$$\left(x-\frac{\sqrt{3}}{2}\right)\left(x+\frac{\sqrt{3}}{2}\right)\left(x-\frac{1}{2}\right)$$

$$\left(x^2-\frac{3}{4}\right)\left(x-\frac{1}{2}\right)$$

$$= x^3-\frac{1}{2}x^2-\frac{3}{4}x+\frac{3}{8}$$

$$= 8x^3-4x^2-6x+3$$

Find all $\theta \in [0, 2\pi)$ satisfying

$$8\sin^3\theta - 4\sin^2\theta - 6\sin\theta + 3 = 0$$

$$\underbrace{8u^3-4u^2}\underbrace{-6u+3} = 0$$

Hint: Factor by grouping.

$$4u^2(2u-1)+3(-2u+1) = 0$$

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37

$$4u^2(2u-1)-3(2u-1) = 0$$

$4u^2$ ☺ -3 ☺ $=$ ☺ $(4u^2-3) = (2u-1)(4u^2-3) = 0$

$2u-1=0$ OR $4u^2-3=0$

$2u=1$

$u=\frac{1}{2}$

M1

$4u^2=3$

$u^2=\frac{3}{4}$

$u=\pm\sqrt{\frac{3}{4}}$

$u=\pm\frac{\sqrt{3}}{2}$

M2

$4u^2+0u-3=0$

$a=4,\ b=0,\ c=-3$

$b^2-4ac = 0^2-4(4)(-3)$

$= +48$

$$\begin{array}{r|l} 2 & 48 \\ 2 & 24 \\ 2 & 12 \\ 2 & 6 \\ & 3 \end{array}$$

$\sqrt{48} = 4\sqrt{3}$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\frac{0\pm4\sqrt{3}}{2(4)} = \pm\frac{4\sqrt{3}}{8}$

$=\pm\frac{\sqrt{3}}{2}$

M3

$4u^2-3 = (2u)^2-(\sqrt{3})^2$

$= (2u-\sqrt{3})(2u+\sqrt{3})$

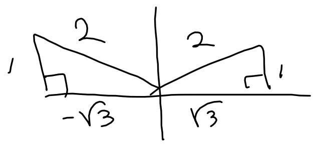
$\Rightarrow 2u-\sqrt{3}=0$ $\quad 2u=-\sqrt{3}$

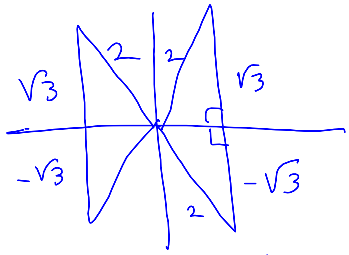
$2u=\sqrt{3}$ $\quad u=-\frac{\sqrt{3}}{2}$

$u=\frac{\sqrt{3}}{2}$

This gives

$\sin\theta = \frac{1}{2}$ $\sin\theta = \pm\frac{\sqrt{3}}{2}$





$4-1=3 \rightsquigarrow \sqrt{3}$

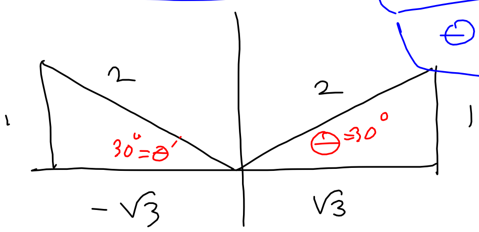
$\boxed{\theta = \frac{\pi}{6}, \frac{5\pi}{6}}$

60°, 120°, 240°, 300°

OR

$\boxed{\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}}$

Quick 'n' dirty.



$\theta \in \left\{ \frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}, \frac{4\pi}{3}, \frac{5\pi}{3} \right\}$

Nice & pretty

Find ALL Solutions

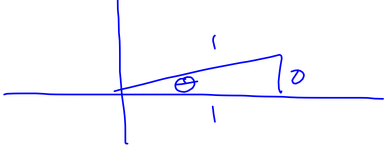
$$\theta \in \left\{ \frac{\pi}{6}+b, \frac{\pi}{3}+b, \frac{2\pi}{3}+b, \frac{5\pi}{6}+b, \frac{4\pi}{3}+b, \frac{5\pi}{3}+b \;\middle|\; b = 2\pi n, n \in \mathbb{Z} \right\}$$

$$= \left\{ x + 2\pi n \;\middle|\; x = \frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}, \frac{4\pi}{3}, \frac{5\pi}{3}, n \in \mathbb{Z} \right\}$$

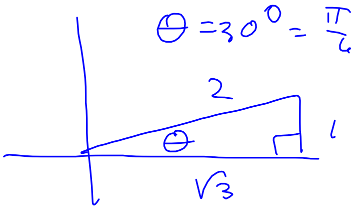
is more elegant.

$\theta = 30^\circ, 150^\circ$

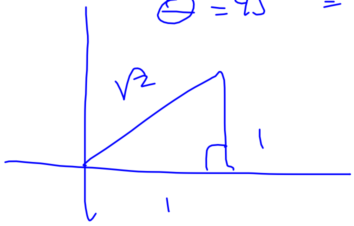
$\theta = 0^\circ = 0$



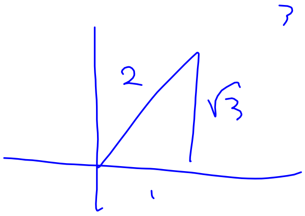
$\theta = 30^\circ = \frac{\pi}{6}$



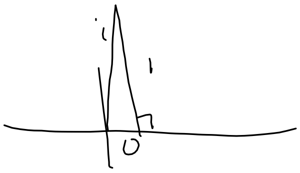
$\theta = 45^\circ = \frac{\pi}{4}$



$\theta = 60^\circ = \frac{\pi}{3}$



$\theta = 90^\circ = \frac{\pi}{2}$



Find all $\theta \in [0, 2\pi)$ satisfying

$8\sin^3 2\theta - 4\sin^2 2\theta - 6\sin 2\theta + 3 = 0$ Trig. Polynomials.

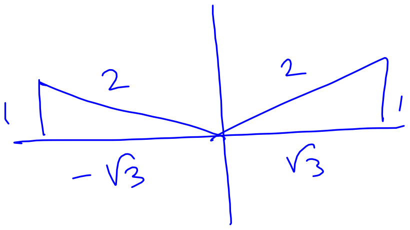
$\theta \in [0, 2\pi)$ means

$0 \le \theta < 2\pi$

$\Rightarrow \quad 0 \le 2\theta < 4\pi \qquad 0 \le 2\theta \le 720^0$

$\sin 2\theta = \frac{1}{2}$

$(510^0)\left(\frac{\pi}{180^0}\right) = \frac{51\pi}{18} = \frac{17\pi}{6}$



$2\theta = 30^0, 150^0, 390^0, 510^0 = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$

$\theta = 15^0, 75^0, 195^0, 255^0$

$\frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$

Angle sum Formulas

$$\sin(x+y) = \sin x \cos y + \sin y \cos x$$

$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$

HALF-ANGLE

$$\sin\left(\frac{x}{2}\right) = \pm\sqrt{\frac{\cos(x)-1}{2}}$$

Which is it? + or − ?

$$\cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{\cos(x)+1}{2}}$$

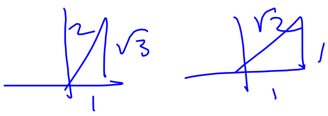
Compute $\cos\left(\frac{7\pi}{12}\right)$ in two ways:

(a) Angle sum

(b) Half-angle.

(a) $\frac{7\pi}{12} = \frac{\pi}{12} + \frac{6\pi}{12} = \frac{2\pi}{12} + \frac{5\pi}{12} = \frac{3\pi}{12} + \frac{4\pi}{12} = \frac{\pi}{4} + \frac{\pi}{3}$

$$\cos\left(\frac{7\pi}{12}\right) = \cos\left(\frac{\pi}{3}+\frac{\pi}{4}\right) = \cos\frac{\pi}{3}\cos\frac{\pi}{4} - \sin\frac{\pi}{3}\sin\frac{\pi}{4}$$

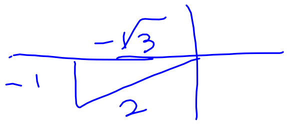


$$= \frac{1}{2}\cdot\frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}} = \frac{1-\sqrt{3}}{2\sqrt{2}}$$

$$= \frac{\sqrt{2}}{\sqrt{2}}\left(\frac{1-\sqrt{3}}{2\sqrt{2}}\right) = \boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$$

(b) $\cos\left(\frac{7\pi}{12}\right)$

$$= \cos\left(\frac{\frac{7\pi}{6}}{2}\right) = \pm\sqrt{\frac{\cos\frac{7\pi}{6}+1}{2}} = \pm\sqrt{\frac{-\frac{\sqrt{3}}{2}+1}{2}}$$



$$= \pm\sqrt{\frac{\frac{-\sqrt{3}+2}{2}}{2}} = \pm\sqrt{\frac{-\sqrt{3}+2}{4}}$$

$$= \pm\frac{\sqrt{-\sqrt{3}+2}}{2}$$

$$= -\frac{\sqrt{-\sqrt{3}+2}}{2} \overset{?!}{=} \frac{\sqrt{2}-\sqrt{6}}{4} \quad ?!$$

Miscellaneous Identities

$\sin(2\theta) = 2\sin\theta\cos\theta$

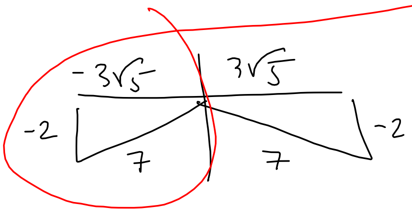
$\cos(2\theta) = 1 - 2\sin^2\theta = \cos^2\theta - \sin^2\theta$

$= 1 - \sin^2\theta - \sin^2\theta = 1 - 2\sin^2\theta$ ✓

Suppose $\sin\theta = -\frac{2}{7}$ & $\cos\theta < 0$.

What quadrant is $\frac{\theta}{2}$ in?

Classical Way with Logic & Reason



$49 - 4 = 45$

$\sqrt{45} = 3\sqrt{5}$

3 | 45
3 | 15
5

This says

$\pi < \theta < \frac{3\pi}{2}$ OR $180^\circ < \theta < 270^\circ$

Then $\frac{\pi}{2} < \frac{\theta}{2} < \frac{3\pi}{4}$ $\qquad 90^\circ < \frac{\theta}{2} < 135^\circ$

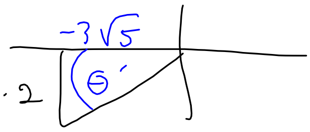
$\boxed{\text{Q II}}$ $\qquad$ Q II

What quadrant is $2u$ in?

$\pi < \theta < \frac{3\pi}{2}$

$2\pi < 2\theta < 3\pi$

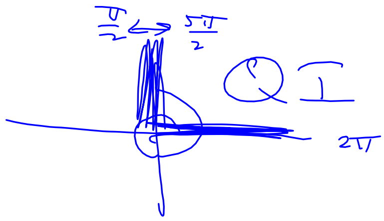
Narrows it down,
but not enough,



So $\theta' < 45^\circ = \frac{\pi}{4}$

So $\pi < \theta < \frac{5\pi}{4}$

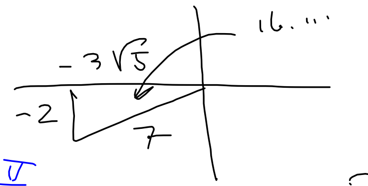
So $2\pi < 2\theta < \frac{5\pi}{2}$



Q I

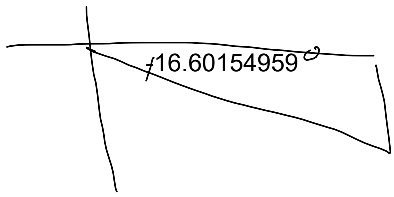
TECHIE WAY

$\sin\theta = -\frac{2}{7}$, $\cos\theta < 0$:



what's θ?

$\arcsin\left(-\frac{2}{7}\right)$



$180^\circ + 16\ldots^\circ \approx 196.6015496^\circ$

$\Rightarrow 2\theta \approx 393.2030992^\circ \in$ Q I