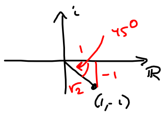
I will add a Section 4.5 Assignment: DeMoivre's Theorem.

$1-i$ in trig form



$\mathbb{C}^3$: Vector Addition, Scalar (Real) multiplication

$\mathbb{C}^4$: Complex # Addition, Scalar Multiplication (Scalar might not be real)

Multiplying Complex #s!

$z = 1 - i = r(\cos\theta + i\sin\theta)$

$= \sqrt{2}\left(\cos(-45^\circ) + i\sin(-45^\circ)\right)$

$= \sqrt{2}\left(\cos(315^\circ) + i\sin(315^\circ)\right)$

$= \sqrt{2}\left(\frac{1}{\sqrt{2}} + \left(-\frac{1}{\sqrt{2}}\right)\right)$

$= 1 - i$!

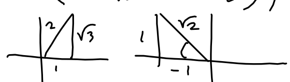
DeMoivre's Multiply r, Add Angles.

$\left(r_1(\cos\theta_1 + i\sin\theta_1)\right)\left(r_2(\cos\theta_2 + i\sin\theta_2)\right)$

$= r_1 r_2\left(\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)\right)$

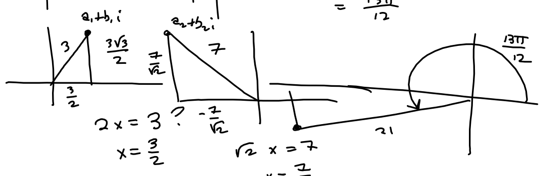
$\left(3\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)\right)\left(7\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)\right)$

$= 21\left(\cos\left(\frac{13\pi}{12}\right) + i\sin\frac{13\pi}{12}\right)$



$\frac{\pi}{3} + \frac{3\pi}{4} = \frac{4}{4}\cdot\frac{\pi}{3} + \frac{3}{3}\cdot\frac{3\pi}{4}$

$= \frac{13\pi}{12}$



$2x = 3$? $-\frac{7}{\sqrt{2}}$

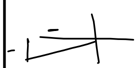
$x = \frac{3}{2}$

$\sqrt{2}\,x = 7$

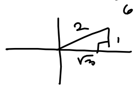
$x = \frac{7}{\sqrt{2}}$

$\sin\frac{13\pi}{12} = -\sqrt{\frac{1-\cos u}{2}} = -\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = -\sqrt{\frac{\frac{2-\sqrt{3}}{2}}{2}} = -\sqrt{\frac{2-\sqrt{3}}{4}}$

$\frac{u}{2} = \frac{13\pi}{12}$



$\Rightarrow u = \frac{13\pi}{6} = \frac{12\pi}{6} + \frac{\pi}{6} \longrightarrow \frac{\pi}{6}$



$\boxed{= -\frac{\sqrt{2-\sqrt{3}}}{2} = \sin\frac{13\pi}{12}}$

$\boxed{\cos\frac{13\pi}{12} = -\frac{\sqrt{2+\sqrt{3}}}{2}}$

Product in Rectangular form:

$21\left(\frac{-\sqrt{2+\sqrt{3}}}{2} + i\left(-\frac{\sqrt{2-\sqrt{3}}}{2}\right)\right)$

$= \frac{-21\sqrt{2+\sqrt{3}}}{2} - \frac{\sqrt{2-\sqrt{3}}}{2}\,i$

DeMoivre's applied to POWERS:

$z = r(\cos\theta + i\sin\theta)$

$z^n = r^n\left(\cos(n\theta) + i\sin(n\theta)\right)$

$x^3+3x^2 = 4$ $\qquad (x+2)^2(x-1)$

x^3+0x^2+3x-4 No, dummy

Try $x=-2$:

$-2 \,|\; 1 \quad 0 \quad 3 \quad -4$
$\quad -2 \quad 4$
$\quad 1 \quad -2 \quad 7$

* sign *

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x^3+3x^2+0x-4

$-2 \,|\; 1 \quad 3 \quad 0 \quad -4$
$\quad -2 \quad -2 \quad 4$
$\quad 1 \quad 1 \quad -2 \quad 0 \leftarrow$ sweet!

Linear Factor (x^1)

This work says
x^3+3x^2-4
$=(x+2)\boxed{(x^2+x-2)}$ Depressed Polynomial

we split off a factor of $x+2$, corresponding to the root $x=-2$

The Fundamental Theorem of Algebra is saying every polynomial of degree n has n roots & splits into linear factors (Count repetitions as "separate" roots)

$-2 \,|\; 1 \quad 3 \quad 0 \quad -4$
$\quad -2 \quad -2 \quad 4$
$\quad 1 \quad 1 \quad -2 \quad 0$
$\quad x^2 \quad x^1 \quad c \quad r$

$(x+2)(x^2+x-2)$
$=(x+2)(x+2)(x-1)$

$2x^2-x-4=0$ Quadratic Formula

$a=2, b=-1, c=-4$

$\Rightarrow b^2-4ac = (-1)^2-4(2)(-4) = 1+32=33$ $\qquad 3\,|\,33 \quad 11$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a} = \dfrac{1 \pm \sqrt{33}}{2(2)} = \dfrac{1 \pm \sqrt{33}}{4}$ David

$\not 5 \; 4, 2 \# 6$

Analyze w/ Discriminant, but don't solve.

$2x^2-x-4=0$

$a=2, b=-1, c=-4 \Rightarrow$

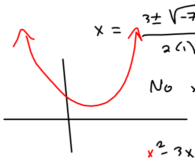
$b^2-4ac = (-1)^2-4(2)(-4) = 1+32=33>0 \Rightarrow$ $\boxed{\text{2 real solns}}$

$x^2-3x+20$

$a=1, b=-3, c=20$

$\Rightarrow b^2-4ac = (-3)^2-4(1)(20)$
$= 9-80 = -71 < 0 \Rightarrow$ $\boxed{\text{2 nonreal solns}}$ Done with it

$x = \dfrac{3 \pm \sqrt{-71}}{2(1)}$



No x-ints in graph

$x^2-3x+20$
$= x^2-3x+\left(\frac{3}{2}\right)^2 - \frac{9}{4} + \frac{80}{4}$
$= \left(x-\frac{3}{2}\right)^2 + \frac{71}{4} \overset{\text{SET}}{=} 0$
$\left(x-\frac{3}{2}\right)^2 = -\frac{71}{4}$
$x-\frac{3}{2} = \pm\sqrt{-\frac{71}{4}} = \pm \frac{i\sqrt{71}}{2}$

$x = \dfrac{3 \pm i\sqrt{71}}{2}$

$x = -1+\sqrt{3}i, -4$

Conjugate Pairs Theorem.

Want Real Coefficients. Build Poly w/ those roots.

Scratch: $(x-(-1+\sqrt{3}i))(x-(-1-\sqrt{3}i))(x+4)$

$(x+1-\sqrt{3}i)(x+1+\sqrt{3}i) =$

$x^2+x+\sqrt{3}i\,x + 1x + 1 + \sqrt{3}i - \sqrt{3}i\,x - \sqrt{3}i - \sqrt{3}i\sqrt{3}i$

$= x^2+2x+1-3(-1)$

$= x^2+2x+4$

Finish.

$= (x+4)(x^2+2x+4)$

$= x^3+2x^2+4x$
$\quad +4x^2+8x+16$

$x^3+6x^2+12x+16$

Apr 2-9:25 AM