

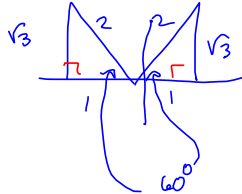
$f(\theta) = 0$ to $f(2\theta) = 0$

Find all solutions in $[0, 2\pi)$

in degrees & radians.

$\sin \theta = \frac{\sqrt{3}}{2}$

$\sin^2 \theta - \frac{\sqrt{3}}{2} = 0$

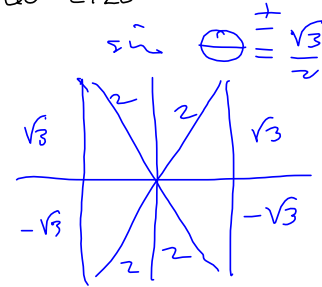


$\frac{\pi}{3}, \frac{2\pi}{3}$

$60^\circ, 180^\circ - 60^\circ = 120^\circ$

$\sin^2 \theta = \frac{3}{4} \Rightarrow$

$\sin \theta = \pm \sqrt{\frac{3}{4}} = \pm \frac{\sqrt{3}}{\sqrt{4}} = \pm \frac{\sqrt{3}}{2}$



$\sin^2(2\theta) = \frac{3}{4}$

$\theta \in [0, 2\pi) \Rightarrow 0 \leq \theta < 2\pi$

$2\theta \in [0, 4\pi) \Rightarrow 0 \leq 2\theta < 4\pi$

$60^\circ, 120^\circ, 240^\circ, 300^\circ$

$\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

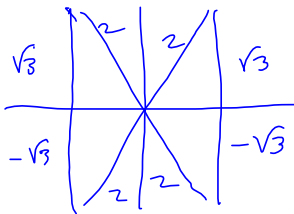
So to see everywhere $\sin(2\theta) = \frac{\sqrt{3}}{2}$ in $[0, 2\pi)$

Find where $\sin \theta = \frac{\sqrt{3}}{2}$ on $[0, 4\pi)$

$\sin(2\theta) = \pm \frac{\sqrt{3}}{2}$

$\sin^2(2\theta) = \frac{3}{4}$

$\sin(2\theta) = \pm \frac{\sqrt{3}}{2}$



$2\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}, \frac{11\pi}{3}$

$2\pi + \frac{\pi}{3}$

$2\pi + \frac{2\pi}{3}$

$2\pi + \frac{4\pi}{3}$

$2\pi + \frac{5\pi}{3}$

$\theta = \frac{\pi}{6}, \frac{2\pi}{6} = \frac{\pi}{3}, \frac{4\pi}{6} = \frac{2\pi}{3}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{11\pi}{6}$

$\sin 2\theta$ is ambiguous.

$(\sin(2\theta)) \theta$ OR $\sin(2\theta)$?

16. -/2 points LarTrig9 2.5.064.

Find all solutions of the equation in the interval $[0, 2\pi)$. (Enter your answers as a comma-separated list.)

$$\sin^2 3x - \sin^2 x = 0$$

Use a graphing utility to graph the equation and verify the solutions.

In the notes, I reduced powers 1^{st}

$$\sin^2(3x) - \sin^2(x) = \frac{1 - \cos(6x)}{2} - \frac{1 - \cos(2x)}{2}$$

$$= \frac{1}{2} \cancel{\frac{1}{2}} \frac{-\cos(6x) + \cos(2x)}{2} = \frac{1}{2} [\cos(2x) - \cos(6x)] \stackrel{SET}{=} 0$$

$$\Rightarrow \cos(2x) - \cos(6x) = 0 \quad \begin{matrix} \cos(u+v) = \cos u \cos v \\ -\sin u \sin v \end{matrix}$$

$$\Rightarrow \cos(2x) - [\cos(4x)\cos(2x) - \sin(4x)\sin(2x)]$$

$$= \cos(2x) - \cos(4x)\cos(2x) + \sin(4x)\sin(2x)$$

$$= \cos(2x) - [\cos(2x)\cos(2x) - \sin(2x)\sin(2x)] + [2\sin(2x)\cos(2x)] \sin(2x)$$

$$= \cos(2x) - \cos^2(2x) + \sin^2(2x) + 2\sin^2(2x)\cos(2x) = 0$$

$$\cos(2x) - \cos^2(2x) + (1 - \cos^2(2x)) + 2[1 - \cos^2(2x)]\cos(2x)$$

$$= \cos(2x) - \cos^2(2x) + 1 - \cos^2(2x) + 2\cos(2x) - 2\cos^3(2x) = 0$$

$$\Rightarrow 3\cos(2x) - 2\cos^2(2x) + 1 - 2\cos^3(2x)$$

$$= -2\cos^3(2x) - 2\cos^2(2x) + 3\cos(2x) + 1 = 0$$

$$\Rightarrow 2\cos^3(2x) + 2\cos^2(2x) - 3\cos(2x) - 1 = 0$$

$$2u^3 + 2u^2 - 3u - 1 = 0$$

$$\frac{p}{q} = \pm 1, \pm \frac{1}{2}$$

$$\begin{array}{r} \underline{1) 2 \quad 2 \quad -3 \quad -1} \\ \underline{ \quad 2 \quad 4 \quad 1} \\ 2 \quad 4 \quad 1 \quad 0 \end{array}$$

so $u=1$ is a root!

$$2u^2 + 4u + 1 = 0$$

$$a=2, b=4, c=1$$

$$b^2 - 4ac = 4^2 - 4(2)(1) = 16 - 8 = 8$$

$$u = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

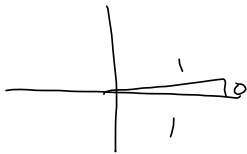
$$= \frac{-4 \pm \sqrt{8}}{2(2)} = \frac{-4 \pm 2\sqrt{2}}{2(2)} = \frac{-2 \pm \sqrt{2}}{2}$$

$$= -1 \pm \frac{\sqrt{2}}{2}$$

$\nearrow -1 + \frac{\sqrt{2}}{2} \approx -0.2928932190$
 $\searrow -1 - \frac{\sqrt{2}}{2} < -1$

$$\sqrt{8} = 2\sqrt{2}$$

$\cos(2x) = 1$ OR $\cos(2x) \approx -.2928932190$

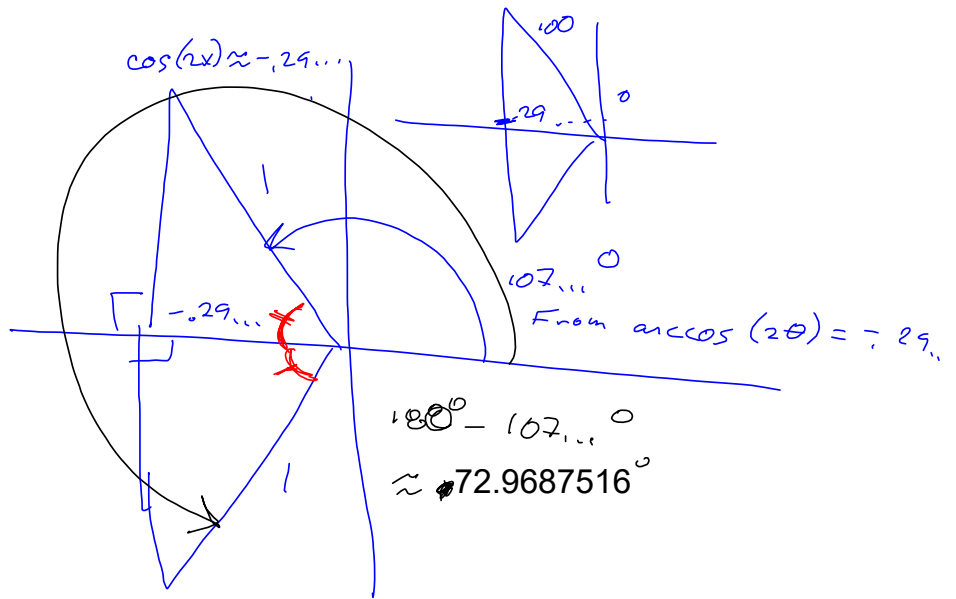


$2x = 0, 2x = 2\pi$

$x = 0$

OR $2x = 2\pi$

$x = \pi$



$180^\circ - 107...^\circ$
 $\approx 72.9687516^\circ$

$2x \approx 107.0312484$ OR

$2x \approx 180^\circ + 72...^\circ \approx 252.9687516^\circ$

$\Rightarrow x \approx \frac{107...^\circ}{2}, \frac{252...^\circ}{2}, 0, 180^\circ$

$107...^\circ + 360^\circ = 467...^\circ \approx 2x$

$252...^\circ + 360^\circ = 612...^\circ \approx 2x$

| | |
|------------------------------|-------------|
| $\cos^{-1}(-1 + \sqrt{2}/2)$ | 73 |
| | 107.0312485 |
| 180-Ans | |
| | 72.96875154 |
| Ans+180 | |
| | 252.9687515 |

| | |
|---------|-----------------|
| | 72.96875154 |
| Ans+180 | |
| | 252.9687515 |
| Ans+360 | |
| | 612.9687515 |
| | 72.96875154+360 |
| | 432.9687515 |

| | |
|-------|---------------|
| | 432.9687515 |
| Ans/2 | |
| | 216.4843758 |
| | 612.9687515/2 |
| | 306.4843758 |
| | 107.0312485/2 |
| | 53.51562425 |

$x \approx 53.51562425, \dots$ etc. Running out of time.