

25. 0/1 points

Find all solutions of the equation in the interval $[0, 2\pi]$. (Enter your answers as a comma-separated list solution, enter NO SOLUTION.)

$\csc x + \cot x = \frac{\sqrt{3}}{3}$

$x = \boxed{\quad} \times \boxed{\frac{2\pi}{3}}$

$\frac{1}{\sin(x)} + \frac{\cos(x)}{\sin(x)} = \frac{\cos(x) + 1}{\sin(x)} = \frac{\sqrt{3}}{3}$

$\cos(x) + 1 = \frac{\sqrt{3}}{3} \sin(x)$

$\cos(x) - \frac{\sqrt{3}}{3} \sin(x) = -1$

$\frac{\sqrt{3}}{3} \sin(x) - \cos(x) = 1$

$\left(\frac{\sqrt{3}}{3} \sin(x) - \cos(x)\right) \left(\frac{\sqrt{3}}{3} \sin(x) + \cos(x)\right) = \frac{\sqrt{3}}{3} \sin(x) + \cos(x)$

$\frac{3}{9} \sin^2(x) - \cos^2(x)$

$(\csc(x) + \cot(x)) (\csc(x) - \cot(x)) = (\csc(x) - \cot(x))^2$

$\csc^2(x) - \cot^2(x) = \frac{\sqrt{3}}{3} (\csc(x) - \cot(x))$

I'm STUMPED!

$(\cos(x) + 1)^2 = \left(\frac{\sqrt{3}}{3} \sin(x)\right)^2$

$\sqrt{1 - \cos^2(x) + 2\cos(x) + 1} = \frac{3}{9} \sin^2(x) = \frac{1}{3} (1 - \cos^2(x)) = \frac{1}{3} - \frac{1}{3} \cos^2(x)$

$\frac{4}{3} \cos^2(x) + 2\cos(x) + 1 = \frac{1}{3}$

$\frac{4}{3} \cos^2(x) + 2\cos(x) + \frac{2}{3} = 0$

$3\left(\frac{4}{3}u^2 + 2u + \frac{2}{3}\right) = 0$ where $u = \cos(x)$

$(4u^2 + 6u + 2 = 0) \div 2$

$2u^2 + 3u + 1 = 0$

$(2u + 1)(u + 1) = 0$

$u = -\frac{1}{2}, -1$

$u = \cos(x); \cos(x) = -1$

$\cos(x) = -1$

$x = \pi$

$\csc x + \cot x \neq \frac{\sqrt{3}}{3}$

$\csc\left(\frac{4\pi}{3}\right) + \cot\left(\frac{4\pi}{3}\right) = -\frac{2}{\sqrt{3}} + \frac{1}{\sqrt{3}}$

$= -\frac{1}{\sqrt{3}} + \frac{\sqrt{3}}{\sqrt{3}} = -\frac{\sqrt{3}}{3} \neq +\frac{\sqrt{3}}{3}$

$\csc\left(\frac{2\pi}{3}\right) + \cot\left(\frac{2\pi}{3}\right) = \frac{2}{\sqrt{3}} - \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

You NEED TO CHECK these solutions!

$A=B \Rightarrow A^2=B^2$

BUT $A^2=B^2 \Rightarrow A=\pm B$

i.e., squaring both sides can introduce solutions that don't solve the original "A=B."

↑ This one required squaring both sides.

THEY AIN'T NO 26!
(OR 27)

$$2^*x^2+x-1$$

$$2\sin^2(x) + \sin(x) - 1 = 0 \quad \text{Let } u = \sin(x)$$

$$(2\sin(x) - 1)(\sin(x) + 1)$$

$$2\sin(x) = 1$$

$$\sin(x) = \frac{1}{2}$$

or

$$\sin(x) = -1$$



$$x = \frac{3\pi}{2}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$2\sin^2\left(\frac{\pi}{6}\right) + \sin\left(\frac{\pi}{6}\right) - 1 = 2\left(\frac{1}{2}\right)^2 + \frac{1}{2} - 1 \\ = \frac{1}{2} + \frac{1}{2} - 1 = 0 \quad \checkmark$$

etc.

We've an extra week (almost).

Good opportunity for Test 1 Re-take & some clinics.

S 2.4 Angle Sum Identity

check your e-mail.

$$\sin\left(\frac{\pi}{4} + \frac{\pi}{6}\right) = \sin\frac{\pi}{4} \cos\frac{\pi}{6} + \sin\frac{\pi}{6} \cos\frac{\pi}{4} = \text{etc.}$$

Smart Notebook Crashed.

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

$$\sin(x+y) = \sin(x)\cos(y) + \sin(y)\cos(x)$$

\tan(x+y) = I never memorized it.

This can be used for Double-Angle Identities.

$$\begin{aligned}\sin(2x) &= \sin(x+x) = \sin(x)\cos(x) + \sin(x)\cos(x) \\ &= \boxed{2\sin(x)\cos(x) = \sin(2x)}\end{aligned}$$

$$\cos(2x) = \cos(x+x) = \cos(x)\cos(x) - \sin(x)\sin(x)$$

$$\begin{aligned}&= \cancel{\cos^2(x) - \sin^2(x)} \quad \rightarrow \\ &= \cos^2(x) - (1 - \cos^2(x)) \quad \boxed{1 - \sin^2(x) - \cos^2(x)} \\ &= \cos^2(x) - 1 + \cos^2(x) \\ &= \boxed{2\cos^2(x) - 1}\end{aligned}$$

$\cos(2x) = 2\cos^2(x) - 1$ is how we bootstrap to half-angle.

Solve this for $\cos(x)$:

$$\begin{aligned} 2\cos^2(x) &= 1 + \cos(2x) \\ \cos^2(x) &= \frac{1 + \cos(2x)}{2} \\ \sqrt{\cos^2(x)} &= \sqrt{\frac{1 + \cos(2x)}{2}} \\ |\cos(x)| &= \sqrt{\frac{1 + \cos(2x)}{2}} \end{aligned}$$

$$\cos(x) = \pm \sqrt{\frac{1 + \cos(2x)}{2}}$$

$$\cos\left(\frac{u}{2}\right) = \pm \sqrt{\frac{1 + \cos(u)}{2}}$$

Deciding which
take a whole period

$$\begin{array}{ll} \sin(a+b) & \cos(a+b) \\ \sin(2x) & \cos(2x) \\ \sin\left(\frac{u}{2}\right) & \cos\left(\frac{u}{2}\right) \end{array}$$

$\frac{1}{2}$ -angle for sine?

$$\text{use } \cos(2x) = 1 - 2\sin^2(x)$$

& solve for $\sin(x)$

$$\sin\left(\frac{u}{2}\right) = \pm \sqrt{\frac{1 - \cos(u)}{2}}$$

FIND WHICH QUADRANT!

35. 0/1 points

LarTrig10 2.3

Use a graphing utility to approximate (to three decimal places) the solutions of the equation in the interval $[0, \pi]$, your answers as a comma-separated list.)

$$\cos(x) = x$$

<https://www.wolframalpha.com/>

has a grapher. There are other graphing utilities on the Internet. Find one that you like and that seems to work for you.