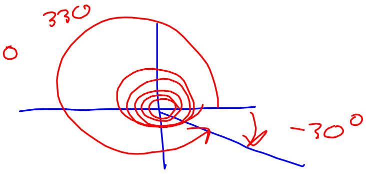
1. (10 pts) Find two angles, between -2π and 2π (i.e., -360^0 and 360^0) that are coterminal with $\frac{71\pi}{6}$. Give exact answers in degrees and radians.

$$\left(\frac{71\pi}{6}\right)\left(\frac{180^0}{\pi}\right) = 2130^0$$

$2130^0 \approx$ 5.916666667 $\Rightarrow$ 5 times around $+ .91\overline{6}$ times around.

$$\left(\left(.91\overline{6}\right)\text{ rotations}\right)\left(\frac{360^0}{1\text{ rotation}}\right) \approx 330.0000001$$

So coterminal with 330^0, -30^0

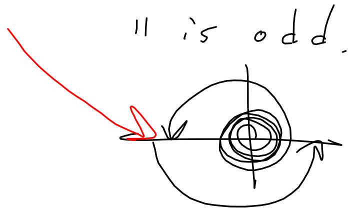


Convert to radians

$$(330^0)\left(\frac{\pi}{180^0}\right) = \frac{11\pi}{6}, \ -\frac{\pi}{6}$$

$$\frac{71\pi}{6} = \frac{66\pi}{6} + \frac{5\pi}{6} = 11\pi + \frac{5\pi}{6}$$

11 is odd. So



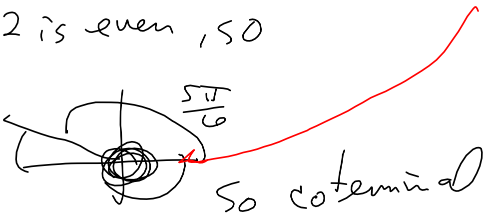
$\frac{5\pi}{6}$

$$\pi + \frac{5\pi}{6} = \frac{6\pi + 5\pi}{6} = \frac{11\pi}{6}$$

$$\frac{77\pi}{6} = \frac{72\pi + 5\pi}{6}$$

$$= 12\pi + \frac{5\pi}{6}$$

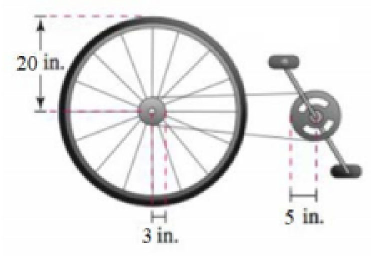
12 is even, so



So coterminal with $\frac{5\pi}{6}$ OR $-\frac{7\pi}{6}$

150^0 OR -210^0

5. (10 pts) The radii of the pedal sprocket, the wheel sprocket, and the wheel of the bicycle in the figure are 5 inches, 3 inches and 20 inches, respectively. A cyclist is pedaling at a rate of 4 revolutions per second. Find the speed of the bicycle in feet per second. Then convert that to miles per hour. Round final answers to 1 decimal place.



6. (10 pts) Sketch the graph of $f(x) = 5\sin\left(\frac{\pi}{\ }x - \frac{7\pi}{\ }\right) + 11$

$$\underbrace{\left(\frac{4 \text{ revs front}}{1 \text{ sec}}\right)\left(\frac{5 \text{ revs rear}}{3 \text{ revs front}}\right)\left(\frac{2\pi \text{ radians}}{1 \text{ rev rear}}\right)}_{\text{Angular Velocity } \frac{\theta}{\text{sec}}}\left(20 \text{ inches}\right)\left(\frac{1 \text{ ft}}{12 \text{ in}}\right)$$

$$= \frac{\cancel{4}^{1}\cdot 5\cdot 2\pi\cdot 20}{3\cdot\cancel{12}_{3}} \qquad \boxed{= \frac{200\pi}{9}\ \frac{\text{ft}}{\text{sec}}}$$

$$\frac{S}{\text{sec}} = r\,\frac{\theta}{\text{sec}}$$

$$\left(\frac{\cancel{200}^{100}\,\pi}{\cancel{9}_{3}}\right)\left(\frac{\text{ft}}{\text{s}}\right)\left(\frac{\cancel{60}^{5}\ \text{mi/hr}}{\cancel{88}_{11}\ \text{ft/sec}}\right)$$

$$\boxed{\frac{500\pi}{33}\ \frac{\text{mi}}{\text{hr}}}$$

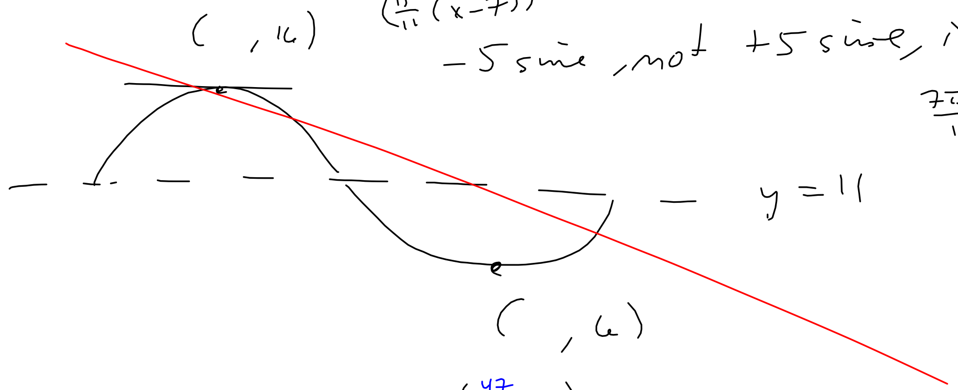
6. (10 pts) Sketch the graph of $f(x) = -5\sin\left(\frac{\pi}{11}x - \frac{7\pi}{11}\right) + 11$.

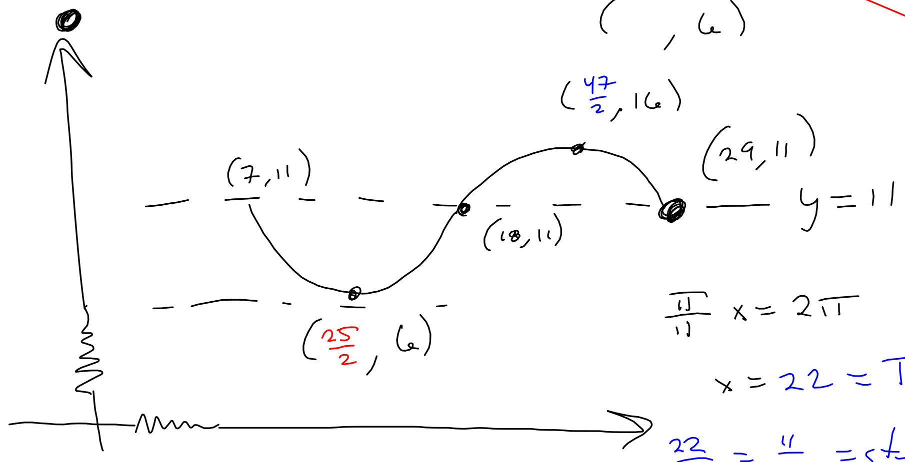
$$\frac{\pi}{11}\left(x - \frac{\frac{7\pi}{11}}{\frac{\pi}{11}}\right)$$

$$\left(\frac{\pi}{11}(x-7)\right)$$

-5 sine, not $+5$ sine, idiot.

$$\frac{7\pi}{11}\cdot\frac{11}{\pi} = 7$$





$$\frac{\pi}{11}x = 2\pi$$

$$x = 22 = T$$

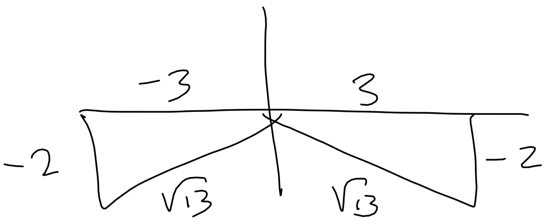
$$\frac{22}{4} = \frac{11}{2} = \text{stepsize}$$

$$7 + \frac{11}{2} = \frac{14+11}{2} = \frac{25}{2}$$

$$8 + \frac{11}{2} = \frac{36+11}{2} = \frac{47}{2}$$

3. Basic concept: Draw the doggone pictures!

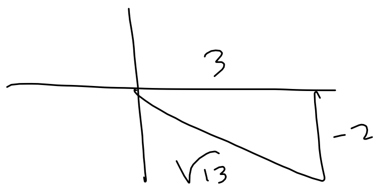
   a. (5 points) Sketch two triangles that satisfy $\sin(\theta) = \frac{-2}{\sqrt{13}}$.



$\sqrt{13}^{\,2} - (-2)^2 = 13 - 4 = 9$

$\rightsquigarrow \sqrt{9} = 3$

b. (5 pts) Assume the terminal side of the angle θ lies in the 4th quadrant (Quadrant IV). Find the other five trigonometric functions of θ.



$\sin\theta = -\frac{2}{\sqrt{13}}$ $\quad \csc\theta = -\frac{\sqrt{13}}{2}$

$\cos\theta = \frac{3}{\sqrt{13}}$ $\quad \sec\theta = \frac{\sqrt{13}}{3}$

$\tan\theta = -\frac{2}{3}$ $\quad \cot\theta = -\frac{3}{2}$

c. (5 pts) Again, assuming θ's terminal side lies in Q IV, and $0 \le \theta < 2\pi$, find θ, in radians *and* degrees, rounded to 3 decimal places.

DO IN DEGREES MODE

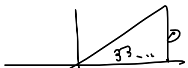
$\arctan\left(-\frac{2}{3}\right) = \tan^{-1}\left(-\frac{2}{3}\right) \approx -33.69006752^\circ \longrightarrow 326.3099325^\circ$

$360^\circ +$

$\arcsin\left(-\frac{2}{\sqrt{13}}\right) = \sin^{-1}\left(-\frac{2}{\sqrt{13}}\right) \approx -33.\ldots^\circ \longrightarrow \boxed{326.310^\circ}$

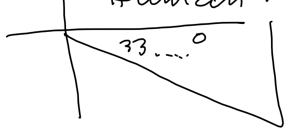
so $360^\circ - 33\ldots^\circ$

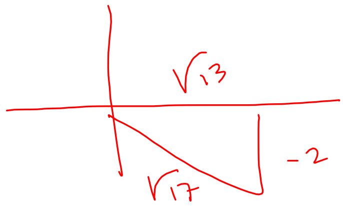
$\arccos\left(\frac{3}{\sqrt{13}}\right) = \cos^{-1}\left(\frac{3}{\sqrt{13}}\right) \approx 33.69006752$



Calculator

Human Knows:



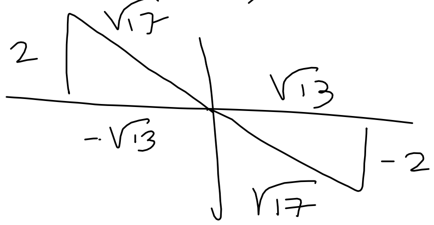


$\sin\theta$ $\quad \csc\theta$

$\cos\theta$ $\quad \sec\theta$

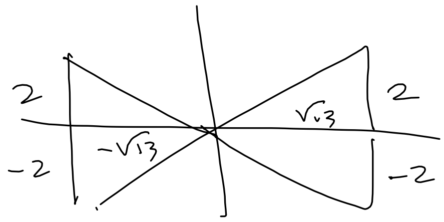
$\tan\theta = -\frac{2}{\sqrt{13}}$ $\quad \cot\theta$

$\tan\theta = \frac{-2}{\sqrt{13}}$



$(-2)^2 + (\sqrt{13})^2 = 4 + 13 = 17$

$\rightsquigarrow \sqrt{17}$



$\tan\theta = \pm\frac{2}{\sqrt{13}}$