos^2 u - (1 - \cos^2 u) = 2\cos^2 u - 1$$

$$\cos(2u) = 2\cos^2 u - 1 \Rightarrow$$

$$2\cos^2 u - 1 = \cos(2u) \Rightarrow$$

$$2\cos^2 u = \cos(2u) + 1$$

$$\cos^2 u = \frac{\cos(2u) + 1}{2}$$

$\frac{1}{2}$ -angle: $\cos(u) = \pm \sqrt{\frac{\cos(2u) + 1}{2}} = \pm \sqrt{\frac{1 + \cos(2u)}{2}}$

$$-1 + 1 - \sin^2 u = \frac{\cos(2u) + 1}{2} - \frac{1}{2} \cdot \frac{2}{2} = \frac{\cos(2u) - 1}{2}$$

$$-\sin^2 u = \frac{\cos(2u) - 1}{2}$$

$$\sin^2 u = \frac{1 - \cos(2u)}{2}$$

$\frac{1}{2}$ -angle $\sin(u) = \pm \sqrt{\frac{1 - \cos(2u)}{2}}$

Use the power-reducing formulas to rewrite the expression in terms of first powers of the cosines of multiple angles.

$$\begin{aligned}
 \sin^4(x) &= (\sin^2(x))^2 = \left(\frac{1-\cos(2x)}{2}\right)^2 \quad \text{write as} \\
 (a-b)^2 &= a^2 - 2ab + b^2 \\
 (a+b)^2 &= a^2 + 2ab + b^2 \quad \left(\frac{c}{d}\right)^2 = \frac{c^2}{d^2} \\
 &= \frac{1 - 2\cos(2x) + \cos^2(2x)}{2^2} = \frac{1}{4} - \frac{\cos(2x)}{2} + \frac{1}{4}\cos^2(2x) \\
 \left(\begin{array}{l} \text{Scratch;} \\ \frac{1}{4}\cos^2(2x) = \frac{1}{4}\left(\frac{1+\cos(4x)}{2}\right) = \frac{1}{8} + \frac{\cos(4x)}{8} \end{array} \right) \\
 &= \frac{1}{4} - \frac{1}{2}\cos(2x) + \frac{1}{8} + \frac{1}{8}\cos(4x) \\
 &= \boxed{\frac{3}{8} - \frac{1}{2}\cos(2x) + \frac{1}{8}\cos(4x)}
 \end{aligned}$$



Solving a Trigonometric Equation In Exercises 61–64, find all solutions of the equation in the interval $[0, 2\pi)$. Use a graphing utility to graph the equation and verify the solutions.

61. $\sin 6x + \sin 2x = 0$

62. $\cos 2x - \cos 6x = 0$

#61

$$\sin 5\theta \approx 3\theta = \frac{1}{2}[\cos(2\theta) - \cos(8\theta)]$$

$$\frac{1}{2}(\cos(u-v) - \cos(u+v))$$

$$\sin(6x) + \sin(2x) = 0$$

$$\Rightarrow \sin(2(3x)) + \sin(2x)$$

$$= 2\sin(3x)\cos(3x) + \sin(2x)$$

2 (ugh!) Looks messy!

Scratch

$$\sin(6x) = \sin(2x+4x) = \sin(2x)\cos(4x) + \sin(4x)\cos(2x)$$

$$= \sin(2x)[1 - 2\sin^2(2x)] + 2\sin(2x)\cos(2x)\cos(2x)$$

$$= \sin(2x) - 2\sin^3(2x) + 2\sin(2x)\cos^2(2x)$$

$$= \sin(2x)[1 - 2\sin^2(2x) + 2[1 - \sin^2(2x)]]$$

$$= \sin(2x)[1 - 2\sin^2(2x) + 2 - 2\sin^2(2x)]$$

= DON'T LOSE SIGHT OF THE

= ORIGINAL PROBLEM!

$$\sin(2x) = 0 \quad \text{OR} \quad -4\sin^2(2x) + 3 = 0$$

$$2x \in [0, 2\pi)$$

$$2x \in [0, 4\pi)$$

where's the original
 $\sin(2x)$?



$$2x = 0, \pi, 2\pi, 3\pi$$

$$x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$$

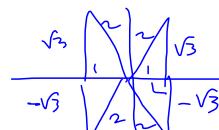
OR

$$4\sin^2(2x) - 3 = 0$$

$$4\sin^2(2x) = 3$$

$$\sin^2(2x) = \frac{3}{4}$$

$$\sin(2x) = \pm \sqrt{\frac{3}{4}} = \pm \frac{\sqrt{3}}{2} = \pm \frac{\sqrt{3}}{2}$$



$$2x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}, \frac{11\pi}{3}$$

$$2x = \frac{2\pi \cdot 7}{3} > \frac{6\pi}{3}$$

$$x = \frac{\pi}{6}, \frac{2\pi}{6} = \frac{\pi}{3}, \frac{4\pi}{6} = \frac{2\pi}{3}, \frac{5\pi}{6}, \frac{8\pi}{6} = \frac{4\pi}{3},$$

$$\frac{10\pi}{6} = \frac{5\pi}{3}, \frac{11\pi}{6}$$

So,

$$\sin(4x) = \sin(2x) [3 - 4\sin^2(2x)] \text{ & the equation is:}$$

$$\sin(2x) [3 - 4\sin^2(2x)] + 3\sin(2x) = 0$$

$$\Rightarrow \sin(2x) (3 - 4\sin^2(2x)) + 1 = \sin(2x) [4 - 4\sin^2(2x)] = 0$$

$$\Rightarrow \sin(2x) = 0$$

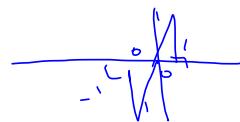
DONE

$$x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$$

$$\text{OR } 4 - 4\sin^2(2x) = 0$$

$$\sin^2(2x) = 1$$

$$\sin(2x) = \pm 1$$



$$2x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$$

$$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

See e-book
for more exercises
along these lines -
#561-64

OR

(An arrow points from the text above to the box containing the solutions for x.)