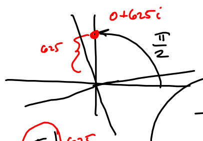
Questions from Chapters 3 or 4?

Beginning Chapter 6 if no one stops me!

4th roots of 625i

$z = 625\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$



$\sqrt[4]{z} = \boxed{\sqrt[4]{625}\left(\cos\frac{\pi}{8} + i\sin\frac{\pi}{8}\right)}$

$\frac{\frac{\pi}{2}}{4} = \frac{\pi}{2} \div 4 = \frac{\pi}{2} \cdot \frac{1}{4} = \frac{\pi}{8}$

5 | 625
5 | 125
5 | 25
5

$= \boxed{5\left(\cos\frac{\pi}{8} + i\sin\frac{\pi}{8}\right)} = \sqrt[4]{z}$

$\frac{\theta + 2\pi k}{n}, \; k = 0, 1, 2, 3$

$\frac{\theta + 2\pi k}{n} = \frac{\theta}{n} + \frac{2\pi k}{n} = \frac{\frac{\pi}{2}}{4} + \frac{2\pi k}{4}$

$k=1: \frac{\pi}{8} + \frac{2\pi}{4} = \frac{5\pi}{8}$

$k=2: \frac{\pi}{8} + \frac{2\pi \cdot 2}{4} = \frac{\pi + 8\pi}{8} = \frac{9\pi}{8}$

etc:

$\frac{\frac{\pi}{2}}{4} = \frac{\pi}{8}$ Increment $= \frac{2\pi}{4} = \frac{\pi}{2} = \frac{4\pi}{8}$

$\frac{\pi}{8} + \frac{4\pi}{8} = \frac{5\pi}{8}$

$\frac{5\pi}{8} + \frac{4\pi}{8} = \frac{9\pi}{8}$

$\frac{9\pi}{8} + \frac{4\pi}{8} = \frac{13\pi}{8}$

$\frac{13\pi}{8} + \frac{4\pi}{8} = \frac{17\pi}{8} = 2\pi + \frac{\pi}{8} \longleftrightarrow \frac{\pi}{8}$

$\boxed{5\left(\cos\frac{5\pi}{8} + i\sin\frac{5\pi}{8}\right), \; 5\left(\cos\frac{9\pi}{8} + i\sin\frac{9\pi}{8}\right), \; 5\left(\cos\frac{13\pi}{8} + i\sin\frac{13\pi}{8}\right)}$

$5\left(\cos\frac{5\pi}{8} + i\sin\frac{5\pi}{8}\right)$

$\sin\frac{5\pi}{8}i$ is ambiguous;

Is it $\sin\left(\frac{5\pi}{8}i\right)$? OR $\left(\sin\left(\frac{5\pi}{8}\right)\right)i$

$a+bi$, rather than $a+ib$?

Review of some standard graphs

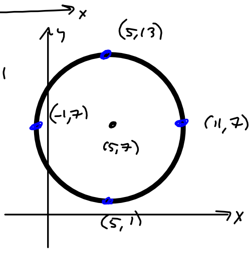
$y = a(x-h)^2 + k$ Parabola

$y = 3(x-5)^2 + 7$ $(h,k) = (5,7)$

$$\frac{(x-h)^2}{r^2} + \frac{(y-k)^2}{r^2} = 1$$

$$\frac{(x-5)^2}{36} + \frac{(y-7)^2}{36} = 1$$

$r = 6$, $(h,k) = (5,7)$

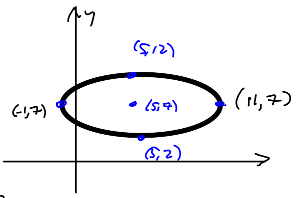


Ellipses:

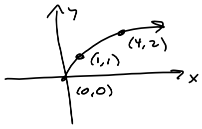
$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(x-5)^2}{36} + \frac{(y-k)^2}{25} = 1$$

$(h,k) = (5,7)$, $a = 6$, $b = 5$



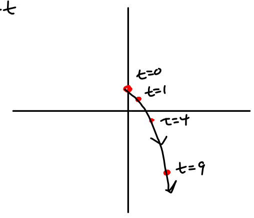
$y = \sqrt{x}$



$x = \sqrt{t}, \quad y = 3 - t$

Plot Some Points

t	x	y
0	0	3
1	1	2
4	2	-1
9	3	-6

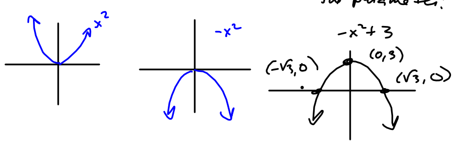


Eliminate Parameter

$x = \sqrt{t} \Rightarrow x^2 = (\sqrt{t})^2 = t$

$\Rightarrow y = 3 - t = 3 - x^2$

Graph after eliminating the parameter:

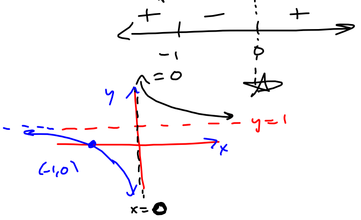


$x = t-1 \implies x+1 = t$ $\qquad (\quad)'$

$y = \dfrac{t}{t-1} \implies \dfrac{x+1}{(x+1)-1} = \dfrac{x+1}{x} \xrightarrow{|x|\to\infty} \dfrac{x}{x} \boxed{= 1 = y \quad H.A.}$

$D = \mathbb{R} \setminus \{0\}$ $\quad$ V.A. = Vertical Asymptote $\boxed{x = 0}$ V.A.

x-int: $\dfrac{x+1}{x} = 0 \implies x+1 = 0 \implies \boxed{x = -1}$ x-int



$x = 6\cos\theta \Rightarrow \frac{x}{6} = \cos\theta$

$y = 5\sin\theta \Rightarrow \frac{y}{5} = \sin\theta$

$\cos^2\theta + \sin^2\theta = 1$

$\left(\frac{x}{6}\right)^2 + \left(\frac{y}{5}\right)^2 = 1$

$\frac{x^2}{36} + \frac{y^2}{25} = 1$ Ellipse!

Capture Direction of increasing t.

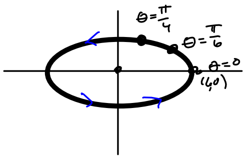
θ	x	y
0	6	0
$\frac{\pi}{6}$	$3\sqrt{3}$	$\frac{5}{2}$
$\frac{\pi}{4}$	$3\sqrt{2}$	$\frac{5\sqrt{2}}{2}$

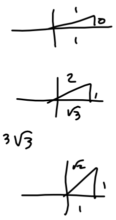
$x = 6\cos\theta$

$y = 5\sin\theta$

$\frac{\sqrt{3}}{2}\cdot 6 = 3\sqrt{3}$

$\frac{\sqrt{2}}{2}\cdot 6 = 3\sqrt{2}$





$x = 1 + \cos\theta \longrightarrow x - 1 = \cos\theta$

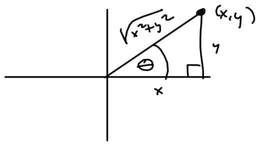
$y = 1 + 2\sin\theta \longrightarrow y - 1 = 2\sin\theta \longrightarrow$

$\frac{y-1}{2} = \sin\theta$

$(x-1)^2 + \frac{(y-1)^2}{2^2} = 1$ from Pythagorean Identity.

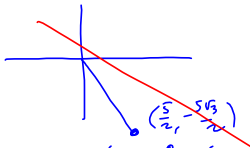
§6.7 $(x,y) \longmapsto (r,\theta)$ = Polar Coordinates.

Spirograph



$r = \sqrt{x^2+y^2}$

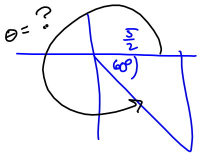
$\theta = \arctan\left(\frac{y}{x}\right)$ IF YOU ARE IN QI. Otherwise, you need to some interpretation, using $\tan\theta = \frac{y}{x}$



Don't Look

$5(\cos 300^\circ, \sin 300^\circ)$

$= 5\left(\frac{1}{2}, \left(-\frac{\sqrt{3}}{2}\right)\right) = \left(\frac{5}{2}, -\frac{5\sqrt{3}}{2}\right)$



$r^2 = \left(\frac{5}{2}\right)^2 + \left(-\frac{5\sqrt{3}}{2}\right)^2$

$= \frac{25}{4} + \frac{25\cdot 3}{4} = \frac{100}{4} = 25$

$\Rightarrow \boxed{r = 5}$

Want $0 \le \theta \le 2\pi$, just because

$\tan\theta = \frac{-\frac{5\sqrt{3}}{2}}{\frac{5}{2}} = -\frac{5\sqrt{3}}{2} \cdot \frac{2}{5} = -\sqrt{3}$

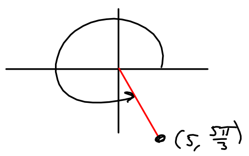
$\rightarrow \arctan(-\sqrt{3}) = -60^\circ = -\frac{\pi}{3} \ne \theta$ if $0 \le \theta \le 2\pi$

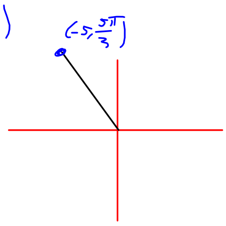
$360^\circ - 60^\circ = 300^\circ = (300^\circ)\left(\frac{\pi}{180^\circ}\right) =$

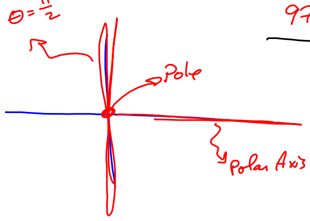
$(r,\theta) = \left(5, \frac{5\pi}{3}\right)$ (or $\left(5, -\frac{\pi}{3}\right)$ if no restriction)

For 1st time, we let $r < 0$.

$(r,\theta) = \left(-5, \frac{5\pi}{3}\right)$







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