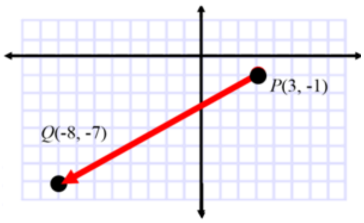
Chapter 4 ?

Consider the directed line segment $\overrightarrow{PQ}$ in the figure on the right. I want you to provide some basic facts about the vector $\vec{u}$:

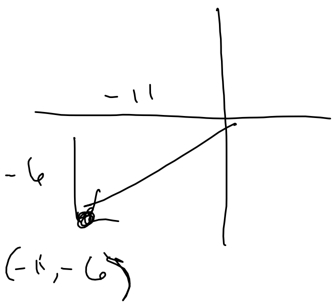


a. (10 pts) Express the vector $\vec{u} = \overrightarrow{PQ}$ in component form.

b. (10 pts) Compute the magnitude of $\vec{u}$. Leave your answer in simplified radical form.

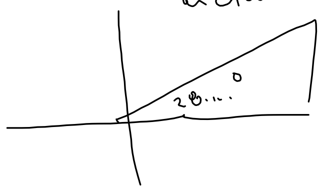
c. (5 pts) Find the direction angle of $\vec{u}$ (the positive angle measured from the positive x-axis). Use degrees, rounded to 4 places.

(a) $\vec{u} = \langle -8-3, -7-(-1) \rangle = \langle -11, -6 \rangle$

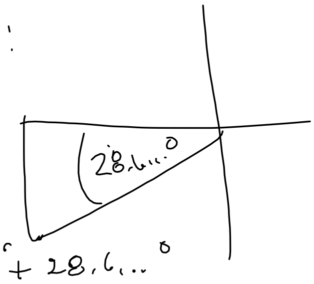


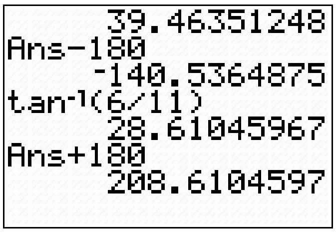
(c) $\arctan\left(\frac{6}{11}\right) \approx 28.61045967^\circ$

arctan



Our pic:





$180^\circ + 28.6\ldots^\circ$

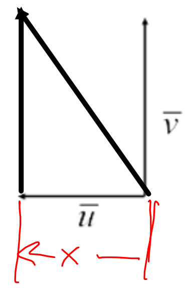
$= 208.61045967^\circ$

$\approx \boxed{208.6105^\circ}$

Calculator gives you a reference angle and you need to put it together through pictures.

(b) $\|\vec{u}\| = \sqrt{11^2 + 6^2} = \sqrt{121+36} = \sqrt{157}$

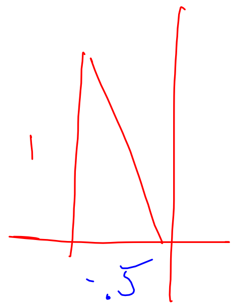
4. The current in a river is flowing at 5 miles per hour, due West. $\left(\|\bar{u}\| = 5 \text{ mph}\right)$. A man in a boat points his boat due North to attempt a crossing. His boat's speed is 10 miles per hour $\left(\|\bar{v}\| = 10 \text{ mph}\right)$.

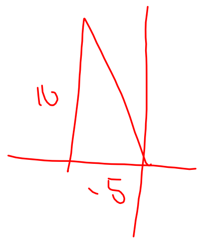


a. (5 pts) Express $\bar{u}$ and $\bar{v}$ in component form.

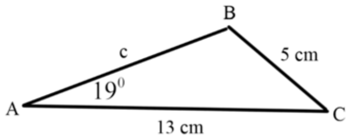
b. (5 pts) How far downstream will the current take the boat, if the river is 1 mile wide?

$\frac{1}{2}$-mile, by similar triangles!





1. Consider the triangle in the figure. Do not use rounded results in your calculations for new results. Only round in the final answer to each of the following:



   a. (10 pts) This triangle is oriented a bit differently than others you've seen for this SSA situation. But you can still show there are 2 solutions to this triangle. Do so.

   b. (10 pts) Draw the picture for the case where angle B is acute and angle C is obtuse. You probably have already drawn it, in your answer to the previous question. In the figure, I've given the picture for an obtuse angle B and acute angle C. Don't worry about the length c, yet. That's part e.

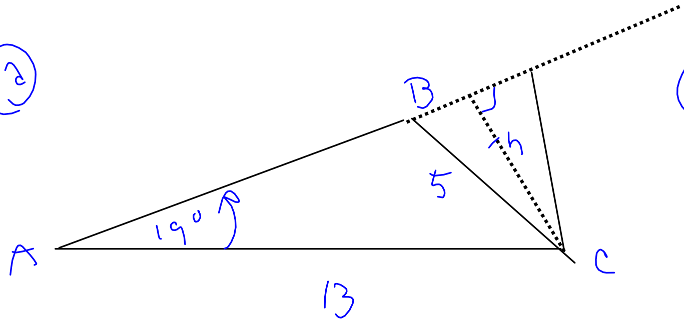
   c. (10 pts) Choose the picture where the angle B is obtuse (the one I've drawn), and use the Law of Sines (and pictures and logic) to find (obtuse) angle B, to 4 decimal places.

   d. (10 pts) Use angle A and your (un-rounded) result for angle B to find angle C in the obvious way (subtraction!), to 4 decimal places.

   e. (5 pts) Use your (un-rounded) result for angle C and the Law of Cosines to find the length of side c. You can check your answer using the Law of Sines, but I insist on seeing the Law of Cosines, here. Give the length c rounded to 4 decimal places.

(a)

(b) is the pic



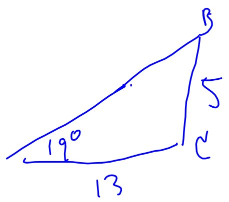
$\frac{h}{13} = \sin 19° \Rightarrow h = 13 \sin 19° \approx 4.$

So $h < 5$ says 5 is big enough.

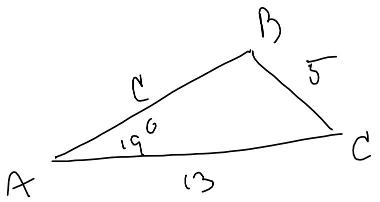
So, obviously there's an "obtuse C" solution, by picture

$5 < 13$ says the obtuse B pic makes sense

(b)



(c)

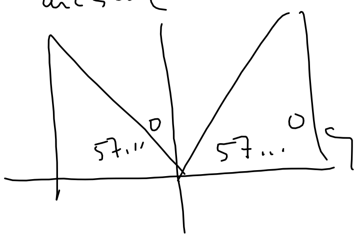


$$\frac{\sin B}{13} = \frac{\sin 19^\circ}{5}$$

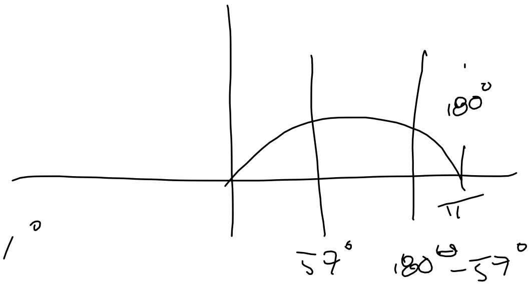
$$\sin B = \frac{13 \sin 19^\circ}{5}$$

$\approx$

$\arcsin(\sin B)$



$\sin x$



$B = 180^\circ - 57\ldots^\circ$

```
13sin(19)/5
         .8464772016
sin⁻¹(Ans)
         57.83055292
Ans-180
        -122.1694471
█
```

$\approx 122.16944471^\circ$

$\approx 122.1694^\circ$

122.1694°

(d) $A + B + C = 180^\circ \Longrightarrow$

$C = 180^\circ - 122.\ldots^\circ - 19^\circ \approx 38.83055292^\circ$

$\approx \boxed{38.8306^\circ \approx C}$

```
         .8464772016
sin⁻¹(Ans)
         57.83055292
Ans-180
        -122.1694471
180+Ans-19
         38.83055292
```

$$\begin{array}{r} 169 \\ 25 \\ \hline 194 \end{array}$$

```
        -122.1694471
180+Ans-19
         38.83055292
196-130cos(Ans)
         94.72951663
Ans^.5
          9.73290895
█
```

(e) $c^2 = a^2 + b^2 - 2ab\cos C$

$= 5^2 + 13^2 - 2(5)(13)\cos(38\ldots^\circ)$

$= 196 - 130\cos(38\ldots^\circ)$

≈ 94.72951663

$\Longrightarrow c \approx 9.73290895$

$\approx \boxed{9.7329 \text{ cm} \approx C}$

5. Let $f(x) = 6x^4 - 35x^3 + 70x^2 + 25x - 26$.

a. (5 pts) Use synthetic division to show that $x = 3 + 2i$ is a solution of the equation $f(x) = 0$.

b. (5 pts) Find the linear factorization of f that is promised to us in the Fundamental Theorem of Algebra.

(a) Divide $f(x)$ by $x-(3+2i)$:

$3+2i$	6	-35	70	25	-26
		$18+12i$ (1)	$-75+2i$ (2)	$-19-4i$ (3)	26
(b) $3-2i$	6	$-17+12i$	$-5+2i$	$6-4i$	0
	x^3	x^2	x	c	r
		$18-12i$	$3-2i$	$-6+4i$	
	6	1	-2	0	
	x^2	x	c	r	

$\Rightarrow 3+2i$ is a zero,

so,

This says:

$f(x) = (x-(3+2i))(x-(3-2i))(6x^2+x-2)$

Linear means everything's to the 1st power.

$6x^2+x-2 = 6x^2+4x-3x-2 = 2x(3x+2)-1(3x+2)$

$= (3x+2)(2x-1)$

Split f into linear factors!

$$\Rightarrow \boxed{f(x) = (x-(3+2i))(x-(3-2i))(3x+2)(2x-1)}$$

(1) $(3+2i)(-17+12i) = -51+36i-34i-24 = -75+2i$

(2) $(3+2i)(-5+2i) = -15+6i-10i-4 = -19-4i$

(3) $6-4i = 2(3-2i) \Rightarrow$
$(3+2i)(6-4i) = 2(3+2i)(3-2i) = 2(3^2+2^2) = 2(13) = 26$

$$\begin{array}{r|rrrrr} 3+2i & 6 & -35 & 70 & 25 & -26 \\ & & 18+12i & -75+2i & -19-4i & 26 \\ \hline & 6 & -17+12i & -5+2i & 6-4i & 0 \end{array}$$

$\Rightarrow 3+2i$ is a zero.

This says

$$f(x) = (x-(3+2i))\left(6x^3 + (-17+12i)x^2 + (-5+2i)x + (6-4i)\right) + 0$$

$$\Rightarrow f(3+2i) = (3+2i-(3+2i))(\sim\sim\sim) = 0 !$$

$$\frac{28}{9} = \frac{27}{9} + \frac{1}{9} = 3 + \frac{1}{9}$$

$$\begin{array}{r} 3 \text{ r} 1 \\ 9\overline{)28} \\ -27 \\ \hline 1 \end{array}$$

$28 = 9 \cdot 3 + 1$

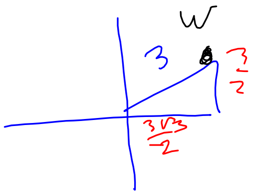
$28 = 9 \cdot 3 + 1$

$\frac{28}{9} = 3 + \frac{1}{9}$

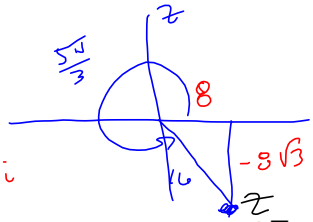
Multiplication of complex numbers:

length: the product of the two lengths.

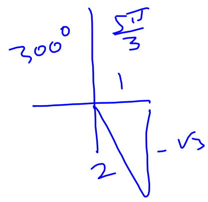
direction angle (also called the "argument"): the sum of the two arguments.

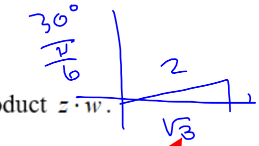


6. Let $z=-4-4\sqrt{3}\,i$
   a. (5 pts) Find $z+\bar{z}$ and $z\bar{z}$, where $\bar{z}$ is the complex conjugate of z.
   b. (5 pts) Express z in trigonometric form.



7. Let $z=16\left(\cos\left(\frac{5\pi}{3}\right)+i\sin\left(\frac{5\pi}{3}\right)\right)$.
   a. (5 pts) Express z in standard form. $= z = 8-8\sqrt{3}i$
   b. (5 pts) Find the trigonometric form of the principal 4th root of z, i.e., find $\sqrt[4]{z}$.
   c. (5 pts) Now, find *all* the 4th roots of z, in trigonometric form.
   d. (5 pts) Find the trigonometric form of z^2.
   e. (5 pts) Finally, let $w=3\left(\cos\left(\frac{\pi}{6}\right)+i\sin\left(\frac{\pi}{6}\right)\right)$, and find the trigonometric form of the product $z\cdot w$.





$zw = 16\cdot 3\left(\cos\left(\frac{5\pi}{3}+\frac{\pi}{6}\right)+i\sin\left(\frac{5\pi}{3}+\frac{\pi}{3}\right)\right)$

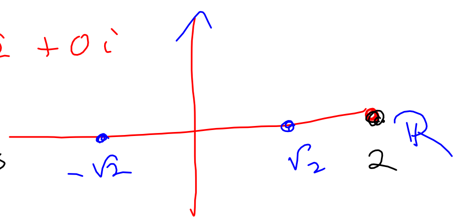
$= 48\left(\cos\frac{11\pi}{6}+i\sin\frac{11\pi}{6}\right) = zw$

$\sqrt{2} = \sqrt{2}+0i$

$2 = 2+0i$

$\Theta = 0^\circ = 0$ radians

= argument



$-\sqrt{2}$: $\frac{2\pi}{2} = \pi$ = increment: Add π to the 1st (principal) square root.

$z = 2(\cos 0 + i\sin 0)$

$\sqrt{z} = \sqrt{2}\left(\cos\frac{0}{2}+i\sin\frac{0}{2}\right)$

the other 2nd root is found by adding $\frac{2\pi}{2}$ to the 1st:

$\sqrt{2}\left(\cos(0+\pi)+i\sin(0+\pi)\right)$

$\sqrt{2}\left(\cos\pi + i\sin\pi\right)$

$= \sqrt{2}(-1+0) = -\sqrt{2}$

$z = 16\left(\cos\left(\frac{5\pi}{3}\right) + i\sin\left(\frac{5\pi}{3}\right)\right)$

$\sqrt[4]{z} = \sqrt[4]{16}\left(\cos\left(\frac{\frac{5\pi}{3}}{4}\right) + i\sin\left(\frac{5\pi}{12}\right)\right)$

$= 2\left(\cos\left(\frac{5\pi}{12}\right) + i\sin\left(\frac{5\pi}{12}\right)\right) = \sqrt[4]{z}$

Increment: $\frac{2\pi}{4} = \frac{\pi}{2}$

$\frac{5\pi}{12} + \frac{\pi}{2}\cdot\frac{6}{6} = \frac{11\pi}{12}$

$\frac{11\pi}{12} + \frac{6\pi}{12} = \frac{17\pi}{12}$

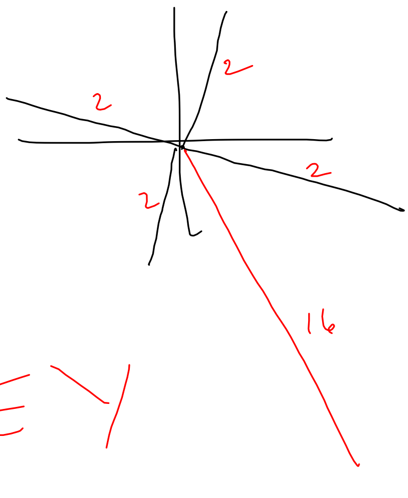
$\frac{17\pi}{12} + \frac{6\pi}{12} = \frac{23\pi}{12}$

$2\left(\cos\frac{11\pi}{12} + i\sin\frac{11\pi}{12}\right)$

$2\left(\cos\frac{17\pi}{12} + i\sin\frac{17\pi}{12}\right)$

$2\left(\cos\frac{23\pi}{12} + i\sin\frac{23\pi}{12}\right.$

FRACTALS
Julia Sets



PAISLEY