MAT 122 Test 1 – Spring, 2020 Name KEY

100 Points Covers Chapter 1 NO GRAPHING CALCULATORS!!!

Do all your work and submit answers with your work, on the separate paper provided. Organize your work for efficient grading and feedback. Leave a margin, especially in the top left, where the staple goes!

Leave space between problems. No prizes for saving paper, here. Figure this stuff out, and use your smarts to plant trees! Only use one column of work. Don't start a 2nd column to save paper. ALL I WANT ON THIS PAGE IS YOUR NAME.

1. (10 pts) Find two angles, between -2π and 2π (i.e., -360^0 and 360^0) that are coterminal with $\frac{89\pi}{6}$. Give exact answers in degrees and radians.

2. Arc Length and Area of Sector. Suppose we have a circle of radius $r = 25$ cm.

   a. (5 pts) Find the arc length on the circle, that is intercepted by an angle of 1332^0. Round to 3 decimal places.

   b. (5 pts) Find the *exact* area of the sector that is intercepted (swept through) by an angle of $\theta = 135^0$

3. Basic concept: Draw the doggone pictures!

   a. (5 points) Sketch two triangles that satisfy $\tan(\theta) = \frac{\sqrt{3}}{\sqrt{13}}$.

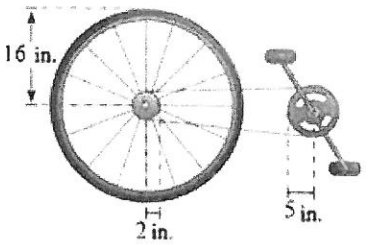
   b. (5 pts) Assume the terminal side of the angle θ lies in the 3rd quadrant (Quadrant III). Find the other five trigonometric functions of θ.

   c. (5 pts) Again, assuming θ's terminal side lies in Q III, and $0 \le \theta < 2\pi$, find θ, in radians *and* degrees, rounded to 3 decimal places.

   d. (5 pts) Give *all* solutions to the equation $\tan(\theta) = \frac{\sqrt{3}}{\sqrt{13}}$, in degrees *and* radians, rounded to three (3) decimal places. (Good one to skip and come back to, if time permits).

4. (10 pts) Sketch one period of the graphs of $y = \cos(x)$ and $y = \sec(x)$ on the same set of coordinate axes.

5. (10 pts) The radii of the pedal sprocket, the wheel sprocket, and the wheel of the bicycle in the figure are 5 inches, 2 inches and 16 inches, respectively. A cyclist is pedaling at a rate of 4 revolutions per second. Find the speed of the bicycle in feet per second. Then convert that to miles per hour. Round final answers to 1 decimal place.



122 TEST 1 SPRING, 2020

(1) (10 pts)

$$\frac{89\pi}{6} \div 2\pi = \frac{84\pi}{6\cdot 2\pi} + \frac{5\pi}{6\cdot 2\pi}$$

$$= 7 \text{ revs} + \frac{5}{12} \text{ revs}.$$

$$\frac{5}{12}\cdot 2\pi = \boxed{\frac{5\pi}{6} \text{ OR } 150^\circ}$$

$$-2\pi + \frac{5\pi}{6} = \boxed{-\frac{7\pi}{6} \text{ OR } -210^\circ}$$

(2a) (5 pts) $s = r\theta = (25)(1332^\circ)\left(\frac{\pi}{180^\circ}\right) = 185\pi \text{ cm}$

$$\approx 581.1946409 \text{ cm} \approx \boxed{581.195 \text{ cm}}$$

(b) (5 pts) $A = \frac{1}{2}r^2\theta = \frac{1}{2}(25)^2(135^\circ)\left(\frac{\pi}{180^\circ}\right) \text{ cm}^2$

$$= \frac{(625)(135)}{(2)(180^\circ)}\pi \qquad \frac{1875}{8}\pi$$

$$= \frac{5^{\cancel{4}^{3}}\cdot 3^{\cancel{3}^{1}}\cdot 5\,\pi}{(2)(2^2)(\cancel{3^2})(\cancel{5})}$$

$$= \boxed{\frac{1875\pi}{8} \text{ cm}^2}$$

$$\approx 736.3107782 \text{ cm}^2$$

$180 = 2^2\cdot 3^2\cdot 5$: $2\,|\,180$, $2\,|\,90$, $3\,|\,45$, $3\,|\,15$, 5

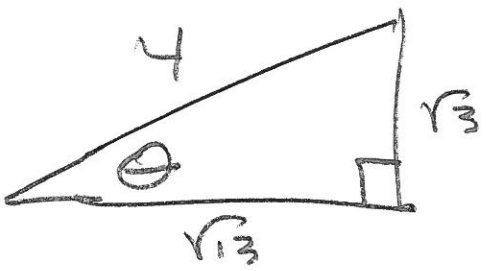
$625 = 5^4$: $5\,|\,625$, $5\,|\,125$, $5\,|\,25$, 5

$135 = 3^3\cdot 5$: $3\,|\,135$, $3\,|\,45$, $3\,|\,15$, 5

122 #1 (2)

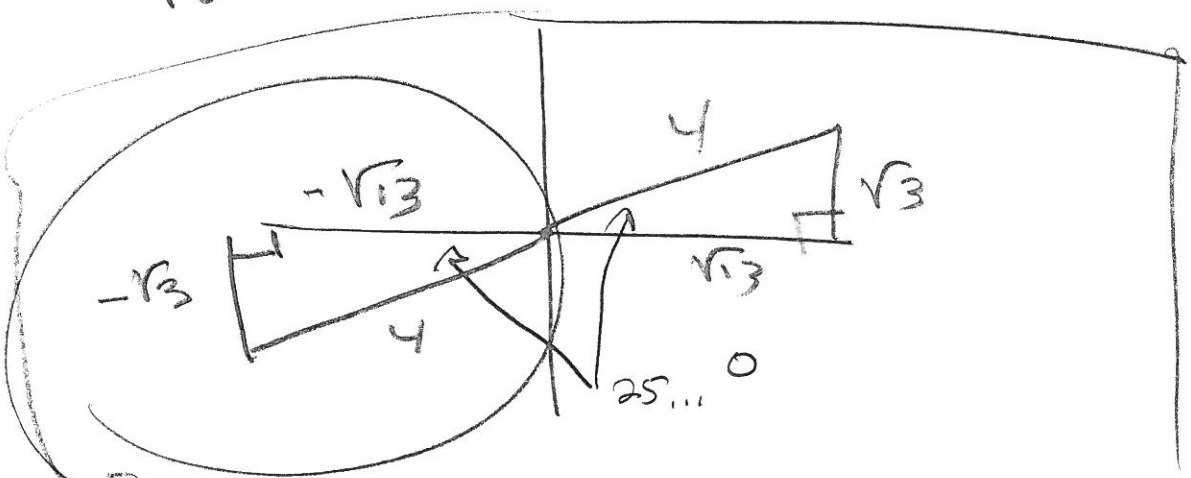
3a) 5pts

$\tan\theta = \frac{\sqrt{3}}{\sqrt{13}}$



$3+13=16$

$\sqrt{16}=4$



b) 5pts

$\theta \in$ QIII given

$\sin\theta = -\frac{\sqrt{3}}{4}$ $\quad \csc\theta = -\frac{4}{\sqrt{3}}$

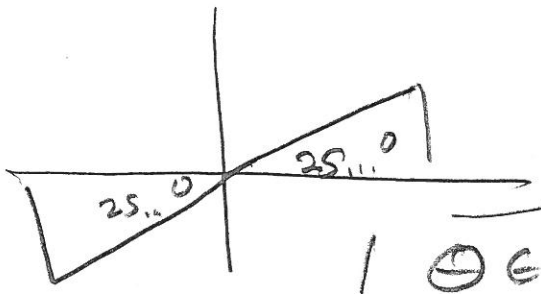
$\cos\theta = \frac{-\sqrt{13}}{4}$ $\quad \sec\theta = -\frac{4}{\sqrt{13}}$

$\tan\theta = \frac{\sqrt{3}}{\sqrt{13}}$ $\quad \cot\theta = \frac{\sqrt{13}}{\sqrt{3}}$

c) 5pts $\arctan\left(\frac{\sqrt{3}}{\sqrt{13}}\right) \approx 25.65890627$

So $\theta = 180^\circ + 25...^\circ \approx 205.659^\circ$

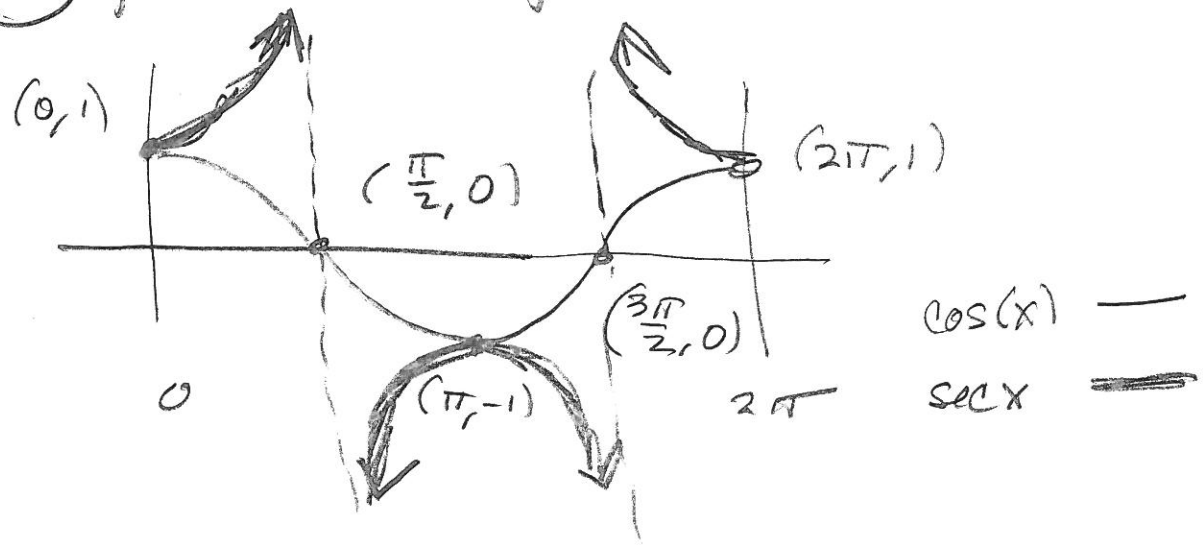
$\theta \approx 3.589$ radians



3d (5pts)

$$\theta \in \{3.589 + n\pi \mid n \in \mathbb{Z}\}$$
$$= \{205.659^\circ + n\pi \mid n \in \mathbb{Z}\}$$

(4) (10pts) $y = \cos(x)$ & $y = \sec(x)$



(5) (10pts) $\left(\frac{4 \text{ revs front}}{1 \text{ sec}}\right)\left(\frac{5 \text{ revs back}}{2 \text{ revs front}}\right)\left(\frac{2\pi}{1 \text{ rev back}}\right)(16 \text{ in})\left(\frac{1 \text{ ft}}{12 \text{ in}}\right)$

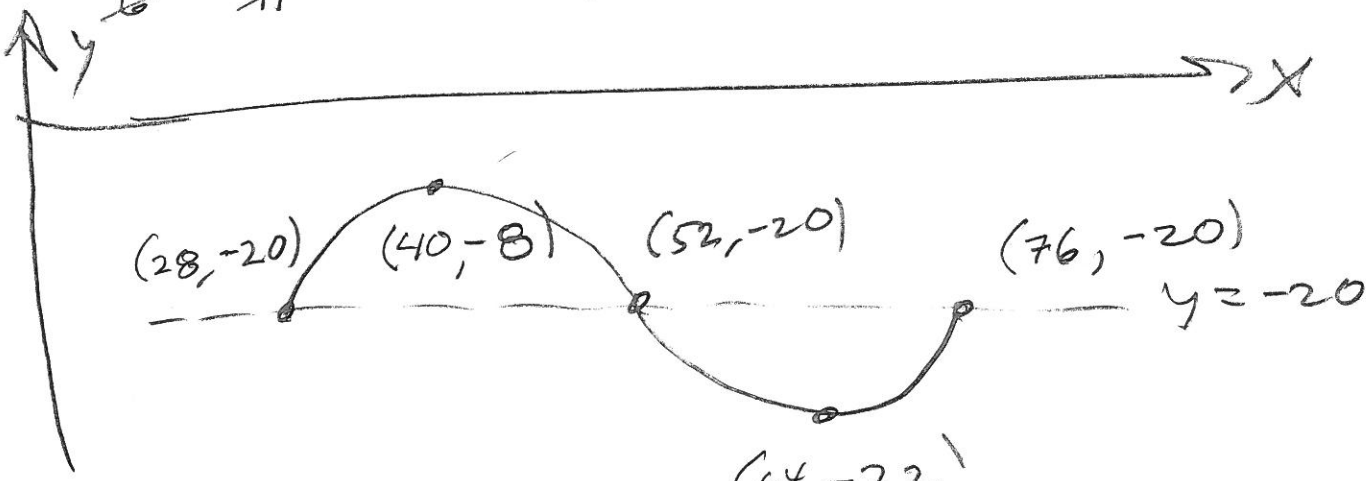
$$= \frac{(4)(5)(2\pi)(16)}{(2)(12)} = \frac{80\pi}{3} \frac{\text{ft}}{\text{s}} \approx 83.77580411 \frac{\text{ft}}{\text{s}}$$

$$\approx 83.8 \frac{\text{ft}}{\text{s}} \approx (83\ldots)\left(\frac{60 \text{ mi/hr}}{88 \text{ ft/s}}\right) \simeq 57.11986643 \text{ mi/hr}$$

$$\approx 57.1 \frac{\text{mi}}{\text{hr}}$$

(6) (12 pts) $f(x) = 12\sin\left(\frac{\pi}{24}x - \frac{7\pi}{6}\right) - 20$

$\frac{7\pi}{6} \cdot \frac{24}{\pi} = 28 \Rightarrow f(x) = 12\sin\left(\frac{\pi}{24}(x-28)\right) - 20$



$\frac{1}{2}T = 24$

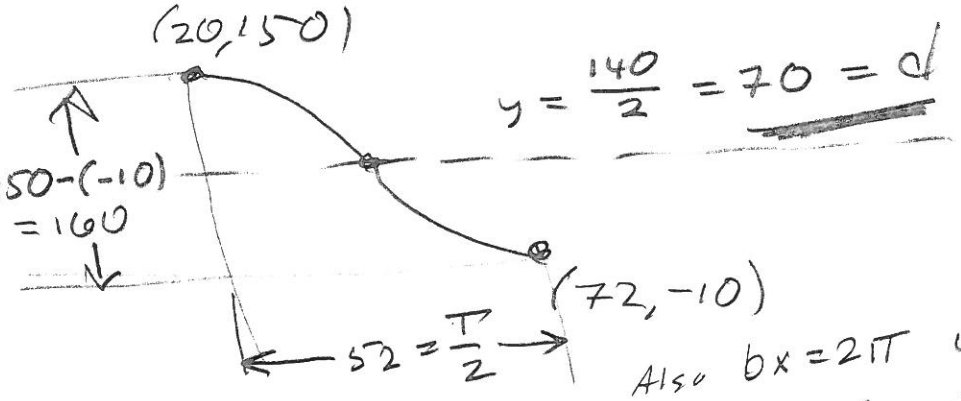
$T = 48$

OR $\frac{\pi}{24}x = 2\pi \rightarrow$

$x = 48$ ✓

$\frac{48}{4} = 12$

(7) (10 pts)



So $a = \frac{160}{2} = 80$

$c = \text{start} = 20$

$\frac{\pi}{52} = b$

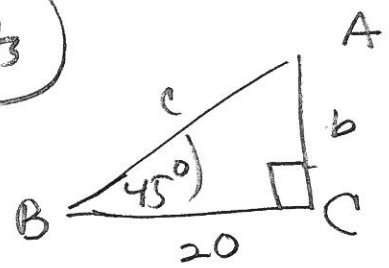
Also $bx = 2\pi$ when $x = 104 \rightarrow$

$b = \frac{2\pi}{104} = \frac{\pi}{52}$

$a\cos(b(x-c)) + d$

$= 80\cos\left(\frac{\pi}{52}(x-20)\right) + 70$

(8) 5pts



$a = 20$ $A = 45^\circ$
$b = 20$ $B = 45^\circ$
$c = 20\sqrt{2}$ $C = 90^\circ$

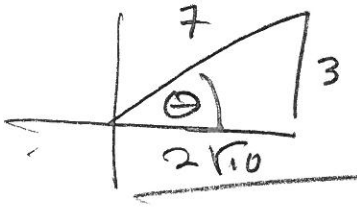
$20^2 + 20^2 = 2(20)^2 \rightarrow c = 20\sqrt{2}$

Check: $\frac{20}{c} = \cos 45^\circ$

$20 = c \cos 45^\circ$

$c = \frac{20}{\cos 45^\circ} = \frac{20}{\frac{1}{\sqrt{2}}} = 20\sqrt{2}$

(9) (a) 5pts $\cot(\arcsin(\frac{3}{7})) = \cot\theta$

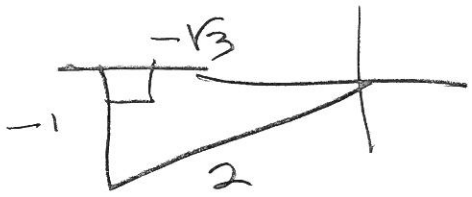


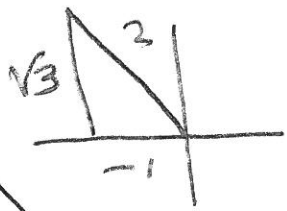
$7^2 - 3^2 = 49 - 9 = 40$

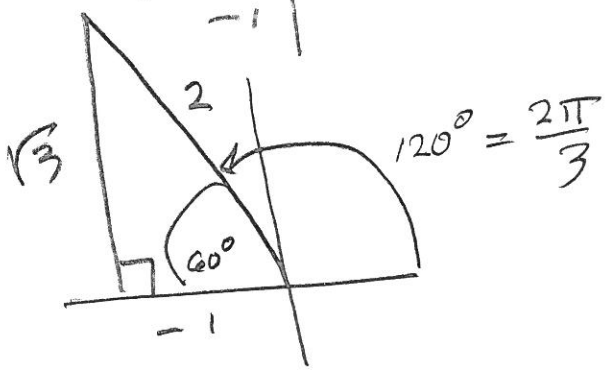
$\sqrt{40} = 2\sqrt{10}$

$\cot\theta = \frac{2\sqrt{10}}{3}$

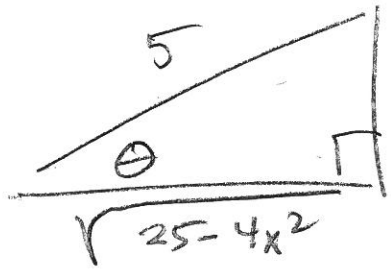
(b) 5pts $\arccos(\sin(\frac{7\pi}{6})) = \arccos(-\frac{1}{2}) = \frac{2\pi}{3}$ OR 120°







(10) (5pts) $\cot\left(\arccos\left(\frac{\sqrt{25-4x^2}}{5}\right)\right) = \cot\theta$



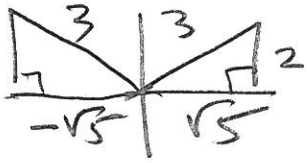
$5^2 - \left(\sqrt{25-4x^2}\right)^2$

$= 25 - (25-4x^2) = 4x^2 = y$

$\Rightarrow y = 2x$

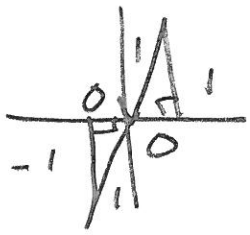
$$\cot\theta = \frac{\sqrt{25-4x^2}}{2x}$$

(B1) (a) $\sin x = \frac{2}{3}$

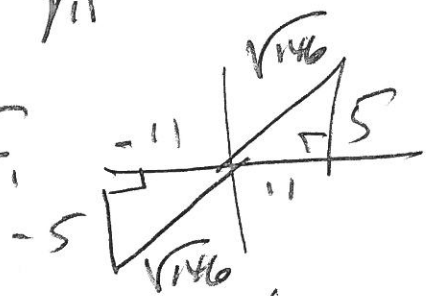


$9-4=5$

(b) $\cos x = 0$



(c) $\tan x = \frac{5}{11}$



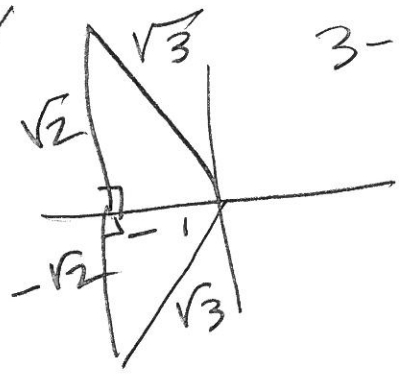
$121+25 = 146$

$2 \overline{)146}$

73

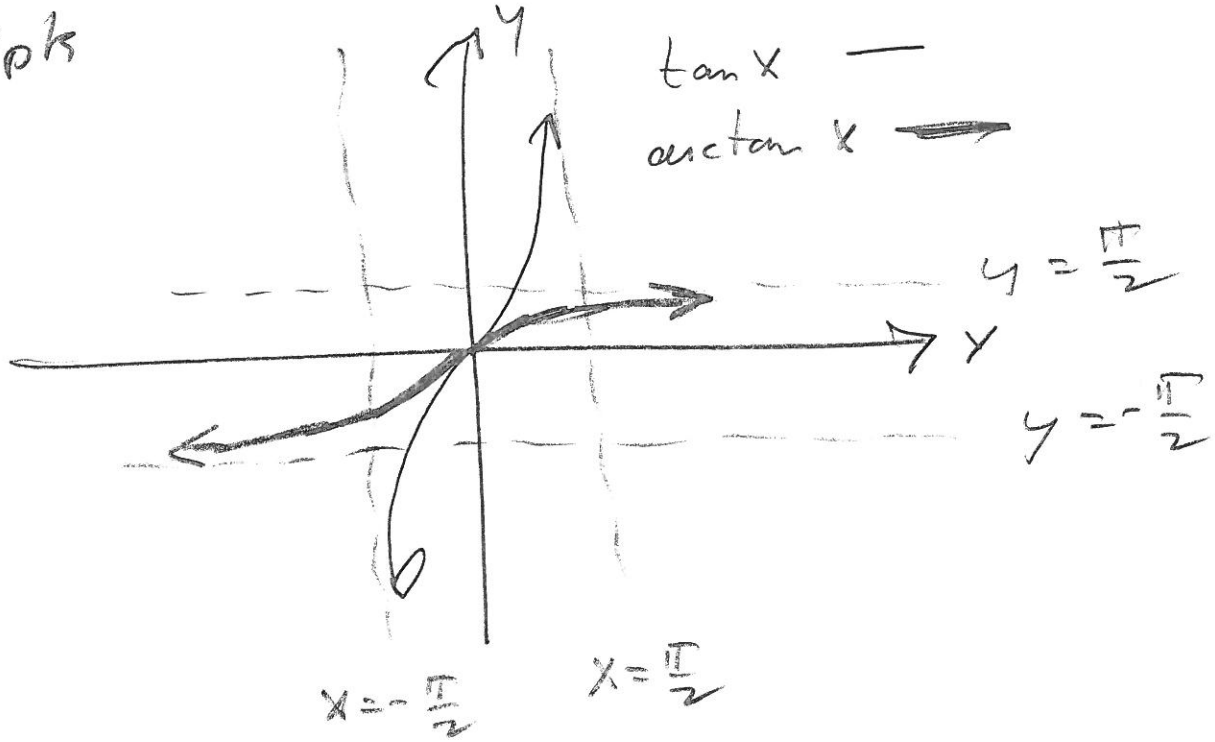
(d) $\cos x = -\sqrt{3} < -1$ ∄

(e) $\sec x = -\sqrt{3}$



$3-1=2$

(B2) 5pk



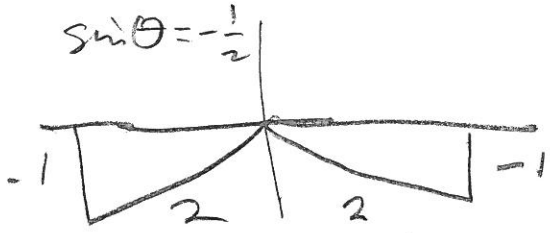
(B3) $8\sin^3\theta + 4\sin^2\theta - 6\sin\theta - 3 = 0$

$8x^3 + 4x^2 - 6x - 3 = 4x^2(2x+1) - 3(2x+1)$

$= (2x+1)(4x^2-3) = (2x+1)(2x-\sqrt{3})(2x+\sqrt{3}) = 0$

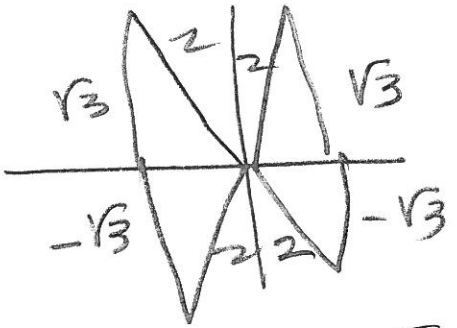
$\Rightarrow x = \sin\theta = -\frac{1}{2}, \pm\frac{\sqrt{3}}{2}$

$\sin\theta = -\frac{1}{2}$



$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

$\sin\theta = \pm\frac{\sqrt{3}}{2}$

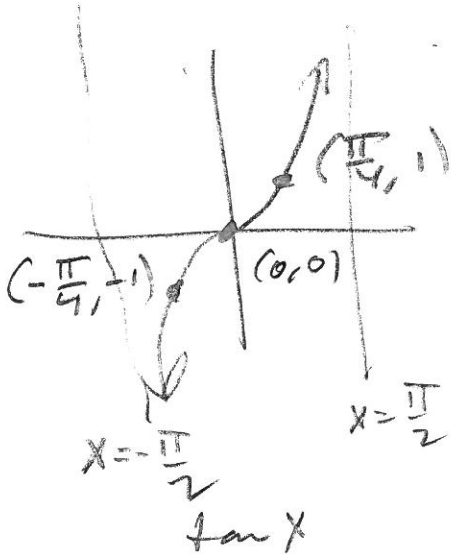


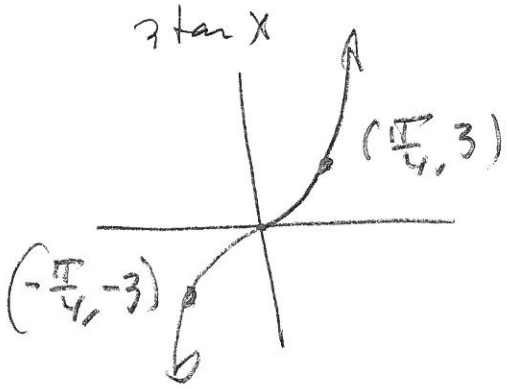
$\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

$\theta \in \left\{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{7\pi}{6}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{11\pi}{6}\right\}$

122 T1 (8)

B4 SpB $f(x) = 3\tan\left(\frac{\pi}{2}x\right) - 4$

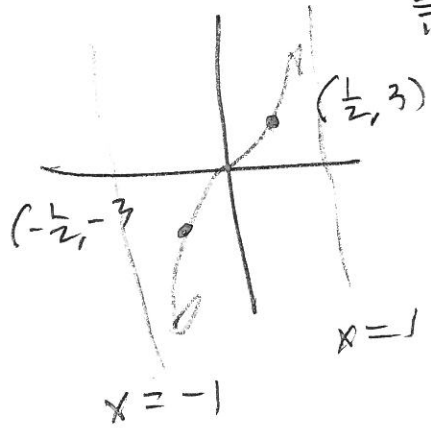




$3\tan\left(\frac{\pi}{2}x\right)$ $\quad x \longmapsto \frac{2}{\pi}x$

$\frac{2}{\pi} \cdot \frac{\pi}{4} = \frac{1}{2}$

$\frac{\pi}{2} \cdot \frac{2}{\pi} = 1$



$f(x) = 3\tan\left(\frac{\pi}{2}x\right) - 4$