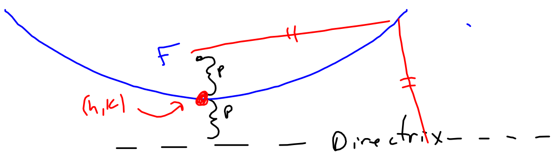
Conics in Rectangular Coordinates

Deadline for all test-taking is Thursday, 5/7, midnight.

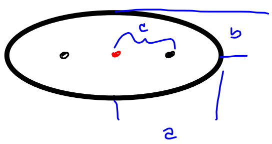
$$9x^2+4y^2-36x+16y+52=36$$

$$9x^2-4y^2-36x-16y+20=36$$

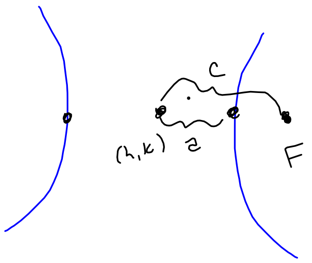
$$x^2-8y+6x+9=0$$

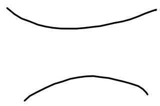


$$4p(y-k) = (x-h)^2$$



$$c^2 = a^2 - b^2$$





$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

$$c^2 = a^2 + b^2$$

$9x^2+4y^2-36x+16y \quad +52=36$

$9x^2-36x \quad + 4y^2+16y \quad = -16$

$9(x^2-4x \quad) + 4(y^2+4y \quad) = -16$

$9(x^2-4x+2^2) + \boxed{4}(y^2+4y+2^2) = -16 + 4(9) + 4(4)$

$\frac{4}{2} \rightsquigarrow 2^2=4$

$9(x-2)^2 + 4(y+2)^2 = 36$

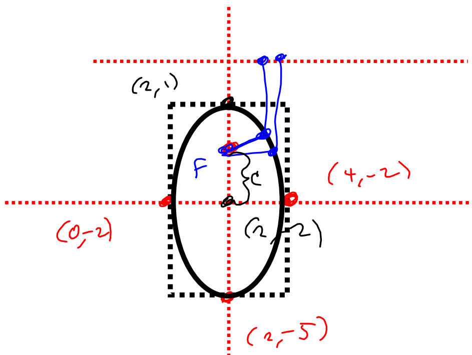
$\frac{9(x-2)^2}{36} + \frac{4(y+2)^2}{36} = 1$

$\frac{(x-2)^2}{4} + \frac{(y+2)^2}{9} = 1$

$4=2^2 \qquad 9=3^2$

$(h,k)=(2,2)$

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$



c = distance from center to foci:

$c^2=a^2-b^2 \qquad (a>b)$

OR

$c^2=b^2-a^2 \qquad (b>a)$

$9x^2-4y^2-36x-16y+20=36$

$9x^2-36x \quad -4y^2-16y \quad = \cancel{+}16$

$9(x^2-4x+2^2)-4(y^2+4y+2^2) = \cancel{+}16+36-16$

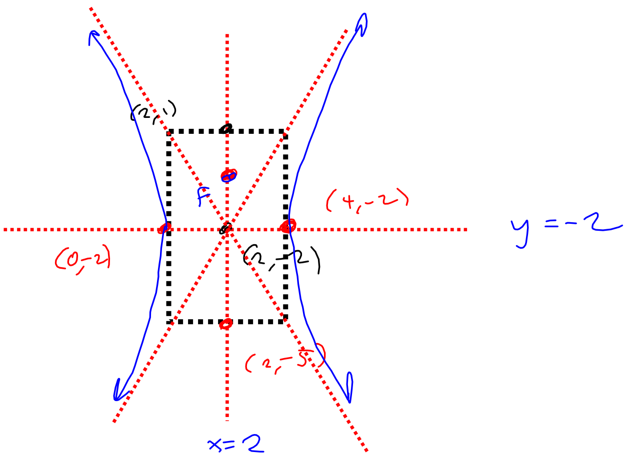
$9(x-2)^2-4(y+2)^2=36$

$\dfrac{9(x-2)^2}{36}-\dfrac{4(y+2)^2}{36}=1$

$\dfrac{(x-2)^2}{4}-\dfrac{(y+2)^2}{9}=1$

$$\boxed{c^2=a^2+b^2}$$

we'll talk about its role in the graph, next time.



$x^2 - 8y + 6x + 9 = 0$

$-8y = -x^2 - 6x - 9$

$8y = x^2 + 6x + 9$

$8y = (x+3)^2$

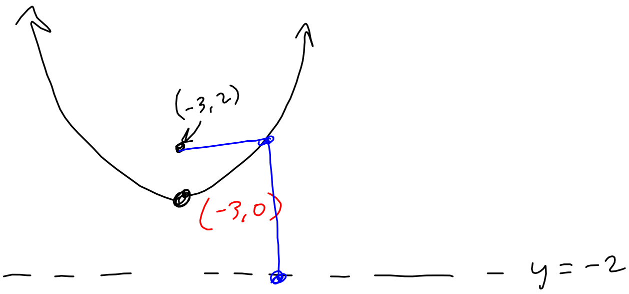
$4p(y-k) = (x-h)^2$

vertex $= (h, k) = (-3, 0)$

$p =$ distance from vertex to focus

$=$ " " " directrix

$4p = 8 \Rightarrow p = 2$



$$r = \frac{ep}{1 \pm e\cos\theta}$$

$0 < e < 1 \Rightarrow$ ellipse

$e = 1 \Rightarrow$ parabola

$e > 1 \Rightarrow$ hyperbola

$$r = \frac{ep}{1 \pm e\sin\theta}$$

$$\frac{ep}{1 \pm e\cos\theta}$$

$$\frac{2}{2 - \cos\theta} = \frac{2}{2\left(1 - \frac{1}{2}\cos\theta\right)}$$

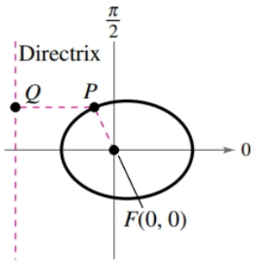
$$= \frac{1}{1 - \frac{1}{2}\cos\theta} = \frac{\left(\frac{1}{2}\right)(2)}{1 - \frac{1}{2}\cos\theta} \rightarrow p = 2$$

$$e = \frac{1}{2}$$

$$1 = \frac{1}{2} \cdot 2$$

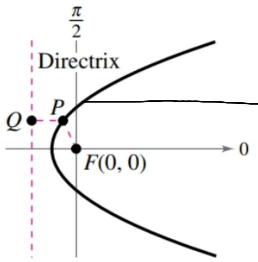
Alternative Definition of a Conic

The locus of a point in the plane that moves such that its distance from a fixed point (focus) is in a constant ratio to its distance from a fixed line (directrix) is a **conic.** The constant ratio is the *eccentricity* of the conic and is denoted by e. Moreover, the conic is an **ellipse** when $0 < e < 1$, a **parabola** when $e = 1$, and a **hyperbola** when $e > 1$. (See the figures below.)



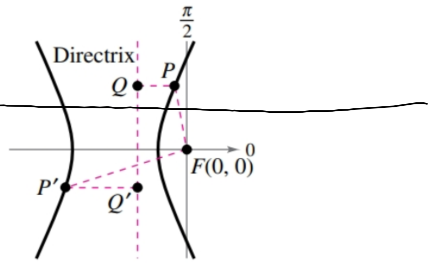
Ellipse: $0 < e < 1$

$\frac{PF}{PQ} < 1$



Parabola: $e = 1$

$\frac{PF}{PQ} = 1$



Hyperbola: $e > 1$

$\frac{PF}{PQ} = \frac{P'F}{P'Q'} > 1$

Apr 28-8:46 AM