I worked Test 3 Re-Take in video.

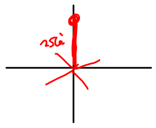
Thing is way too long!

If you took it and got a good score, then fine.

But it'll be available (in shortened form) for at least one more week.

Apr 9-8:23 AM

4^{th} roots of $256i$:



90°

$\sqrt[4]{256}$

2 | 256
2 | 128
2 | 64
2 | 32
2 | 16
2 | 8
2 | 4
2

$= 2 \cdot 2$

$= 4$

$256i =$

$256\left(\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)\right)$

$\dfrac{\frac{\pi}{2}}{4} = \frac{\pi}{8} = 22.5^\circ$

$\boxed{\sqrt[4]{256i} = 4\left(\cos\frac{\pi}{8} + i\sin\frac{\pi}{8}\right)}$

Increment is $\frac{2\pi}{4} = \frac{\pi}{2}$

$\frac{\pi}{8} + \frac{\pi}{2} = \frac{\pi + 4\pi}{8} = \frac{5\pi}{8}$

$\frac{5\pi}{8} + \frac{4\pi}{8} = \frac{9\pi}{8}$

$\frac{9\pi}{8} + \frac{4\pi}{8} = \frac{13\pi}{8}$

$\frac{13\pi}{8} + \frac{4\pi}{8} = \frac{17\pi}{8} = \frac{16\pi}{8} + \frac{\pi}{8} \xleftrightarrow{\text{coterminal}} \frac{\pi}{8}$

$\sqrt[2]{2}$

$4\left(\cos\frac{\pi}{8} + i\sin\frac{\pi}{8}\right)$, $4\left(\cos\frac{5\pi}{8} + i\sin\frac{5\pi}{8}\right)$, $4\left(\cos\frac{9\pi}{8} + i\sin\frac{9\pi}{8}\right)$,

$4\left(\cos\frac{13\pi}{8} + i\sin\frac{13\pi}{8}\right)$

$z = 8\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$ Find cube roots.

$\sqrt[3]{z} = 2\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

& $2\left(\cos\frac{11\pi}{12} + i\sin\frac{11\pi}{12}\right)$

$2\left(\cos\frac{19\pi}{12} + i\sin\frac{19\pi}{12}\right)$ are the other 2.

Increment $= \frac{2\pi}{3}$

$\frac{\pi}{4} + \frac{2\pi}{3} = \frac{3\pi + 8\pi}{12} = \frac{11\pi}{12}$

$\frac{19\pi}{12}$

$\frac{27\pi}{12} = \frac{24\pi}{12} + \frac{3\pi}{12}$

$\xleftrightarrow{\text{coterm}} \frac{3\pi}{12} = \frac{\pi}{4}$

1()

$z = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} \Rightarrow z^6 = \cos(3\pi) + i\sin(3\pi)$

Find z^6

$\left(r_1\left(\cos(a) + i\sin(a)\right)\right)\left(r_2\left(\cos(b) + i\sin(b)\right)\right)$

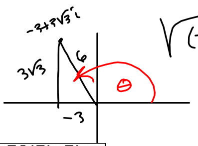
$= r_1 r_2\left(\cos(a+b) + i\sin(a+b)\right)$

$r_1 = r_2$ $a = b$, then $z^2 = r_1^2\left(\cos(a+a) + i\sin(a+a)\right)$

$= r_1^2\left(\cos(2a) + i\sin(2a)\right)$

$z^6 = r_1^6\left(\cos(6a) + i\sin(6a)\right)$

Trig fam of $-3+3\sqrt{3}\,i$ $\qquad (3\sqrt{3})^2 = 3^2\sqrt{3}^2$



$$\sqrt{(-3)^2+(3\sqrt{3})^2} = \sqrt{9+9\cdot 3} = \sqrt{36} = 6$$

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tan⁻¹(-3√(3)/3)
    -1.047197551
tan⁻¹(-3√(3)/3)
             -60
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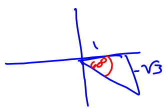
Find the argument θ

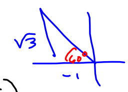
$$\tan\theta = \frac{3\sqrt{3}}{-3} = -\sqrt{3}$$

$\arctan(-\sqrt{3}) = -60^\circ$



So $\theta = 180^\circ - 60^\circ = 120^\circ$

$= 180^\circ + \arctan(-\sqrt{3})$



TRIG FORM:

$6(\cos 120^\circ + i\sin 120^\circ)$ $\qquad (120^\circ)\left(\frac{\pi}{180^\circ}\right)$

$= 6\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$ $\qquad = \frac{2\pi}{3}$

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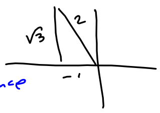
using cosine:

$\cos^{-1}\left(-\frac{1}{2}\right) = 120^\circ$

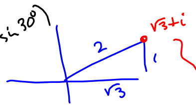
$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60^\circ$ reference angle in Q II



$\left(3(\sqrt{3}+i)\right)^{10}$ $\quad 3^{10}$

$3(\sqrt{3}+i) = 3\,[$

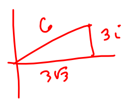
$\sqrt{3}+i$
$= 2(\cos 30^\circ + i\sin 30^\circ)$



$z = 3(\sqrt{3}+i)$

$(\sqrt{3}, i)$ as point in plane.

$3(\sqrt{3}+i)$



Trig form:

$6\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

$6\left(\frac{\sqrt{3}}{2} + i\frac{1}{2}\right)$

Trust the "book" & they put parens exactly where they meant to

$$3(\sqrt{3}+i)^{10} = 3\left[2(\cos 30^\circ + i\sin 30^\circ)\right]^{10}$$
$$= 3\left[2^{10}(\cos 300^\circ + i\sin 300^\circ)\right]$$

$$3 = \left(\sqrt[10]{3}\right)^{10}$$
$$3(\sqrt{3}+i)^{10} = \left(\sqrt[10]{3}\sqrt{3} + i\sqrt[10]{3}\right)$$

$$2\sqrt{5} = \sqrt{2^2}\sqrt{5} = \sqrt{4\cdot 5} = \sqrt{20}$$
$$2(5)^{\frac{1}{2}} = (2^2)^{\frac{1}{2}}(5)^{\frac{1}{2}} = (2^2\cdot 5)^{\frac{1}{2}}$$

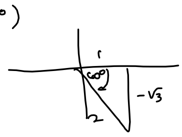
$$3(\sqrt{3}+i)^{10}$$

$$= 3\left[2(\cos 30^\circ + i\sin 30^\circ)\right]^{10}$$

$$= 3\left[2^{10}(\cos 300^\circ + i\sin 300^\circ)\right]$$

$$= 3072(\cos 300^\circ + i\sin 300^\circ)$$

$$= 3072\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$$



$$= 1536 + 1536\sqrt{3}\,i$$

$$\approx 1536 + 2660.43004\,i$$

```
          60466176
2^10
              1024
Ans*3
              3072
1536*√(3)
        2660.43004
```

roots $-2\pm\sqrt{3}\,i$, -4

Build polynomial w/ real coefficients with those roots

$$(x-(-2+\sqrt{3}i))(x-(-2-\sqrt{3}i))(x-(-4))$$

$$(x+2-\sqrt{3}i)(x+2+\sqrt{3}i)(x+4)$$

$$=(x+4)(x^2+2x+\sqrt{3}xi+2x+4+2\sqrt{3}i-\sqrt{3}ix-2\sqrt{3}i-\sqrt{3}\sqrt{3}i^2)$$

$$=(x+4)(x^2+4x+7)$$

$$\begin{array}{r} =x^3+4x^2+7x \\ 4x^2+16x+28 \\ \hline x^3+8x^2+23x+28 \end{array}$$

$$4-\sqrt{3}\sqrt{3}i^2$$
$$=4-3i^2$$
$$=4+3$$
$$=7$$

$3x^3 - x^2 + 96x - 32$

Rational Roots Theorem (zeros)

$\frac{32}{3}$ $32 = 2^5$

$\pm(1, 2, 4, 8, 16, 32)$

$\pm\left(\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{8}{3}, \frac{16}{3}, \frac{32}{3}\right)$

$x = \frac{1}{3}$ is a root?

Divide $f(x)$ by $x - \frac{1}{3}$

$$\begin{array}{r|rrrr} \frac{1}{3} & 3 & -1 & 96 & -32 \\ & & 1 & 0 & 32 \\ \hline & 3 & 0 & 96 & 0 \\ & x^2 & x^1 & c & r \end{array}$$

$(x - \frac{1}{3})(3x^2 + 96)$

$3x^2 + 96 = 0$

$3(x^2 + 32) = 3(x^2 - (-32))$

$= 3(x^2 - (\sqrt{32}\,i)^2)$

$= 3(x - \sqrt{32}\,i)(x + \sqrt{32}\,i) = 0$

$\Rightarrow x = \pm\sqrt{32}\,i = \pm 2 \cdot 2\sqrt{2}\,i = \pm 4\sqrt{2}\,i$

$$\begin{array}{r|l} 2 & 32 \\ 2 & 16 \\ 2 & 8 \\ 2 & 4 \\ & 2 \end{array}$$

$x = \frac{1}{3}, \pm 4\sqrt{2}\,i$

Factored Form

$f(x) = 3(x - \frac{1}{3})(x - 4\sqrt{2}\,i)(x + 4\sqrt{2}\,i)$

$f(x)$ split into linear factors