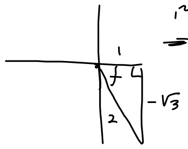
Test 4 questions

$x^5 - (1-\sqrt{3}i) = 0$

$x^5 = 1-\sqrt{3}i$

Principle Root!

$\sqrt[5]{x^5} = x = \sqrt[5]{1-\sqrt{3}i}$



$1^2+\sqrt{3}^2 = 1+3=4=c^2$

$\Rightarrow c = \pm\sqrt{4} = 2$

$\theta = -60^0 = -\frac{\pi}{3}$

OR

$\theta = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$= 300^0$

Generally prefer (convention)

$0 \le \theta < 2\pi$

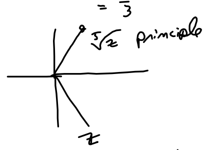
300^0 OR $\frac{5\pi}{3}$

inc. $= \frac{2\pi}{5} = \frac{6\pi}{15}$

Principle Root:

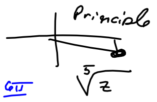
$\frac{\frac{5\pi}{3}}{5} = \frac{5\pi}{3}\cdot\frac{1}{5}$

$= \frac{\pi}{3}$



$r = 2$

$\theta = -\frac{\pi}{3}$ OR $\frac{5\pi}{3}$



$\boxed{\frac{\theta}{5} = -\frac{\pi}{15}}$ 1

Increment is $\frac{2\pi}{5} = \frac{6\pi}{15}$

$-\frac{\pi}{15} + \frac{6\pi}{15} = \frac{5\pi}{15} = \boxed{\frac{\pi}{3}}$ 2

$\frac{5\pi}{15} + \frac{6\pi}{15} = \boxed{\frac{11\pi}{15}}$ 3

$\frac{11\pi}{15} + \frac{6\pi}{15} = \boxed{\frac{17\pi}{15}}$ 4

$\frac{17+6}{15}\pi = \boxed{\frac{23\pi}{15}}$ 5

$\frac{23+6}{15}\pi = \boxed{\frac{29\pi}{15}}$ 6 coterminal w/ $-\frac{\pi}{15}$

$\frac{\pi}{3} + \frac{2\pi}{5} = \frac{5\pi+6\pi}{15} = \frac{11\pi}{15}$

$\frac{11+6}{15}\pi = \frac{17\pi}{15}$

$\frac{17+6}{15}\pi = \frac{23\pi}{15}$

$\frac{23+6}{15}\pi = \frac{29\pi}{6}$

$z = 2\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)$

$\sqrt[5]{z} = \sqrt[5]{2}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right),$

Then $\sqrt[5]{2}\left(\cos\frac{11\pi}{15} + i\sin\frac{11\pi}{15}\right)$

etc.

$\sqrt[5]{\sqrt{2}}$

$\sqrt[5]{\sqrt{2}} = \left(2^{\frac{1}{2}}\right)^{\frac{1}{5}}$

$= 2^{\frac{1}{2}\cdot\frac{1}{5}} = 2^{\frac{1}{10}}$!

$\left(3\left(\cos(12°)+i\sin(12°)\right)\right)^4 =$

$3^4\left(\cos(48°)+i\sin(48°)\right)$

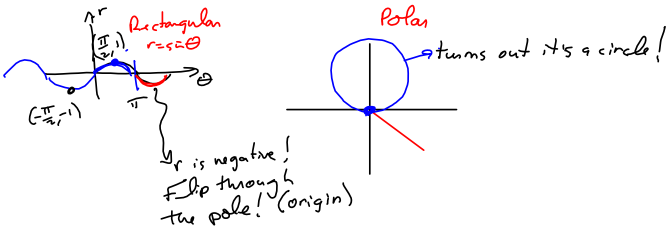
Re-takes for Test 4 coming soon!

Graphing in Polar Coordinates.

KEY: Graphing in Rectangular coordinates & seeing the "loops!"

$r=\sin\theta$ in polar coords.

First graph in rectangular coords



See Symmetry, S6.8 pg 2 (Pg 478 in 10th Ed.)

Symmetry about $\theta=\frac{\pi}{2}$ (Symmetry about y-axis)

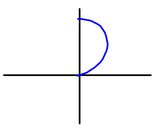
$r=\sin\theta \quad (r,\theta)\longmapsto(-r,-\theta)$

$-r=\sin(-\theta)=-\sin\theta$

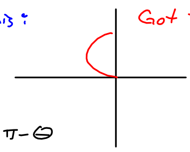
$\Rightarrow r=\sin\theta=$ Same!

(Equivalent Equation!)

That means all I needed to graph was from 0 to $\frac{\pi}{2}$ for this one.



gives us this:



Got this for free!

Alternate: Replace θ by $\pi-\theta$

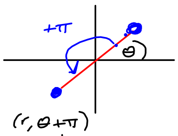
$r=\sin\theta \longmapsto r=\sin(\pi-\theta)=\sin\pi\cos(-\theta)+\sin(-\theta)\cos\pi$

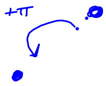
$=0\cdot\overset{\text{EVEN}}{\cos\theta}-(\sin\theta)(-1)=\sin\theta$! Same!

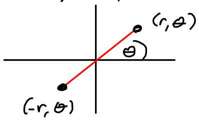
Symmetry through the pole:

$(r,\theta) \longmapsto (r, \theta+\pi)$

$(r,\theta) \longmapsto (-r, \theta)$







You see this kind of symmetry in something like

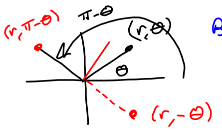
$r^2 = \sin\theta$

$(-r)^2 = \sin\theta$

$(-1)^2 r^2 = r^2 = \sin\theta$ Same!

3rd Kind of Symmetry:

Thru the polar axis (x-axis)



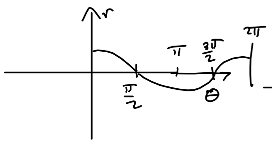
Bottom half of picture from top half.

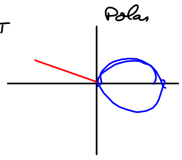
$(r,\theta) \longmapsto (r,-\theta)$

OR

$(r,\theta) \longmapsto (-r, \pi-\theta)$

$r = \cos\theta \leadsto r = \cos(-\theta) = \cos\theta$!





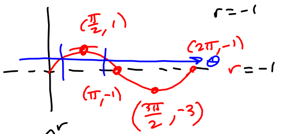
Circle!

Amp = 2

$r = 2\sin\theta - 1$

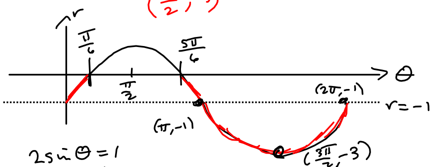
$y = -1$ midline

$r = -1$



$-1+2 = 1$

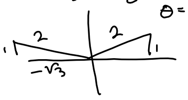
$-1-2 = -3$

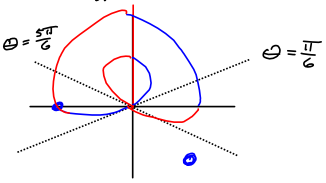


$2\sin\theta = 1$

$\sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$





$r = 2\sin\theta - 1$

$r = 2\sin(\pi - \theta) - 1$

$= 2(\sin\pi\cos(-\theta) + \sin(-\theta)\cos\pi) - 1$

$= 2\sin\theta - 1$! woo-hoo!

Symmetry thru $\theta = \frac{\pi}{2}$

$r = 2\sin(-\theta) - 1$ Polar Axis?

$= -2\sin\theta - 1$ not the same

$-r = 2\sin(\pi - \theta) - 1$

$-r = 2\sin\theta - 1$ not same.

Thru pole?

$r = 2\sin(\pi + \theta) - 1$

$= 2(\sin\pi\cos\theta + \sin\theta\underbrace{\cos\pi}_{-1}) - 1$

$= 2(-\sin\theta) - 1$ No

Try $(-r, \theta)$

$-r = 2\sin\theta - 1$ not same