

$$r = \frac{ep}{1 \pm e \cos \theta} \quad r = \frac{ep}{1 \pm e \sin \theta}$$

$0 < e < 1 \Rightarrow$ ellipse

$e = 1 \Rightarrow$ parabola

$e > 1 \Rightarrow$ hyperbola



$$\frac{ep}{1 \pm e \cos \theta}$$

$$\frac{2}{2 - \cos \theta} = \frac{2}{2(1 - \frac{1}{2} \cos \theta)}$$

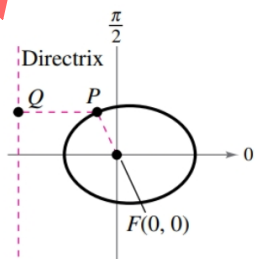
$$= \frac{1}{1 - \frac{1}{2} \cos \theta} = \frac{(\frac{1}{2})(2)}{1 - \frac{1}{2} \cos \theta} \rightarrow p = 2$$

$$e = \frac{1}{2}$$

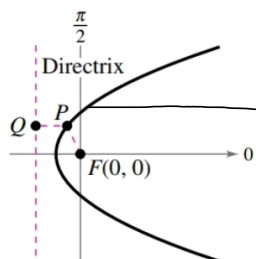
$$1 = \frac{1}{2} \cdot 2$$

Alternative Definition of a Conic

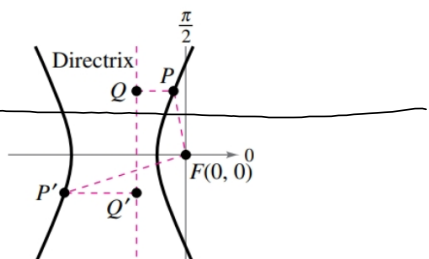
The locus of a point in the plane that moves such that its distance from a fixed point (focus) is in a constant ratio to its distance from a fixed line (directrix) is a **conic**. The constant ratio is the *eccentricity* of the conic and is denoted by e . Moreover, the conic is an **ellipse** when $0 < e < 1$, a **parabola** when $e = 1$, and a **hyperbola** when $e > 1$. (See the figures below.)



Ellipse: $0 < e < 1$
 $\frac{PF}{PQ} < 1$



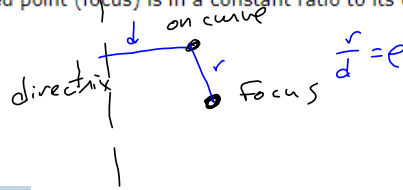
Parabola: $e = 1$
 $\frac{PF}{PQ} = 1$



Hyperbola: $e > 1$
 $\frac{PF}{PQ} = \frac{P'F}{P'Q'} > 1$

The locus of a point in the plane that moves such that its distance from a fixed point (focus) is in a constant ratio to its distance from a fixed line (directrix) is a 1a **LarTrig10 6.9.001. (3884513)** 3884513

$0 < e < 1 \rightarrow$ ellipse
 $e = 1 \rightarrow$ parabola
 $e > 1 \rightarrow$ hyperbola



The constant ratio is the 2a **2. LarTrig10 6.9.002.** conic and is denoted by 2b

$e =$ eccentricity
 (Not Euler's 'e')

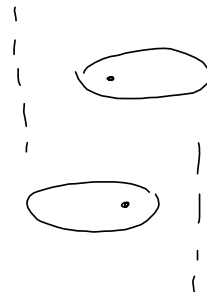
An equation of the form $r = \frac{ep}{1 - e \cos(\theta)}$ has a 3a vertical directrix

3b to the left of the pole .

$\frac{ep}{1 \pm e \sin \theta}$ Horizontal directrix

Match the conic with its eccentricity.

- (a) $0 < e < 1$ ellipse
- (b) $e = 1$ parabola
- (c) $e > 1$ hyperbola



3. LarTrig10 6.9.003.

Write the polar equation of the conic for each value of e . Identify the type of conic represented by each equation. Verify your answers with a graphing utility.

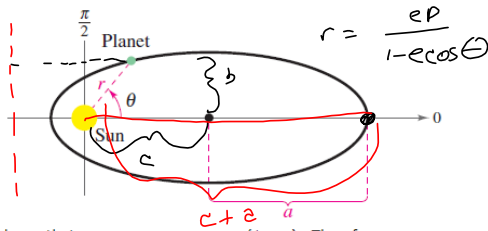
$$r = \frac{2e}{1 - e \cos(\theta)}$$

$$e = 1 \quad r = \frac{2}{1 - \cos \theta} \quad \text{parabola}$$

$$e = 0.5 \quad r = \frac{2e}{1 - .5 \cos \theta} = \frac{1}{1 - .5 \cos \theta} \quad \text{ellipse}$$

$$e = 1.5 \quad \frac{2e}{1 - 1.5 \cos \theta} = \frac{3}{1 - 1.5 \cos \theta} \quad \text{hyperbola}$$

The planets travel in elliptical orbits with the sun at one focus. Assume that the focus is at the pole, the major axis lies on the polar axis, and the length of the major axis is $2a$ (see figure). Show that the polar equation of the orbit is $r = a(1 - e^2) / (1 - e \cos \theta)$, where e is the eccentricity.



$$r = \frac{ep}{1 - e \cos \theta}$$

#52 webAssign

#53 in book

want: $r = \frac{a(1 - e^2)}{1 - e \cos \theta}$

want: $ep = a(1 - e^2) = a - ae^2$

$$a^2 - b^2 = c^2 = a^2 e^2$$

$$ep = a(1 - e^2) ?$$

$$ep = a(1 - \frac{c^2}{a^2})$$

$$= ep$$

$$e = \frac{c}{a}$$

$$a^2 - b^2 = c^2$$

$$ae = c$$

$$r = \frac{ep}{1 - e \cos \theta}$$

$$a(1 - e^2) = a(1 - \frac{c^2}{a^2})$$

$$= a(\frac{a^2 - c^2}{a^2}) = \frac{a^2 - c^2}{a} = \frac{b^2}{a}$$

hint: $\theta = 0 \Rightarrow$ on major axis & distance from focus to point on ellipse is $r = c + a = ea + a = a(1 + e)$

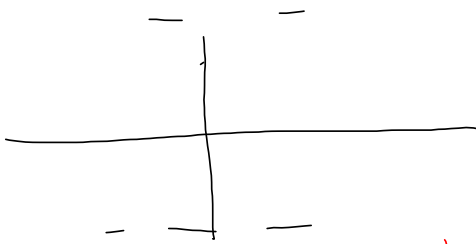
$$a(1 + e) = \frac{ep}{1 - e \cos 0} = \frac{ep}{1 - e} \Rightarrow$$

$$a(1 + e)(1 - e) = ep \Rightarrow$$

$$a(1 - e^2) = ep$$

$$r = \frac{2}{2 + 6\sin(\theta + \frac{2\pi}{3})}$$

$\cos(\theta + \frac{2\pi}{3})$ what's happening?



$$\begin{aligned} \frac{2}{2 + 6\sin\theta} &= \frac{2}{2(1 + 3\sin\theta)} \\ &= \frac{1}{1 + 3\sin\theta} = \frac{3 \cdot \frac{1}{3}}{1 + 3\sin\theta} \end{aligned}$$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ gives you the endpoints of the major axis for an ellipse or vertices for a hyperbola.

$\theta = 0$ & π give you the same as above.

$$\frac{1}{1 + \frac{1}{2}\sin\theta}$$

$$ep = \frac{1}{2}p = 1$$

$$p = 2$$

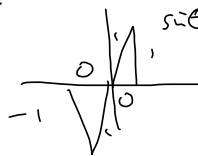
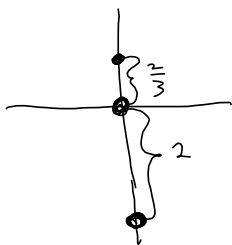
$$\theta = \frac{\pi}{2} : \frac{1}{1 + \frac{1}{2}} = \frac{1}{\frac{3}{2}} = \frac{2}{3}$$

$$\theta = \frac{3\pi}{2} : \frac{1}{1 - \frac{1}{2}} = \frac{1}{\frac{1}{2}} = 2$$

$$\frac{1}{1 + 3\sin\theta}$$

$$\frac{ep}{1 \pm e\cos\theta}$$

$$\sin\theta = \pm 1$$



Length of major axis

$$= 2 + \frac{2}{3} = \frac{6+2}{3} = \frac{8}{3} = 2a$$

$$\frac{x^2}{2^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{b^2} + \frac{y^2}{2^2} = 1$$

$$\frac{3}{1 - 1.5\cos\theta}$$

Graphing Calculator

MODE: Polar

connected - vs - dot Mode

↓ Includes stuff you don't want

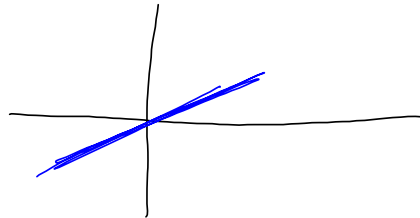
↓ Leaves out stuff you want.

$$\frac{eD}{1 \pm \cos \theta}$$

-vs-

$$\frac{eD}{1 \pm \cos(\theta + \frac{\pi}{4})}$$

Rotate
clockwise $\frac{\pi}{4}$ radians



$$\frac{eD}{1 \pm \cos(\theta - \frac{\pi}{4})}$$

Counter-clockwise
rotation by
 $\frac{\pi}{4}$